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1918

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ALFRED N. GOLDSMITH, Ph.D.

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THE COLLEGE OF THE CITY OF NEW YORK

NEW YORK, N. Y.

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The Officers and Committees of the Institute for 1918 will appear in a forthcoming issue.

The Institute of Radio Engineers announces with the deepest regret the death of

Commandant Camille Tissot.

M. Tissot was born in 1868, and was in succession a student at the Naval School, Officer in the Navy, Doctor of Science, Professor at the Naval School, Professor at the Institute of Electricity, Laureate of the Academy of Science at Paris, and Member of the Scientific Committee on Radio Telegraphy.

Most of the problems relating to radio communication were the object of careful studies on his part. At the same time as Mr. W. Duddell (who survived him but a few weeks), he introduced into radio telegraphic measurements the element of precision. He measured weak currents of radio frequency, using the bolometer, and was thus enabled to make quantitative measurements in radio reception. As a result of these experiments, he was able to throw much light on the laws of resonance, the types of oscillations in antennas, the damping influence of the ground, and the laws governing propagation of waves over short distances. He also devised a classic method of measuring decrements. His book, "Etude sur la resonance des systemes d'antennes dans la Telegraphie sans fil" ("Study of Resonance in Radio Antennas"), (Paris, Gauthier-Villars, 1906), set forth the results of his investigations in 1904-1905, and was a landmark in radiotelegraphic development.

His investigations were then extended in the direction of the mode of operation of the crystal and electrolytic detectors. A paper by him on "The Influence of Alternating Currents on Certain Melted Metallic Salts," was presented before The Institute of Radio Engineers in 1913, and published in Volume 2, number 1 of the "PROCEEDINGS." His researches in radio telephony, in methods of using alternating currents for the charging of condensers, in the transmission of time signals, and in other directions, merit close attention.

Two other important works were published by him: a treatise on "Electric Oscillations," which constitutes a valuable contribution on the theoretical side of this subject, and a more technical "Manual of Radio Telegraphy."

After placing himself at the entire disposal of the French Navy, he contracted on board a Mediterranean mine sweeper the illness which led to his death.

Paris, November 17, 1917. L. B.

THE DYNATRON

A Vacuum Tube Possessing Negative Electric Resistance*

By

Albert W. Hull, Ph.D.

(Research Laboratory, General Electric Company, Schenectady, New York)

1. Definition

The dynatron belongs to the kenotron family of high vacuum, hot cathode devices which the Research Laboratory has developed. Two members of this family, the kenotron rectifier and the pliotron, have already been described in this journal.[1] The fundamental characteristic of kenotrons is that their operation does not depend in any way upon the presence of gas.

In construction, the dynatron resembles the kenotron rectifier and the pliotron. In principle and operation, however, the three are fundamentally different. Each utilizes a single important principle of vacuum conduction. The kenotron rectifier utilizes the uni-directional property of the current between a hot and cold electrode in vacuum. The pliotron utilizes the space charge property of this current, which allows the current to be controlled by the electrostatic effect of a grid. The dynatron utilizes the secondary emission of electrons by a plate upon which the primary electrons fall. It is, as its name indicates, a generator of electric power, and feeds energy into any circuit to which it is connected. It is like a series generator, in that its voltage is proportional to the current thru it, but it is entirely free from the hysteresis and lag that are inherent in generators and in all devices which depend upon gaseous ionization.

2. Construction

The dynatron consists essentially of an evacuated tube containing a filament, a perforated anode and a third electrode, called the plate. The essential construction is shown in Figure 1. The plate must be situated near the anode, in such a

* Received by the Editor, January 30, 1917.

[1] "Proc. I. R. E.," September, 1915.

position that some of the electrons, set in motion by the anode voltage, will fall upon it. A battery is provided for maintaining the filament at incandescence and for maintaining the anode at a constant positive voltage of 100 volts or more, with respect to the filament. This voltage is not varied during operation, and the anode plays no part in the operation of

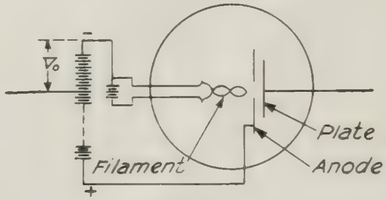

FIGURE 1

the tube except to set in motion a stream of primary electrons, and to carry away the secondary electrons from the plate; that is, to supply the power.

Figure 2 shows the construction of one of the practical types of dynatron that have been developed. The plate has been bent into the form of a cylinder (Figure 2, a) in order to utilize more fully the electron emission from the filament, and the anode has been provided with a large number of holes, instead of one. This is accomplished by using a perforated cylinder (Figure 2, b), or spiral of stout wire (Figure 2, c), or a network of fine tungsten wires (Figure 2, d). The filament is a spiral of tungsten wire (Figure 2, e). The filament may be further provided with a heavy insulated wire along its axis (Figure 2, f), or surrounded by an insulated spiral grid (Figure 2, g), making a "four member" tube, which is called a *pliodynatron.* The characteristics of the pliodynatron are discussed in Section 8.

3. Characteristics—Negative Resistance

Electrons from the filament F (Figure 1) are set in motion by the electric field between F and the anode A. Some of them go thru the holes in the anode and fall upon the plate P. If P is at a low potential with respect to the filament, these electrons will enter the plate and form a current of negative electricity in the external circuit. If the potential of P is raised, the velocity with which the electrons strike it will increase,

and when this velocity becomes great enough they will, by their impact, cause the emission of secondary electrons from the plate. These secondary electrons will be attracted to the more positive anode A. The net current of electrons,

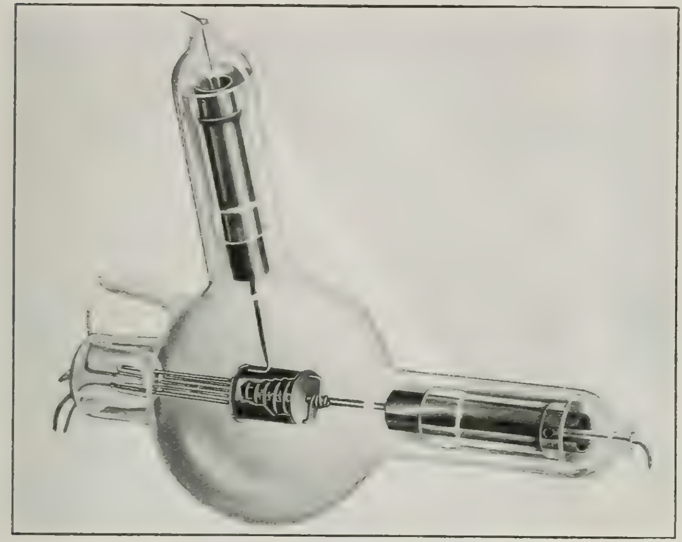

Figure 2—Dynatron

received by the plate, is the difference between the number of primary electrons that strike and enter it and the number of secondary electrons which leave it. The number of primary electrons depends on the temperature of the filament and is practically independent of the voltage of the plate. The number of secondary electrons, however, increases rapidly with the voltage difference between plate and filament, and may become very much larger than the number of primary electrons; that is, each primary electron may produce several secondary electrons, as many as twenty in some cases.

The result is the characteristic voltage current relation shown in Figure 3. The abscissas represent voltages of the plate with respect to the negative end of the filament. The ordinates represent current in the plate circuit, reckoned positive for electrons passing from filament to plate, i. e., in the direction that is equivalent to positive electricity flowing from high potential to low across the vacuum. It is seen that, for low volt-

ages, the current is very small, since only those electrons which come from the most negative end of the filament are able to reach the plate. As the voltage is increased, the current increases rapidly, and at about 25 volts, the plate is receiving the full primary current from the whole filament. For all higher voltages, this primary current remains essentially constant.

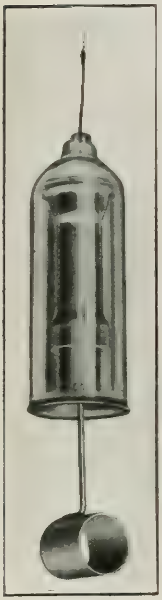

FIGURE 2, a

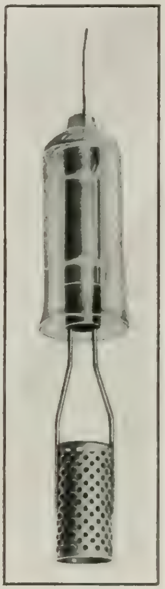

FIGURE 2, b

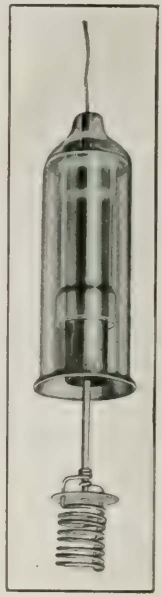

FIGURE 2, c

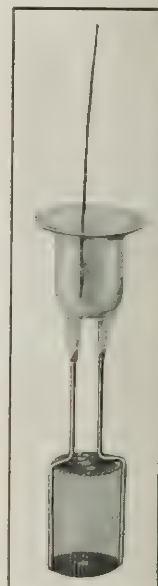

FIGURE 2, d

When the voltage is raised above 25 volts, however, the second factor becomes important. The primary electrons strike the plate with sufficient energy to cause the emission of secondary electrons, and this emission increases rapidly with the voltage, hence the *net* current to plate decreases rapidly. At 100 volts the number of secondary electrons leaving the plate is equal to the number of primary electrons entering it, so that the net current received by the plate is zero. As the voltage further increases, the number of secondary electrons becomes greater than the number of primary electrons, and the plate suffers a net loss of electrons; that is, the current is in the opposite direction to the impressed voltage. When the voltage is still further increased, a point is reached at which the anode

is no longer sufficiently positive to carry away all the secondary electrons from the plate, and the current to the plate again becomes zero, and then rapidly rises to a value corresponding to the number of primary electrons.

It is evident from Figure 3 that over the range A to C, that is, between 50 and 150 volts in the case here represented, the current in the dynatron decreases almost linearly with increase

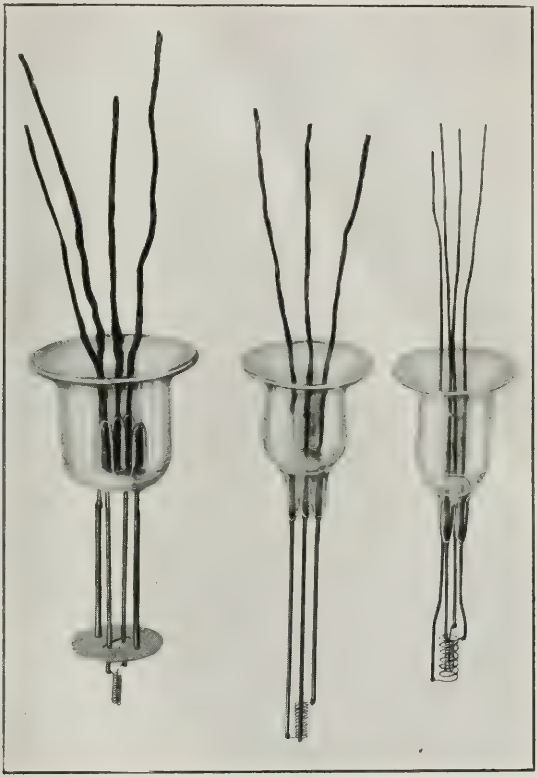

FIGURE 2, e FIGURE 2, f FIGURE 2, g

of voltage, and obeys the equation $i = \frac{E}{\bar{r}} + i_o$, where i_o and $\bar{r}$ are constants, $\bar{r}$ being negative. Since the constant i_o does not affect the variable part of the current in any of the applications for which the dynatron has been used, it is convenient to characterize the dynatron by the constant $\bar{r}$, which will be called its *negative resistance*. The justification for this name is that

the behavior of the dynatron in any circuit containing resistance, capacity, inductance and electromotive force can be accurately calculated by treating the dynatron as a linear conductor with negative resistance $\bar{r}$. Examples of such calculations are given below.

The term i_o in the above equation disappears if the dynatron is connected in series with a battery, of voltage equal to that at which the dynatron current is zero (point B, Figure 3). The combination is a *true* negative resistance, for which $i = \frac{E}{\bar{r}}$. For example, if the dynatron of Figure 1 be put, with its batteries, in a box, and two wires be brought out thru the box as terminals,

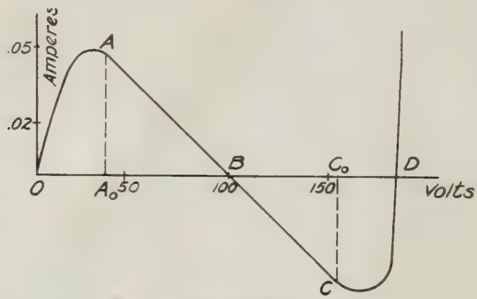

FIGURE 3

one from the plate P and one from a point V_o of the battery corresponding to the point B of Figure 3, this "negative resistance box" would behave in all respects like a conductor with negative resistance, over the range of voltage, positive and negative, represented by $B\,C_o$ and $B\,A_o$ in Figure 3.

The magnitude of the negative resistance, which is the slope of the current voltage curve, Figure 3, and the range of voltage $A_o - C_o$ over which it can be used, depends upon the anode voltage, the temperature of the filament, and, to some extent, on the shape and material of the electrodes. The effect of varying anode voltage alone is shown for two different types of tube in Figures 4 and 5, and the effect of varying filament temperature in Figure 6. It is seen that the effect of varying anode voltage is, in general, to shorten or lengthen the range of the negative resistance part of the curve, without changing the value of the negative resistance. A slight shift in the voltage V_o at which the curves cross the axis is, for one tube, to the right with increasing voltage, and for the other, to the left. It is

therefore to be anticipated that with proper construction, this shift could be made accurately zero, and the operation of the tube be independent of the value of anode voltage over a wide range. Varying the filament temperature, on the other hand, changes the negative resistance only, without affecting the range or the value of V_o. This affords a simple means of adjusting the negative resistance to any desired value, but at the same time imposes a condition upon the uniform operation of the tube, namely, that the temperature of the filament be kept constant.

It will be noticed that the negative slope of the curves in Figure 4 is less straight than those of Figure 5. This is a disadvantage where exact balancing of positive and negative resistance is desired, but for some of the purposes of radio work to be described later, it is an advantage. The degree of curvature depends upon the construction of the tube, and may be made anything that is desired.

4. Dynatron in Circuit Containing Positive Resistance

A. Series Connection. Circuit with Zero Resistance

If the dynatron is connected in series with a circuit containing positive resistance, the total resistance of the circuit is the

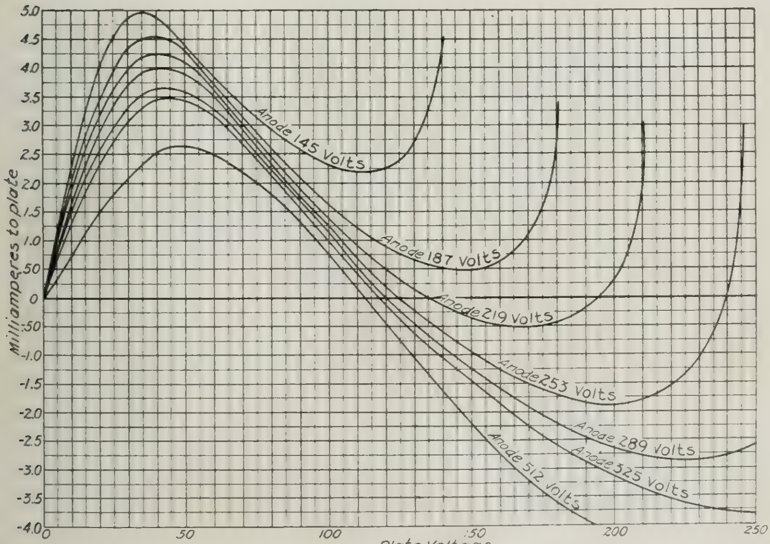

Figure 4

algebraic sum of the positive and negative resistances, and may be made as small as desired by making the positive and negative resistances nearly equal. Such a circuit has very interesting properties. For, while the total resistance of the circuit is very small, that of its parts, individually, is not. Hence a small change in the e.m.f. applied to the whole circuit will cause a

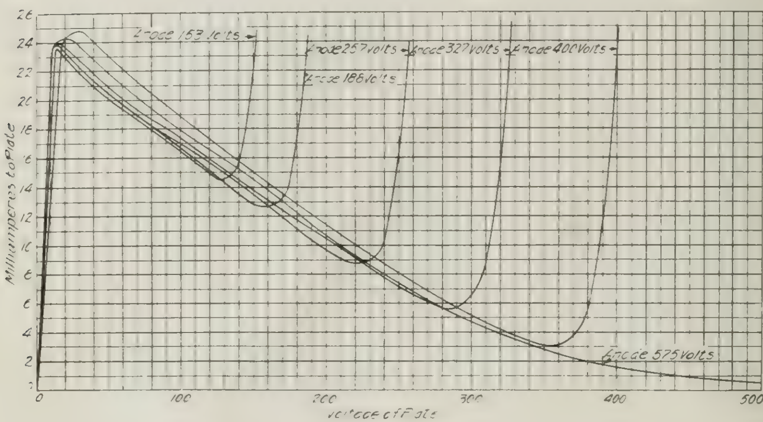

FIGURE 5

comparatively large change in current, and therefore in the iR drop across each part separately: i. e., the circuit acts as a voltage amplifier.

The connections are shown in Figure 7. An ohmic resistance R is connected in series with a dynatron of negative resistance $\bar{r}$, the battery terminal of the dynatron being connected at the point V_o corresponding to the voltage at which the dynatron current is zero.[2] (B, Figure 3.) If an e.m.f. E be impressed across the combination, causing a current I to flow and a voltage drop e_1 in the ohmic resistance and e_2 in the dynatron, then

[2] The amplification of *voltage changes* remains the same if the battery terminal of the dynatron is at some other point than that corresponding to the point B in Figure 3, provided it be in the range $A-C$ (Figure 3) over which the dynatron curve is straight. In that case the equations are

$$e_1 = I\,R$$

$$e_2 = I\,\bar{r} - I_o\,\bar{r}, \text{ where } I_o \text{ is a constant}$$

hence
$$E = I\,(\bar{r}+R) - I_o\bar{r}$$

$$\frac{d\,e_1}{d\,E} = \frac{R}{R+\bar{r}}, \text{ that is}$$

voltage changes are amplified in the ratio $\dfrac{R}{R+\bar{r}}$.

$$e_1 = I\,R$$

$$e_2 = I\,\bar{r}$$

Hence $$E = I\,(\bar{r} + R),$$

and $$\frac{e_1}{E} = \frac{R}{\bar{r} + R}$$

is the ratio of voltage across the ohmic resistance to total voltage, that is, the voltage amplification. This can evidently be made as large as desired by making $\bar{r}$ and R nearly equal, since $\bar{r}$ is negative.

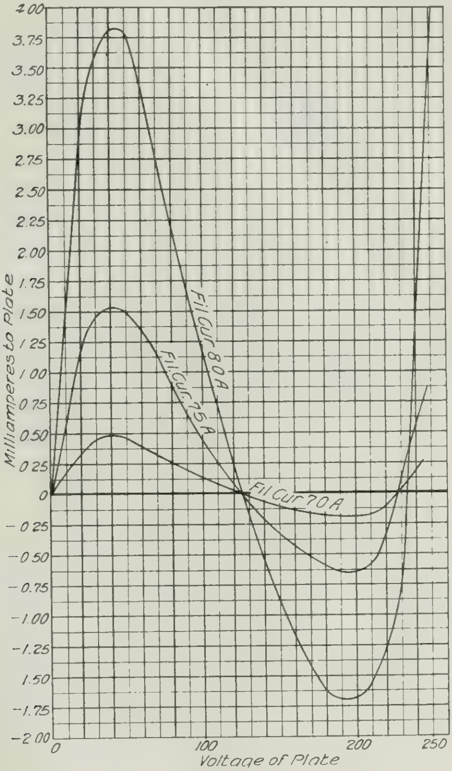

FIGURE 6

These relations may be clearly seen in the graphical representation of Figure 8, where the three curves marked e_1, e_2 and E represent the current-voltage relation in the ohmic resistance, the dynatron, and the total circuit respectively.

With constant batteries, an amplification ratio of 1000-fold

can easily be maintained. For example, if R represents a high resistance galvanometer of 2,000 ohms or more, an e.m.f. of 0.01 volt impressed at the terminals of the combination will cause an e.m.f. of 10 volts across the galvanometer, with corresponding amplification of galvanometer current.

Further examples and applications of this principle to radio work are given in a later section.

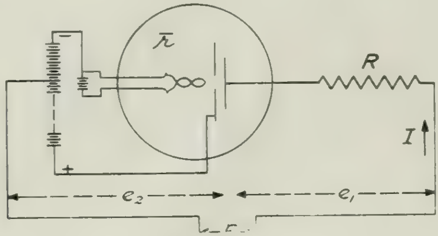

FIGURE 7

B. PARALLEL CONNECTION

If the dynatron is connected in parallel with a circuit containing positive resistance, the total conductivity of the circuit which is the sum of the positive and negative conductivities of

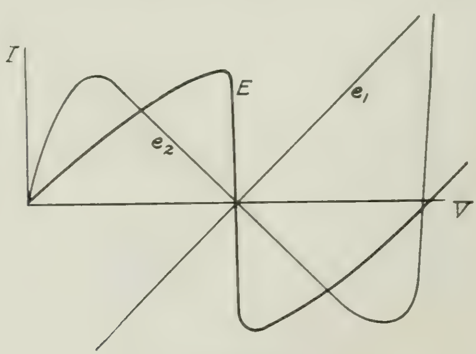

FIGURE 8

its parts, can be made very small. The circuit then acts as a current amplifier. The connections are shown in Figure 9. The total current I is the sum of the current i_1 thru the positive resistance and i_2 thru the dynatron.

Hence $$I = i_1 + i_2 = E\left(\frac{1}{\bar{r}} + \frac{1}{R}\right)$$

$$\frac{i_1}{I} = \frac{\bar{r}}{\bar{r}+R},$$ which may be made very large by making $\bar{r}$ and R nearly equal.

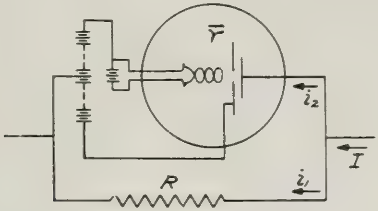

FIGURE 9

These relations are shown graphically in Figure 10, where the curves marked i_1, i_2 and I represent the current-voltage relation in the positive resistance, the dynatron, and the total circuit respectively.

The current I to be amplified may be that thru a photo-electric cell, a kenotron, or any other non-inductive device the current of which is independent of voltage.

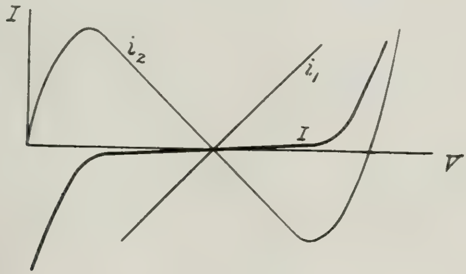

FIGURE 10

5. Dynatron in Circuit Containing Resistance, Inductance, and Capacity

If the dynatron be left open-circuited, as in Figure 1, it is unstable. This was to be expected as a necessary accompaniment of "negative resistance," and can easily be seen from the current-voltage relation in Figure 3. For when the voltage is

greater than that corresponding to the point B, the plate is losing electrons, and hence becoming more positive; and the more positive it becomes, the more rapidly it loses electrons, until the point C is reached. Above C it continues to lose electrons, but more slowly, until it reaches the potential D at which it is in equilibrium. In like manner if the initial potential of the plate is less than B, it will continue to receive electrons until its potential has fallen to 0. At B the plate is in equilibrium, but the equilibrium is unstable, and if slightly disturbed, it will go to 0 or D.

The same instability occurs if the circuit of Figure 1, instead of being left open, is closed thru too high a resistance, so that the rate at which the plate receives electrons is greater than the rate at which these electrons can flow away thru the resistance. In this case the equilibrium voltages will not be D and 0, but some voltage in the range $D\,C_o$, and $O\,A_o$ respectively. This behavior may be strikingly shown by connecting a voltmeter between filament and plate, and opening the circuit. In this case the stable positions are 0 and a point just below D, and if the plate is originally at B, it will jump to either one or the other of these positions, depending on chance.

If the circuit contains inductance and capacity, as well as resistance, a similar action takes place. The plate charges up thru the vacuum, at a rate depending on the capacity and negative resistance, and discharges thru the circuit at a rate depending on the inductance and positive resistance. If the inductance is too high, the plate will receive electrons more rapidly than they can flow away thru the inductance, and will charge up to some point beyond A or C at which the rate of charge and discharge are instantaneously equal. The inertia of the inductance will then carry it backward toward B, and if the resistance is not too great it will pass thru B and oscillate continuously. Whether the circuit will oscillate continuously, or come to rest at B, or come to rest at some other voltage between 0 and D, depends on the relations between inductance, positive and negative resistance, and capacity. These relations can best be given by mathematical analysis, as follows:—

Let the dynatron, with negative resistance $\bar{r}$, be connected in series with a circuit containing inductance, L, resistance R, and capacity C, as shown in Figure 11. Then, calling the instantaneous e.m.f. across either part of the circuit E, we have:

$$\text{For inductive part of circuit } I = \frac{E}{R} - \frac{L}{R}\frac{dI}{dt}$$

For condenser $$I+i=-C\frac{dE}{dt}$$

For dynatron $$i=\frac{E}{\bar r}+i_o$$

which gives, eliminating E and i,

$$\frac{d^2I}{dt^2}+\left(\frac{R}{L}+\frac{1}{\bar r C}\right)\frac{dI}{dt}+\frac{1}{LC}\left(1+\frac{R}{\bar r}\right)I+\frac{i_o}{LC\bar r}=0$$

the solution of which is

$$I=\frac{i_o}{R+\bar r}+A\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{L}-\frac{1}{\bar r C}\right)t}\cos\left(\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2\bar r C}\right)^2}\,t-\alpha\right) \qquad (1)$$

if $$\left(\frac{R}{L}-\frac{1}{\bar r c}\right)^2-\frac{4}{LC}<0$$

and

$$I=-\frac{i_o}{R+\bar r}+A\,\varepsilon^{\left[-\left(\frac{R}{2L}-\frac{1}{2\bar r C}\right)+\sqrt{\left(\frac{R}{2L}-\frac{1}{2\bar r C}\right)^2-\frac{1}{LC}}\right]}$$
$$+B\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2\bar r C}\right)-\sqrt{\left(\frac{R}{2L}-\frac{1}{2\bar r C}\right)^2-\frac{1}{LC}}\right]} \qquad (2)$$

if $$\left(\frac{R}{L}-\frac{1}{\bar r C}\right)^2-\frac{4}{LC}>0$$

where i_o, A, B, α are constants.

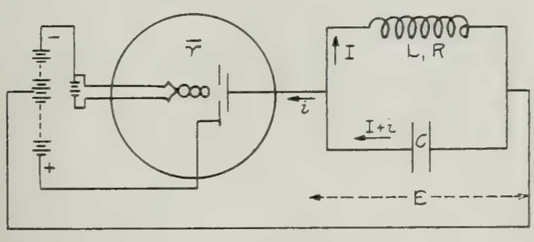

FIGURE 11

The case of most interest is the oscillatory solution, given by equation 1. This differs from the equation of a simple oscillatory circuit in that the damping factor is decreased from $\frac{R}{2L}$ to $\frac{R}{2L}-\frac{1}{2rC}$, where r represents the positive numerical value of

$\bar{r}$, and the period is increased by increasing the damping correction from $\left(\frac{R}{2L}\right)^2$ to $\left(\frac{R}{2L}+\frac{1}{2\bar{r}C}\right)^2$. It is identical in form with the equation of a circuit containing a leaky condenser, the positive leakage resistance of the condenser being replaced by the negative resistance $\bar{r}$ of the dynatron.

Two oscillatory cases are to be distinguished according as the damping factor is positive or negative. In the first case the circuit is stable, but its damping may be made as small as desired, so that an impressed oscillation will persist for a very long time. In the second case, the circuit will oscillate continuously, with an amplitude that would become infinite if the negative resistance held over an infinite range, and which is therefore limited by the length of the straight portion of the negative resistance curve.

The criterion that the circuit shall generate oscillations is that

$$\frac{R}{L}+\frac{1}{\bar{r}C}<0$$ or, if r denote the positive numerical value of $\bar{r}$,

$$Rr<\frac{L}{C} \qquad (3)$$

In order to test this relation, the inductance L in Figure 11 was made an air-core coil, and a secondary coil in series with a telephone was coupled loosely with it, in order to detect when the circuit was oscillating. With a definite value of negative resistance (determined by a separate experiment from the slope of the current-voltage curve) different capacities were introduced, and the maximum value of positive resistance was determined with which the circuit would still oscillate. The results are given in Table 1.

Table 1

R Ohms	r Ohms	L Henries	C Farads	Rr	L/C
75	3,000	0.689	2.90×10^{-6}	225×10^3	237×10^3
85	3,000	0.689	2.56×10^{-6}	255×10^3	269×10^3
96	3,000	0.689	2.26×10^{-6}	288×10^3	304×10^3
108	3,000	0.689	2.05×10^{-6}	324×10^3	334×10^3
126	3,000	0.689	1.75×10^{-6}	379×10^3	392×10^3
158	3,000	0.689	1.41×10^{-6}	475×10^3	487×10^3
204	3,000	0.689	1.12×10^{-6}	614×10^3	615×10^3
253	3,000	0.689	0.930×10^{-6}	760×10^3	725×10^3
78	6,520	0.689	1.27×10^{-6}	510×10^3	543×10^3
90	6,520	0.689	1.14×10^{-6}	587×10^3	602×10^3
116	6,520	0.689	0.90×10^{-6}	757×10^3	767×10^3
162	6,520	0.689	0.636×10^{-6}	$1{,}060\times10^3$	$1{,}080\times10^3$
354	6,520	0.689	0.294×10^{-6}	$2{,}310\times10^3$	$2{,}340\times10^3$
674	6,520	0.689	0.150×10^{-6}	$4{,}400\times10^3$	$4{,}600\times10^3$

According to theory, the maximum value of Rr should be very near to, but always less than $\frac{L}{C}$. It is seen that this relation is satisfied within the limits of experimental error. The values of Rr are all about 3 per cent. less than $\frac{L}{C}$, which is the limit set by the sensitiveness of the telephone with the permissible coupling.

The frequency of oscillation is given by the equation

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L} - \frac{1}{rC}\right)^2},$$

in which the bracketed term under the radical is, for most practical circuits, negligible. The range of possible frequencies which can be generated is determined by the above equation, together with the relation (3) between resistance, inductance, and capacity. The limit of radio frequency is set by the minimum value of capacity, positive resistance, and negative resistance, and can be calculated if the distributed capacity and inductance of the coils and connecting wires are known. An ordinary dynatron short-circuited by a couple of turns of heavy wire will give a frequency of about 20,000,000 cycles per second, and it is possible to go continuously from this to a frequency of less

than 1 cycle per second by simply changing inductance and capacity.

The wave-form depends on the ratio of inductance to capacity and resistance. According to theory we should expect a perfect sine wave when $\frac{L}{C}$ is very nearly equal to Rr (since in this case the circuit fulfills the condition of simple harmonic motion), with increasing distortion as the ratio of $\frac{L}{C}$ to Rr increases. As this is a question of considerable importance, a series of oscillograms was taken with different ratios of $\frac{L}{C}$ to $R\,r$. They are shown in Figure 12. The circuit is that of Figure 11, except that a secondary circuit is coupled inductively with the primary in order to show the form of the wave in a coupled circuit. In each photograph the upper curve gives the current in the coupled circuit, the middle curve the current in the primary circuit, and the lower curve a 40 cycle timing wave. Air inductance and paraffin condensers were used.

Films A to D show the effect of increasing the ratio $\frac{L}{C}$, keeping R and r constant. As $\frac{L}{C}$ increases the primary wave changes from a pure sine wave (film A) to a very slightly distorted wave (film B) and finally to a very badly distorted wave (film D). For comparison with curve D, film E was taken under the same conditions and the same frequency, but with a proper ratio of $\frac{L}{C}$. It is a good sine wave. It is to be noted that the oscillation in the coupled circuit is a fair sine wave, even when the primary is badly distorted.

6. Dynatron In Inductive Circuit With Impressed Periodic Electromotive Force

If a periodic e.m.f., represented by $e_0 \cos \omega t$ be impressed upon the circuit of Figure 11, the forced oscillations which it impresses upon the circuit may attain a much greater value than in a circuit containing no dynatron. This can best be seen from mathematical analysis. The equations of the circuit are:

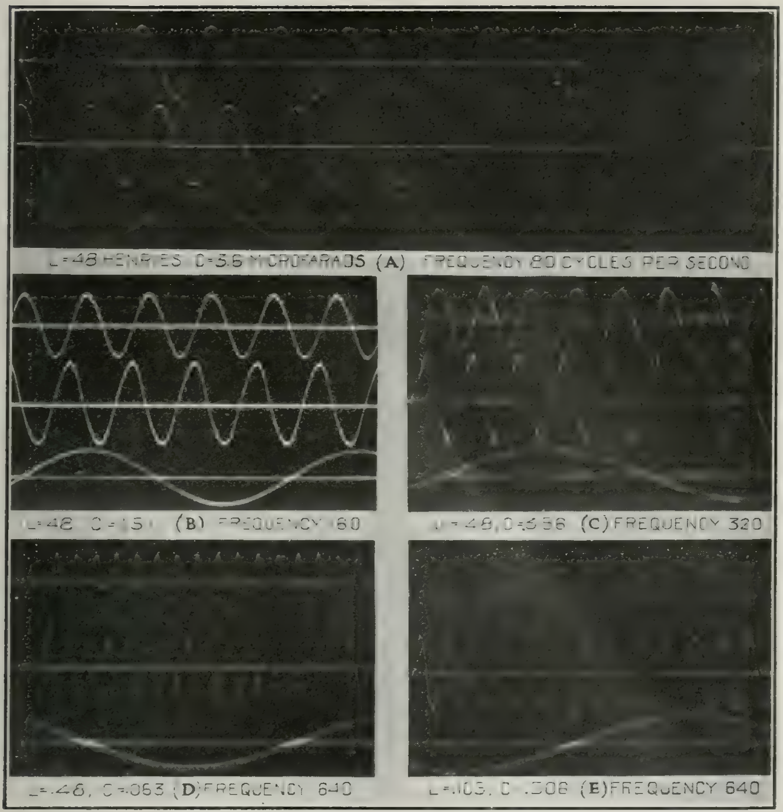

FIGURE 12—Effect of Capacity on Wave Form in Oscillating Dynatron

The middle curve in each film is the current thru the dynatron, the upper curve the current in the coupled circuit, the lower curve a 40-cycle wave for comparison

$$I\,R+L\frac{d\,I}{d\,t}=E-e_o\cos\omega\,t$$

$$i=\frac{E}{\bar{r}}+i_o$$

$$I+i=-C\frac{d\,E}{d\,t}$$

whence

$$\frac{d^2E}{d\,t^2}+\left(\frac{R}{L}+\frac{1}{C\,\bar{r}}\right)\frac{d\,E}{d\,t}+\frac{1}{L\,C}\left(1+\frac{R}{\bar{r}}\right)E+\frac{i_o}{L\,C}(R+L)=\frac{e_o}{L\,C}\cos\omega\,t$$

and

$$E = -\frac{i_0\,(R+L)}{1+\frac{R}{\bar r}} - A\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{L}-\frac{1}{C\bar r}\right)t}\cos\left\{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2C\bar r}\right)^2}\;t-\alpha\right\}$$

$$+\frac{e_0\cos(\omega t-\theta)}{\sqrt{\left(1+\frac{R}{\bar r}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar r}\right)^2}} \qquad (4)$$

if

$$\frac{1}{LC} > \left(\frac{R}{2L}-\frac{1}{2C\bar r}\right)^2$$

or

$$E = -\frac{i_0\,(R+L)}{1+\frac{R}{\bar r}} + A\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2C\bar r}\right)+\sqrt{\left(\frac{R}{2L}-\frac{1}{2C\bar r}\right)^2-\frac{1}{LC}}\right]t}$$

$$+B\,\varepsilon^{\left[-\left(\frac{R}{2L}+\frac{1}{2C\bar r}\right)-\sqrt{\left(\frac{R}{2L}-\frac{1}{2C\bar r}\right)^2-\frac{1}{LC}}\right]t}$$

$$+\frac{e_0\cos(\omega t-\theta)}{\sqrt{\left(1+\frac{R}{\bar r}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar r}\right)^2}} \qquad (5)$$

if

$$\frac{1}{LC} < \left(\frac{R}{2L}-\frac{1}{2C\bar r}\right)^2$$

where A, B, α, and θ are constants, with the usual meanings.

In either case, provided $R<r$, the amplitude of the forced oscillations is

$$\frac{e_0}{\sqrt{\left(1+\frac{R}{\bar r}-LC\omega^2\right)^2+\omega^2\left(RC+\frac{L}{\bar r}\right)^2}}$$

and can be made as large as desired (since $\bar r$ is negative) by making

$$\left.\begin{aligned} R\,r &= \frac{L}{C} \\ \frac{R}{r} &= 1-LC\omega^2 \end{aligned}\right\} \qquad (6)$$

and

The first condition is equivalent to zero damping. The second shows that for maximum sensitiveness the frequency ω must be equal to $\sqrt{\frac{1}{LC}\left(1+\frac{R}{\bar r}\right)}$, which is the natural frequency of the system when its damping is zero.

It is to be noted that the sensitiveness of the system is the same whether the damping term $\frac{R}{L}+\frac{1}{C\bar r}$ is positive or nega-

tive. If it is positive, the natural oscillations of the system soon die out, leaving only the forced oscillation given by (4) and (5). If it is negative, the system will generate oscillations of its own of a frequency $\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}-\frac{1}{2C\bar{r}}\right)^2}$ slightly different from ω, in addition to the oscillations of frequency ω given by (5), and these two will produce heterodyne interference. The application of this to radio receiving is discussed below.

7. The Effect of a Magnetic Field

A profound change in characteristics is produced by placing the cylindrical type of dynatron shown in Figure 2 in a magnetic field parallel to the axis of the cylinder. The electrons from the filament, which in the absence of the magnetic field move in nearly straight lines to the anode and pass freely thru its holes (Figure 13 a), are constrained by the field to move in spirals, and strike the anode more or less tangentially (Figure 13 b), so that a much larger proportion are stopped by it. The

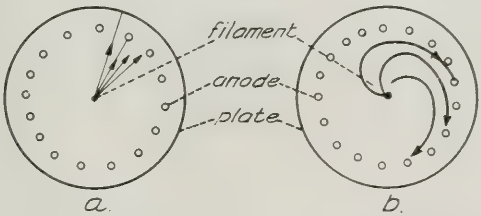

Figure 13 a Figure 13 b

result is to diminish greatly the number of electrons reaching the plate. Superimposed upon this effect is a restraining effect of the field upon the secondary electrons which try to leave the plate, resulting in a change from negative resistance to positive resistance characteristic.

These effects are shown in Figure 14, where each curve rep-

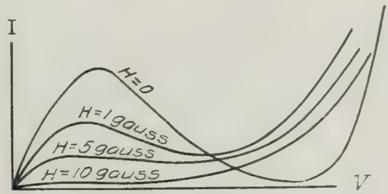

Figure 14

resents the voltage-current relation of the dynatron in a definite field. It will be seen that as the field increases the curves become lower and flatter, and soon lose their negative slope altogether. It is thus possible, by varying the magnetic field, to control the behavior of the dynatron. This method of control is especially applicable to the radiophone, as will be explained later.

8. The Pliodynatron

An electrostatic field may be used instead of a magnetic field to control the number of electrons reaching the plate. It has been shown (see Figure 6) that the effect of changing the number of electrons leaving the filament, by varying its temperature, is to change the negative resistance without affecting the other characteristics of the current voltage relation. If the temperature of the filament could be easily and rapidly changed, this would be an effective means of controlling the dynatron. The same result may be accomplished, however, by the electrostatic action of a grid close to the filament; that is, by the application of the pliotron principle. A dynatron which thus utilizes the pliotron principle is called a *pliodynatron*. Its construction is the same as that of the simple dynatron with the addition of a "control member," which may be a grid surrounding the filament (Figure 2, g) or a metal rod inside the (spiral) filament (Figure 2, f).

Its relation to the pliotron can be most clearly seen in the "plate type" of pliodynatron, a photograph of which is shown in Figure 15. It is identical in construction with the pliotron except for the addition of the perforated anode.

The characteristics of the pliodynatron can be seen from Figure 6, if for filament temperature we substitute grid potentials. The steepness of the curve increases, that is, the negative resistance decreases, with increasing grid potential. The relation is capable of more exact statement: It is known that in the pliotron, with constant anode voltage, the number of electrons leaving the filament is proportional to grid potential over a wide range, and this must be true in the pliodynatron, where the anode voltage is always constant. It may be shown, both theoretically and experimentally, that the negative resistance is inversely proportional, over a wide range, to the total number of electrons leaving the filament. The negative resistance is therefore inversely proportional to grid potential. The behavior of the pliodynatron in circuits containing resist-

ance, inductance and capacity is therefore given by equations (1) to (6) if we replace R in these equations by $\frac{R_o}{v}$, where R_o is a constant and v the potential difference between grid and filament.

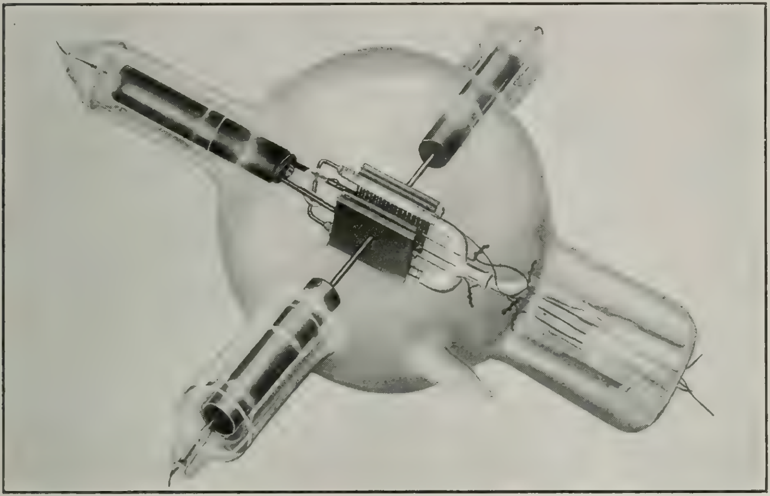

FIGURE 15—General Electric Company Pliodynatron

The negative resistance of the pliodynatron makes it a powerful amplifier. An increase of grid potential, by increasing the current thru the load in the plate circuit and hence the voltage drop over the load, lowers the voltage of the plate. In the pliotron this lowering of plate voltage tends to decrease the plate current, and thus opposes the effect of the grid. In the pliodynatron, however, a decrease in plate voltage means an increase in current, which may be very large if positive and negative resistance are nearly equal. This will be clear from Figure 16, where the curves marked v_1 and v_2 represent the current voltage relation for the grid voltages v_1 and v_2 respectively of a pliodynatron and a pliotron. If we start with an initial current of i_1, corresponding to plate voltage E_1, and raise the grid voltage from v_1 to v_2, the current tends to rise to i_2. On account of the decrease in plate voltage, however, the pliotron current will rise to some smaller value i', while the pliodynatron current will rise to a much larger value i''. The advantage to be gained in

this way may be large, if the resistance in the circuit is high. For example, the maximum aperiodic voltage amplification thus far obtained with a pliotron is about 15-fold, while with a pliodynatron we have obtained 1000-fold.

A better method of representing the characteristic behavior

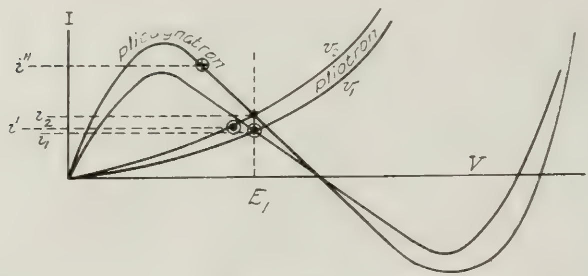

FIGURE 16

of the pliodynatron is, instead of plotting the current to the plate against plate voltage, to plot it against the total voltage across plate and series resistance, as in curve E, Figure 8. A series of such plots, for different grid potentials, is shown in Figure 17. The voltage plotted is now constant, being that of

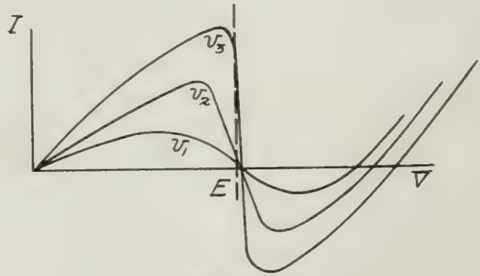

FIGURE 17

the battery, and for any given value E we obtain the currents corresponding to different grid potentials from the intersections of the curves with a vertical line thru E. If E is taken just to the left of the point where the curves cross the axis, the current will increase at first slowly and then very rapidly with grid potential, as shown in Figure 18. The amplification is, under

these circumstances, both asymmetric and high, and the tube should constitute a good radio receiver. This is discussed more fully in Section 14 below.

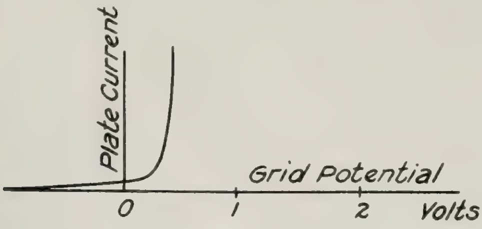

FIGURE 18

APPLICATIONS OF THE DYNATRON TO RADIO WORK

9. DYNATRON AS GENERATOR OF RADIO WAVES

It has been shown in section (5) that the dynatron always oscillates providing $Rr < \frac{L}{C}$, where R and r are the positive and negative resistance, respectively, of the circuit, L the inductance and C the capacity. The frequency of oscillation is approximately $\frac{1}{2\pi\sqrt{LC}}$, and may be given any value from 1 to 10,000,000 by changing inductance and capacity alone. It has also been shown that for low frequencies the oscillations are very nearly pure sine waves provided $\frac{L}{C}$ is not too great compared with Rr. Theory indicates that this should be true for all frequencies, and a search for harmonics at radio frequencies has verified the expectation.

The dynatron therefore satisfies all the requirements of a radio generator, and has the advantage that its operation is invariable and free from lag, and that the frequency may be given any value by changing a single inductance or capacity. Its oscillations may be controlled either by opening and closing the main circuit, or by changing any one of the four factors L, C, R, and r in accordance with the condition of oscillation given above. Its efficiency is low, probably less than 50 per cent. under best conditions. This is not, however, a serious limitation, except as regards the cost of power, since the tubes are capable of running very hot without deterioration. The

maximum output at radio frequency of the tubes thus far constructed is about 100 watts, but no effort has been made to develop a high power tube.

It is generally necessary to transform the radio energy by means of a coupled circuit. In the discussion thus far the effect of such a coupled circuit on the oscillation has been neglected. The calculation for the case of inductively coupled circuits is not easy, but it may be shown experimentally that conditions similar to those derived above hold, even when the coupled circuit absorbs nearly all of the energy.

10. Pliodynatron as Radio Telephone

The simplest method of controlling the oscillations of the dynatron is to vary the negative resistance, by means of a grid around the filament, as in the pliodynatron. It has been shown in Section (8) that the negative resistance of the pliodynatron is inversely proportional to grid potential. Hence if the ratio of inductance to capacity and resistance be initially just large enough to produce oscillation (which is also the condition for producing pure sine waves), a slight decrease in grid potential will stop the oscillations.

If the negative resistance part of the pliodynatron curve, instead of being straight, is curved like that of Figure 4, the oscillations will not fall abruptly from full value to zero when the grid potential is reduced beyond the critical value, but will be gradually reduced in amplitude as the grid potential is decreased. This is exactly what is required for the radiophone and it is easy to make pliodynatrons which have this characteristic.

The connections are shown in Figure 19. The oscillating circuit is the same as Figure 11, except that the dynatron is replaced by a pliodynatron, and is coupled inductively to the antenna. A microphone M, coupled thru the transformer T to the grid circuit of the pliodynatron, serves to control the amplitude of the oscillations. A battery of a few volts, between grid and filament, keeps the grid always negative with respect to filament.

It is found that, with a proper ratio of inductance to capacity, the amplitude of the radio waves is very nearly proportional to the grid potential, and hence to the instantaneous displacement in the vocal wave. This was proved for constant grid potential by means of a hot wire ammeter in the antenna circuit, and for alternating grid potentials by impressing a sine wave on the

transformer T, and observing the form of the rectified radio waves in a coupled circuit containing a kenotron rectifier and oscillograph.

Under these circumstances, it was found that speech transmitted to the microphone M and received at a station a few miles distant suffered very little more distortion than in the ordinary wire telephone. With a small tube giving about 10

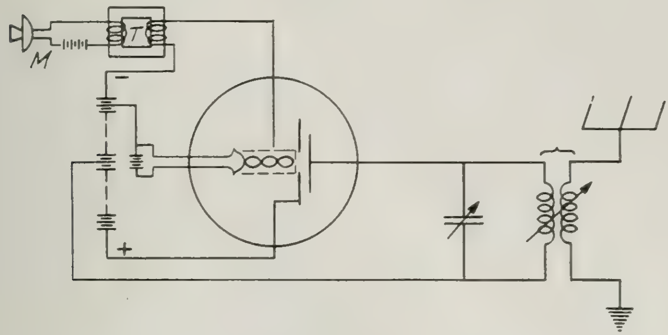

FIGURE 19

watts, it was possible to radiophone 16 miles (26 km.) with good intensity and articulation. No attempt has been made to telephone greater distances, or to develop high power pliodynatrons. The maximum output of a single tube which it has been possible to control thus far is about 60 watts.

11. MAGNETICALLY CONTROLLED DYNATRON AS RADIO TELEPHONE

Instead of controlling the negative resistance by a grid, as in the pliodynatron, we may use a magnetic field, as explained in Section (7). It is seen from Figure 14 that the change both in slope and amplitude of the negative resistance portion of the curves is a continuous function of the magnetic field strength. Hence if the magnetic field coil is connected in series with a microphone, the amplitude of the radio oscillations may be controlled by the voice, as in the pliodynatron. The energy needed to set up a magnetic field of the required strength is small, and can easily be furnished by the microphone circuit, but the impedance of the coil tends to choke out the higher voice frequencies.

12. Dynatron as Amplifier and Detector

It has been shown in Section 6 that a small periodic electromotive force impressed upon a circuit containing a dynatron may be amplified in any desired ratio by properly adjusting the capacity and inductance of the circuit: that is, the resonant value of current or voltage in the dynatron circuit is infinite, except as it is limited by the length and straightness of the dynatron curve. The impressed oscillations may be radio oscillations in an antenna coupled with the dynatron circuit, and the amplified voltage or current be used to operate a detector. It is important to notice that the energy consumed in the detector does not decrease the amplification, since the dynatron can be adjusted just to neutralize this loss, in addition to the other losses in the oscillating circuit. The simplest examples are when the detector losses are of a pure resistance nature, as, for example, when a high resistance galvanometer, such as one of the Einthoven type, is inserted in the oscillating circuit, or an audion with leaky grid, the leakage being proportional to voltage, is connected across any part of the oscillating circuit. In these cases, equation (4) of Section 6 applies directly, the positive resistance R being the total resistance of the circuit, including galvanometer and grid. In the cases where the detector is inductively coupled to the oscillating circuit, the impedance due to the coupling is equivalent to a resistance, so that similar relations hold.

Since the amplitude of the "resonant current" in the dynatron circuit is limited by the length and straightness of the negative resistance curve, it is evident that if we operate the dynatron in a region very near one end of the curve, as at A or C, Figure 3, the current will be asymmetric, and the dynatron may itself be used as a detector. Suitable connections are shown in Figure 20, where a telephone T with condenser C' across its terminals is inserted directly in the dynatron circuit. The distributed capacity between turns of the telephone offers low resistance to radio frequencies, so that the conditions of amplification discussed above still hold. But the high inductance of the telephone will, according to condition (3) of Section 5, cause the circuit to oscillate at audio frequency, unless its resistance be very high, or a condenser C', of suitable capacity, be connected across its terminals.

The circuit shown in Figure 20 has two advantages, in addition to its high amplification, viz.:

1. The ratio of inductance to capacity may be adjusted

so that the circuit oscillates with natural frequency very near that of the radio waves, as explained in Section 6, thus producing heterodyne beats.

2. The capacity C' and negative resistance $\bar{r}$ may be so adjusted as to neutralize the resistance of the telephone for a particular audio frequency, determined by the product of C' and telephone inductance, and if this frequency be made the same as the group frequency of the incoming radio waves, the sensitiveness becomes very great.

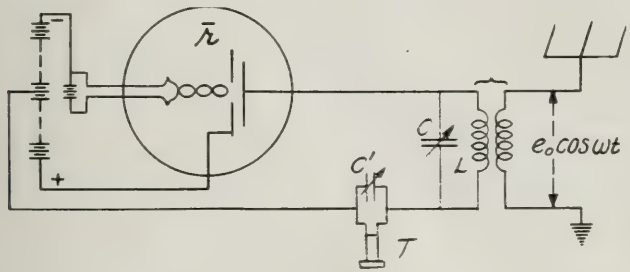

FIGURE 20

These predictions have been verified separately by experiment. In order to test the behavior of the complete circuit, it was set up as in Figure 20, and its reception of signals from a small spark set compared with that of a sensitive audion. For very weak signals the audion was the more sensitive, indicating small asymmetry in the dynatron oscillation. For medium signals, however, the dynatron response was many times stronger, and its intensity could be increased to almost any degree by adjustment of the capacity C'.

It is interesting to note that the coupling in a circuit like that of Figure 19 may be made very close without affecting the selectivity, since the condition for selectivity, viz.: a small damping factor, still holds. This is true both for the antenna coupling and that of the auxiliary detecting circuit, when one is used. The fact that sensitiveness and selectivity are independent of both resistance and coupling coefficient makes it possible to use a much more effective ratio of transformation than has hitherto been practicable.

13. USE OF DYNATRON FOR NEUTRALIZING RESISTANCE IN RADIO CIRCUITS

The negative resistance of the dynatron may be utilized to supply the energy losses of whatever nature, in any circuit, and

the circuit thereupon behaves, as regards selectivity, damping, and sensitiveness to external stimuli, like a circuit having zero resistance. The amount of energy fed into the circuit by the dynatron is i^2r, where r is the negative resistance and i the current (steady value or r.m.s.) thru the dynatron. Examples of this use of the dynatron in simple circuits containing resistance, inductance and capacity have already been given in Sections 3 to 6. Two further examples will illustrate its use in circuits where the resistance characteristic is more complex.

(a) Dynatron in Plate Circuit of Pliotron for Aperiodic Amplification.

The current thru the pliotron, for constant grid voltage, increases with increasing voltage of the plate, that is, it has the characteristic of a positive resistance, which limits its amplifying power as explained in Section 8. This resistance characteristic may be neutralized by connecting a dynatron in parallel with the pliotron, as in Figure 21. Using a pliotron whose "positive resistance" was 100,000 ohms, and a series resistance of 250,000 ohms in the circuit, we were able in this way to increase the d. c. voltage amplification from 12-fold, for the pliotron alone, to 625-fold. A further advantage in this connection is that the dynatron can be operated at such a voltage that its current is just equal and opposite to that of the pliotron, so that the total current thru the circuit is zero. This allows the use of a more sensitive measuring instrument.

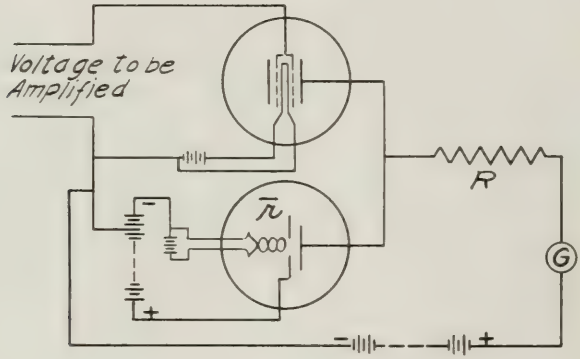

FIGURE 21

(b) Dynatron in Grid Circuit of Pliotron Detector.

The increase in voltage of the grid of a pliotron detector is

opposed by a leakage current which increases with voltage, as in a positive resistance, and also by the counter e. m. f. and losses in its own and the coupled antenna circuit. These losses may be neutralized by a dynatron in parallel with the grid, as in Figure 22. With this arrangement the intensity of weak signals from a spark set was increased from audibility to a roar.

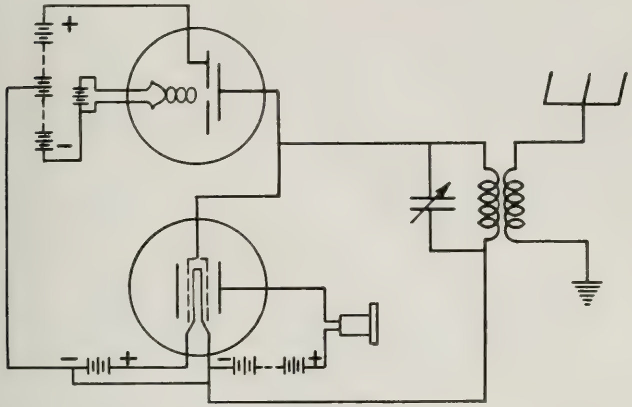

FIGURE 22

The dynatron, instead of being connected directly to the grid of the pliotron, may be in a separate circuit which is inductively coupled to any part of the grid or antenna circuit.

14. PLIODYNATRON AS AMPLIFIER AND DETECTOR

It has been shown in Section 8 that a pliodynatron in series with a suitable resistance is capable of producing an aperiodic voltage amplification of 1000-fold. To maintain this amplification requires constant batteries and continuous attention. A value of 100-fold, is, however, very easy to maintain. By connecting two pliodynatrons in series a total amplification of 10,000-fold has been obtained. With this amplification it should be possible to receive radiograms on an aperiodic antenna.

This arrangement of pliodynatron and positive resistance is equally applicable to a tuned antenna circuit. The connections are shown in Figure 23. The telephone itself furnishes sufficient resistance, and a condenser C' connected across the telephone is adjusted so that its capacity is just sufficient to keep the circuit from oscillating, according to condition 3, Section 5. With this connection, the amplification is asym-

metric, i. e., different for positive and negative variation in grid potential, as shown in Figure 18. To increase the selectivity, a circuit LC, tuned for radio frequency may be included in series with the telephone, and either adjusted to the verge of oscillation, or allowed to generate oscillations for heterodyne work. The telephone should, in case of radiograms, be tuned for the

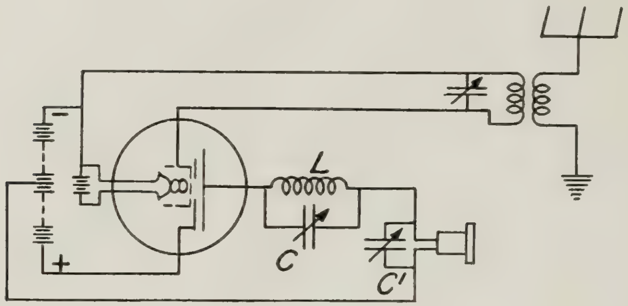

FIGURE 23

group frequency of the signals. It may then be brought to the verge of audio-oscillation by adjusting the negative resistance, and the final adjustment for radio sensitiveness be made by varying the ratio of L to C, keeping their product constant.

In the circuit of Figure 23 all the losses may be compensated, in the manner just described, except those in the grid circuit and antenna. Figure 24 shows a modification of the circuit of Figure 23 in which the grid and antenna losses also are compensated. The modification consists in connecting the grid, not to the filament, but to a properly chosen point P on a resistance R in series with the plate. The pliodynatron is then operated

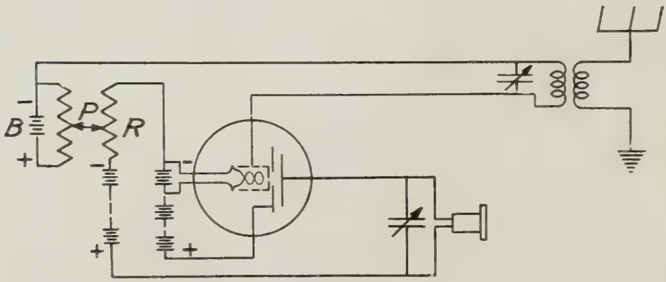

FIGURE 24

at such voltage that the current in the plate circuit is negative (between B and C, Figure 3), that is, positive electricity, or its equivalent, flows from filament to plate across the vacuum and thence thru the battery and resistance R back to filament. Raising the potential of the grid increases the current thru R, and raises the potential of P, which tends still further to increase the potential of the grid. By this mechanism, energy is fed back from the plate circuit, which may be adjusted to furnish any amount of energy desired, into the grid circuit, and by properly adjusting P, the amount of energy thus appropriated may be made just sufficient to neutralize the losses in grid and antenna, without causing oscillation. The antenna coupling should be close, and its resistance may be as large as desired. A potentiometer is shown connected across a battery B the voltage of which is equal to the normal drop in R, for keeping the grid potential constant during adjustment.

SUMMARY: A new, hot cathode, three electrode vacuum tube, the dynatron, is described. A constant, positive voltage is applied between the hot cathode and the perforated rugged anode. A supplementary anode is placed beyond the main anode, and is maintained at a lower positive potential than the main anode.

Because of secondary electronic emission from the supplementary anode, thru a certain range of applied voltages, the supplementary anode-to-filament circuit acts as a true negative resistance. Consequently the dynatron can be used as an oscillator at almost any desired audio or radio frequency or as a voltage or current amplifier. The theory of oscillation therefor is given, and experimentally verified.

The effect of magnetic fields on the value of the negative resistance is studied. The effect of inserting a true grid (thus producing a pliodynatron) is also considered. The latter device is not only an amplifier, but can readily be used as a controlled oscillator for radio telephony. In this connection, experiments are described.

The use of the dynatron as an amplifying detector and as a means for neutralizing circuit resistance is explained, as well as the similar employment of the pliodynatron. All receiver circuit losses can be compensated and selectivity retained at close coupling.

TELEPHONE RECEIVERS AND RADIO TELEGRAPHY*

By

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A general knowledge of the characteristics and operation of the telephone receiver may be obtained by a study of the motional-impedance circle, an account of which was first published by Kennelly and Pierce in September, 1912.[1] Subsequent researches have been completed[2] and others are now being carried on by way of applying the microscope to different parts of the general proposition.

The motional-impedance of the telephone receiver at a given frequency of vibration is the difference between its impedance when the diafram is free to vibrate and its impedance when the vibration of the diafram is prevented; that is, it is the difference between the free and damped impedances of the receiver. Impedance may be measured by means of the ordinary impedance bridge shown in Figure 1. In work already done, vibratory electric current ranging in frequency from about 400∽ to about 2500∽ was supplied by the Vreeland oscillator, the potential across the bridge being kept constant at about 15 volts. The resistance and inductance of the receiver coils were balanced by adjusting the variable resistance and inductance in one of the bridge arms until silence was noted in the indicating telephone of the bridge.

Preparatory to finding the damped impedance, the receiver was mounted so that its diafram was in a vertical plane. The motion of the diafram was damped by connecting its center to the end of a small horizontal brass rod to the other end of which was fixed a suspended brass cylinder weighing 1,000 grams (2.2 pounds) or more. This damping mass was so supported as not to strain the diafram or change the air gap between it and the pole pieces. Impedance observations were then made for a number of frequencies of vibration of the exciting current between 400∽ and

*Received by the Editor July 15, 1917.

[1] Bibliography 2, pages 113–151.

[2] Bibliography 4, pages 421–482; and Bibliography 5, pages 415–460.

1500$\backsim$. The curves between damped resistance and damped reactance as ordinates and frequency of vibratory current as abscissa are given in Figure 2. If the damped reactances for

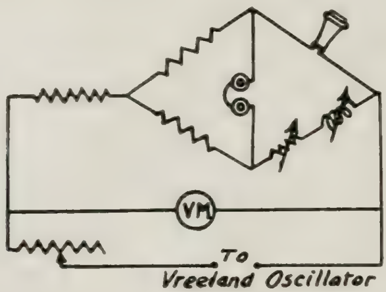

FIGURE 1

Impedance Bridge

given frequencies thruout the range are plotted as ordinates against the corresponding damped resistances as abscissas, the damped impedance curve shown in Figure 3 is obtained.

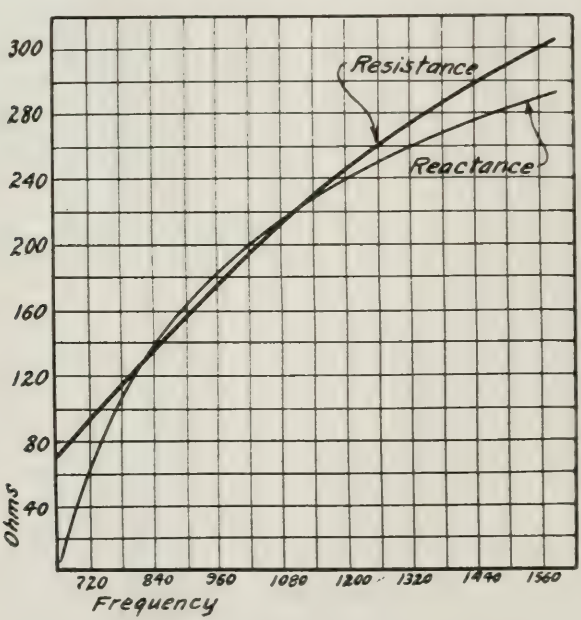

FIGURE 2

Damped Resistance and Reactance of Receiver Against Frequency

For the free impedance of the receiver, the damping mass was detached from the diafram, and impedance observations made at the same frequencies of vibration as before. The curves between free resistance and free reactance as ordinates and frequency of vibratory current as abscissa are given in Figure 4. If the free reactances for given frequencies thruout the range are plotted as ordinates against the corresponding free resistances as abscissas, the free impedance curve shown in Figure 5 is obtained.

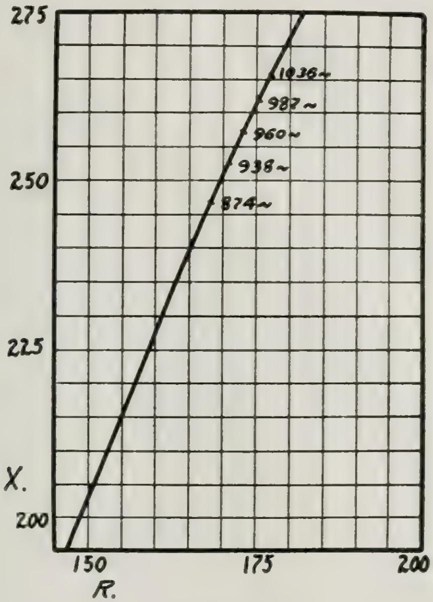

FIGURE 3
Damped Impedance

The differences between the free and damped impedances of the telephone receiver are shown in Figure 6, where the curve of free impedances is superimposed upon that of damped impedances. The straight lines joining the points of each curve corresponding to the same frequency represent the impedance at each frequency due entirely to the vibratory motion of the diafram. Eaeh of these lines then represents in magnitude and direction the motional-impedance of the telephone recevier for the given frequency, and if drawn radially with one end

at a single point as origin, the far ends of the lines will be observed to lie upon a circle. This circle, shown in Figure 7, is the motional-impedance circle of the telephone receiver.

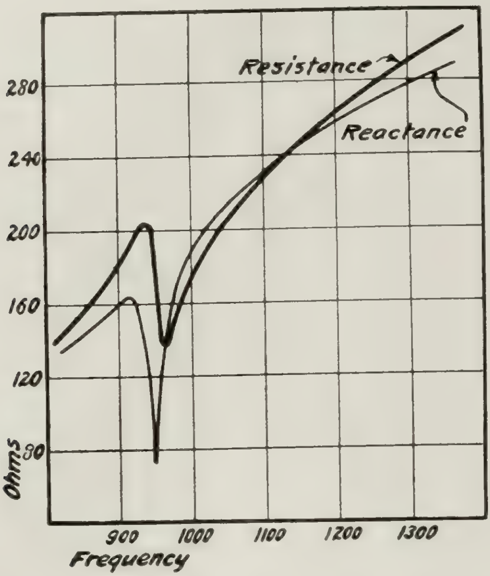

FIGURE 4

Free Resistance and Reactance of Receiver Against Frequency

As usually plotted, the difference between the free and damped reactances for a number of frequencies which include the resonance frequency of the diafram is plotted as ordinates against the difference between the free and damped resistances corresponding to the same frequencies respectively as abscissas. The points so located lie upon a circle and the chords drawn from the origin are the motional-impedances for the various frequencies. The diameter of the circle is the motional-impedance corresponding to the natural frequency of the diafram. The angle thru which this diameter is depressed below the resistance axis represents the lag of the vibration of the diafram behind the current, due to hysteresis and eddy currents and to the vibrational variation of the air gap between the diafram and pole pieces. The quadrantal points lie at the ends of the diameter drawn perpendicular to the impedance diameter and the frequencies, n_1 and n_2, corresponding to these points are used in

the following equation in the computation of the damping factor, Δ, of the diafram:[3]

$$\Delta = \pi (n_2 - n_1) \qquad \text{hyp. rad./sec.} \qquad (1)$$

Since impedance has the dimensions of a velocity, and the curve of the motional-impedances of a telephone receiver is a circle, the inference is natural that the curve of the diafram velocities is also a circle. This conclusion has been found true experimentally. The equation of motion of the center of the diafram is

$$m \ddot{x} + r \dot{x} + s x = f = F \sin \omega t \qquad \text{dynes} \angle \qquad (2)$$

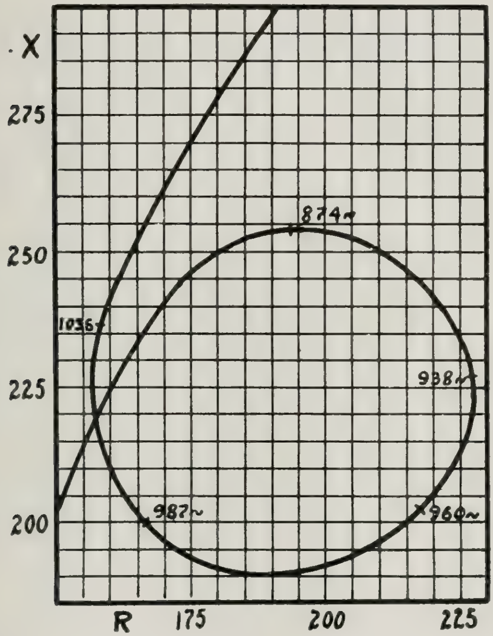

Figure 5
Free Impedance

where	m = equivalent mass of the diafram,	gms.
	r = mechanical resistance constant,	dynes/kine
	s = elastic constant,	dynes/cm.
	f = instantaneous pull,	dynes
	F = maximum pull at any frequency,	dynes
	x = displacement at center of diafram,	cm.

[3]For derivation, see Bibliography, 2, page 146.

The solution of this equation for velocity is

$$\dot{x} = \frac{f}{\sqrt{r^2 + \left(m\,\omega - \frac{s}{\omega}\right)^2}} \quad \text{cm./sec.} \angle \qquad (3)$$

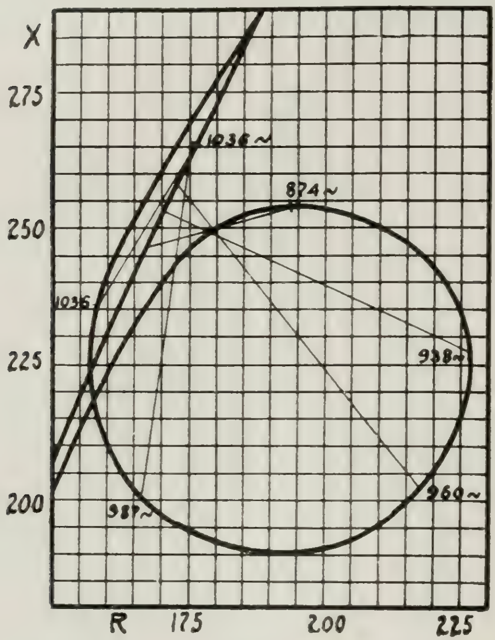

Figure 6

Impedance Due to Vibration of Diafram.

and the phase angle, a, between the pull, f, and the velocity, $\dot{x}$, is given by

$$\tan^{-1} a = \frac{m\,\omega - \frac{s}{\omega}}{r} \quad \text{numeric} \qquad (4)$$

where $\omega = 2\,\pi\,n$, (n = frequency).

If the velocity, $\dot{x}$, and the phase angle, a, are computed for a number of frequencies in the resonance region, and a curve is plotted in polar coordinates between a and $\dot{x}$, the points so found lie upon a circle called the velocity circle of the diafram, Figure 8. Velocity circles have been obtained experimentally

by means of a diafram vibration explorer,[4] and show the same general characteristics as the motional-impedance circle.[5] Thus the immediate cause of the motional-impedance circle is shown to be the diafram velocity, and incidentally the deduction that impedance has the dimensions of velocity is upheld experimentally.

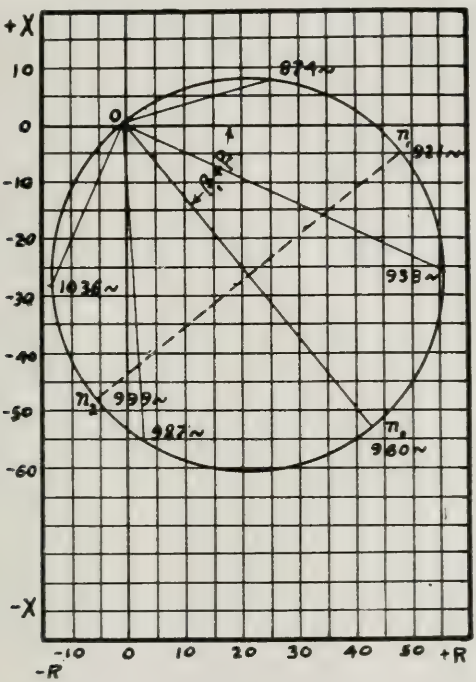

FIGURE 7
Motional-Impedance Circle

The phenomena of the vibration of the telephone diafram as shown by the velocity circle occur so rapidly that the eye cannot follow them. A similar phenomenon may however be produced slowly so that the eye can easily follow it by means of coupled torsion pendulums.[6] In a trial made, one of the coupled units consisted of a heavy bob attached to the end of a stiff wire and so fastened to the support that its frequency could

[4] Bibliography, 3, page 97.
[5] Bibliography, 4, page 451
[6] Bibliography, 5, page 439.

be varied by adjusting the length of suspension. The second coupled unit was a very small torsion pendulum suspended from the axis of the large bob. When vibrating, the steady state was soon reached when both pendulums had the frequency of the large one. As the frequency of the large pendulum was increased in steps by adjusting the length of its suspension, from a frequency less than the natural frequency of the small pendulum to one greater, the phase relation of the displacements of the two bobs passed from co-phase, thru quadrature at resonance, to phase opposition; and the amplitude of the small pendulum increased as resonance was approached, was a maximum at resonance, and then decreased as resonance was passed. The

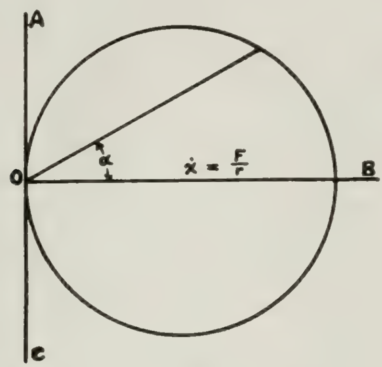

FIGURE 8
Velocity Circle of Telephone Diafram

displacement of the large pendulum corresponds to the force pulling on the diafram, and the velocity of the small pendulum corresponds to the velocity of the diafram. Vibrational velocity leads displacement by an angle of $\frac{\pi}{2}$, therefore, if the observed phase difference between the maximum velocity of the small bob and the maximum displacement of the large bob is plotted to polar coordinates against the velocity of the small bob as radius vector, an approximate circle is obtained—the velocity circle.

The intimate connection between the motion of the diafram and the motional-impedance circle makes it possible to use the latter in the determination of the motional constants, m, r, and s, of the diafram if one of them is known.[7] These constants have

[7]Bibliography, 4, page 472.

been found in so many cases that from the experience gained it is now possible for one to estimate them to a fair degree of accuracy from measurements made on the diafram, its diameter, thickness, and mass.

The equivalent mass, m, is defined as the mass which, when concentrated at the center of the diafram, exhibits the same kinetic energy as is exhibited by the actual distributed mass of the diafram. In the ideal case, the motion of the diafram is expressed by a Bessel's equation,[8] and the distribution of vibration amplitudes over the surface is such that the equivalent mass is given by[9]

$$m = 0.183M \qquad \text{mgs.} \qquad (5)$$

where M is the active mass (that included within the clamping ring).

This amplitude equation was derived by Lord Rayleigh,[10] and is expressed by the curves given in Figure 9. If x_m is the

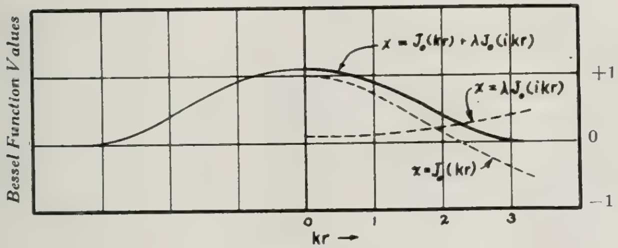

FIGURE 9

Amplitudes Along a Diameter of a Vibrating Diafram

maximum amplitude of a point at a distance r from the center of the diafram, the relation is given by

$$x_m = P[J_o(\mathrm{kr}) + \lambda J_o(i\,\mathrm{kr})] \qquad \text{cms.} \qquad (6)$$

where J_o is the Bessel's function of the zeroth order; k is a constant involving frequency of vibration, density, thickness and elasticity of the diafram; λ is a constant satisfying the boundary conditions; $i = \sqrt{-1}$; and P is the constant of amplitude ratio.

The equivalent mass as already given is found by using this

[8] Bibliography, 3, page 122.
[9] Bibliography, 3, page 131.
[10] Bibliography, 1, page 352.

equation in connection with the equation defining the equivalent mass of the diafram,[11] which is

$$m = \frac{2\pi\rho}{x_o^2}\int_o^a x_r^2\, \mathrm{r}\, dr \qquad \text{gms.} \qquad (7)$$

where ρ = density, a = radius of diafram, x_o = maximum amplitude at the center and x_r = maximum amplitude at a distance r from the center.

In the vibration of telephone diaframs, the ideal distribution of amplitudes over the surface is departed from because of non-uniformities in density, in clamping, in force application, etc. In many cases, the amplitude at the center is reduced, thereby causing an increase in the equivalent mass; a working value for this may be taken approximately as

$$m = 0.3M \qquad \text{gms.} \qquad (8)$$

While individual cases depart from this value, it has nevertheless proved useful in preliminary work.

The elastic constant, s, of the diafram is the pull at the center in dynes required to produce a deflection of 1 cm. It is given by

$$s = (k\, t^3\, Y)/D^2 \qquad \text{dynes/cm.} \qquad (9)$$

where t = thickness of diafram, D = diameter, and Y = Young's modulus.

If the elastic constant and dimensions of one telephone diafram are known, then the elastic constant of any other telephone diafram of the same material may be found approximately by a simple proportion if its dimensions are given. In one instance, the constants of a telephone diafram were found to be

$m = 1.16$	grams
$D = 5$	cms.
$t = 0.\ 03$	cm.
$s = 40 \times 10^6$	dynes/cm.

The natural frequency of the diafram is obtained from equation (3) when the vibrational velocity is a maximum. This occurs when

$$\omega_o^2 = \frac{s}{m} \qquad \text{(rad./sec.)}^2 \qquad (10)$$

where $\omega_o = 2\pi n_o$

(n_o = the natural frequency of the diafram.)

Thus

$$n_o = \frac{1}{2\pi}\sqrt{\frac{s}{m}} \qquad \text{cycles/sec.} \qquad (11)$$

[11] Bibliography, 3, page 131.

In the case of the diafram cited, the natural frequency is

$$n = 935 \sim$$

Means are thus provided for giving an approximate idea of the value of the natural frequency of any telephone diafram when it is clamped in the usual way in a telephone receiver.

At this point it may be well to draw attention to the fact that diaframs do not always vibrate in a simple way but are often affected by non-uniformities in clamping which introduce vibrational systems superimposed upon that of the diafram. If a part of the boundary is not tightly clamped, the effect upon the vibration of the diafram has been found[12] to be the same as tho a piece of spring brass were attached to the center of the diafram, one end of the spring strip being left free to vibrate with the diafram. If the motional-impedance circle of the receiver with this spring attached to the diafram is taken, it is found that a re-entrant loop occurs at the frequency corresponding to the natural frequency of the attached spring; see Figure 10. This re-entrant loop occurs also in the velocity circle, as has been determined experimentally. If the natural frequency of this spring should be that of the diafram itself, then the amplitude at resonance, which should be the largest, would collapse to a value much smaller, the magnitude of the shrinkage depending upon the importance of the superimposed vibrational system; see Figure 11. The motional constants, m, r, and s, of this superimposed system may be computed in a way similar to that for the constants of the diafram. This superimposed vibrational system is related to the vibratory motion of the diafram somewhat as an infinite impedance loop is related to the alternating electric current circuit in which it is placed.

The mechanical resistance constant, r, of the telephone diafram is important as a regulator of the sharpness of tuning and resonant amplitude. The relation between the decrement and the resistance constant of the diafram is found from the well-known solution[13] of equation (2) when the right side is zero. This may be written:

$$\dot{x} = \frac{X\,\omega_o^2}{\omega_r}\,\varepsilon^{-\frac{r}{2m}t}\sin\omega_r t \qquad \text{cm./sec.}\angle \qquad (12)$$

$$= \frac{X\,\omega_o^2}{\omega_r}\,\varepsilon^{-\Delta t}\sin\omega_r t \qquad \text{cm./sec.}\angle \qquad (13)$$

[12]Bibliography, 5, page 434.
[13]Bibliography, 6, page 122.

where X = initial displacement; $\omega_o = 2\pi n_o$ where n_o = natural frequency for forced oscillations; and $\omega_r = 2\pi n_r$, where n_r = natural frequency for free oscillations.

Equations (12) and (13) indicate the relation between the logarithmic decrement[14] per second, Δ, and the resistance constant, r, which is

$$\Delta = \frac{r}{2m} \qquad \text{hyp. rad./sec.} \quad (14)$$

From equation (1),

$$\frac{r}{m} = 2\pi(n_2 - n_1) \qquad \text{cycles/sec.} \quad (15)$$

The range of resonance is defined as that portion of the peak of the resonance curve where the vibrational energy of the diafram is equal to or greater than one-half that at resonance;

[14] Following is a simple derivation:

$$\dot{x} = \dot{x}_o \varepsilon^{-\frac{r}{2m}t} \sin \omega_r t$$

where $\dot{x} = \dfrac{X\omega_o^2}{\omega_r}$ = the initial maximum vibrational velocity.

$$\omega_o = \sqrt{\frac{s}{m}} = \text{resonance angular velocity of forced oscillations.}$$

and

$$\omega_r = \sqrt{\frac{s}{m} - \frac{r^2}{4m^2}} = \text{resonance angular velocity of free oscillations.}$$

$$= \sqrt{\omega_o^2 - \Delta^2}$$

where $\Delta = \dfrac{r}{2m}$.

Let $\dot{x}_1, \dot{x}_2, \ldots$ be the values of $\dot{x}$ at times

$\dfrac{T}{4}, \dfrac{5T}{4}, \ldots$

where $T = \dfrac{2\pi}{\sqrt{\omega_o^2 - \Delta^2}}$ = periodic time.

These values of $\dot{x}$ will be the successive maximum velocities of the oscillatory motion in the same direction; and

$$\dot{x}_1 = \dot{x}_o \varepsilon^{-\frac{rT}{8m}}$$

$$\dot{x}_2 = \dot{x}_o \varepsilon^{-\frac{5rT}{8m}}$$

. . . .

From these equations, we have

$$\log \frac{\dot{x}_1}{\dot{x}_2} = \frac{rT}{2m}$$

This quantity, $\dfrac{rT}{2m}$, is the well-known logarithmic decrement. Dividing by time gives $\dfrac{r}{2m} = \Delta$, which is the damping factor, or logarithmic decrement per second. It is observed that the difference between the squares of the resonance angular velocities of forced and free oscillations is equal to the logarithmic decrement per second, Δ.

thus the range of resonance is the frequency interval defined by $(n_2 - n_1)$[15]. The breadth of tuning is given by[15] $\frac{n_2 - n_1}{n_o}$. By comparing this expression with equation (15), it is seen that the breadth of tuning for a given natural frequency depends upon the ratio of the mechanical resistance, r, and the equivalent mass, m, of the diafram.[16]

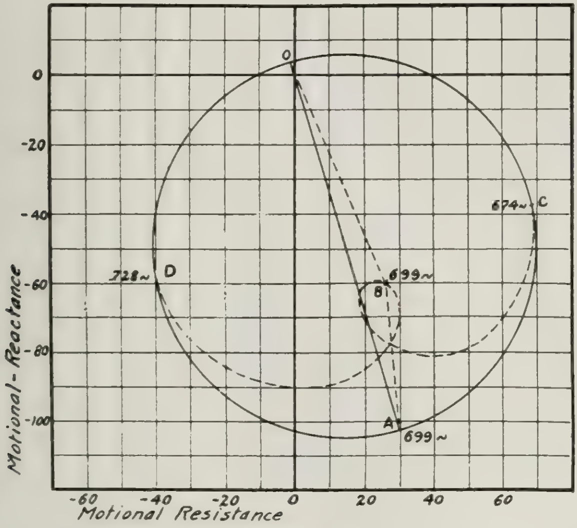

FIGURE 10

Distorted Motional-Impedance Circle Re-entrant Loop

The general effect of the thickness and radius of the diafram upon the breadth of tuning are indicated in Figures 12 and 13, respectively. The assumption will be made that r varies simply as the area of the diafram; this is a rough approximation. Referring to Figure 12, let

$$n_o = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

represent the natural frequency of the diafram of thickness, t; then, for a thickness, $\frac{t}{2}$, we have, from equation (9),

$$n_o' = \frac{1}{2\pi}\sqrt{\frac{s}{8} \div \frac{m}{2}} = \frac{1}{2} \times \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

or the natural tone is lowered by one octave when the thickness

[15] Bibliography, 5, page 449.
[16] Bibliography, 7, pages 15–16.

is reduced one-half while the resonance velocity $\dot{x}=F/r$ remains the same (within our approximation).

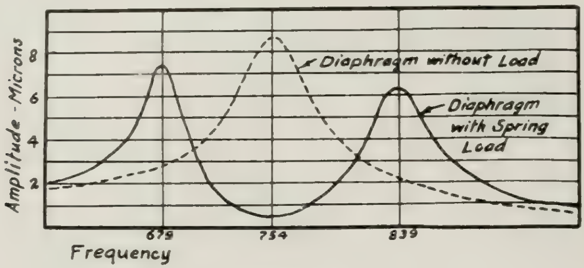

FIGURE 11
Vibration of Diafram with Spring Load

The breadth of tuning is given by

$$(n_2-n_1)=\frac{r}{2\pi m}$$

for the diafram of thickness, t; then, by making the thickness $\frac{t}{2}$, the breadth of tuning becomes

$$(n_2-n_1)=\frac{r}{2\pi\frac{m}{2}}=2\times\frac{r}{2\pi m},$$

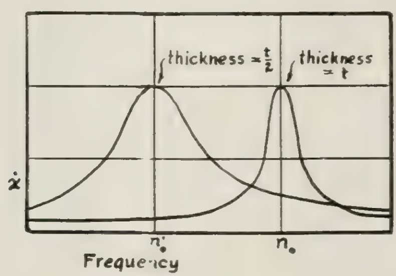

FIGURE 12
Effect of Change in Diafram Thickness on Resonance and Breadth of Tuning

or the breadth of tuning is doubled when the thickness is reduced one-half. Thus, the thinner the diafram, the broader the tuning and the lower the natural tone.

Referring to Figure 13, let

$n_o = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$ represent the natural frequency of the diafram of diameter, D;

then, for a diameter of $\frac{D}{2}$, we have from equation (9),

$$n_o' = \frac{1}{2\pi}\sqrt{\frac{4\,s}{m/4}} = 4 \times \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

or the natural tone is raised by two octaves when the diameter

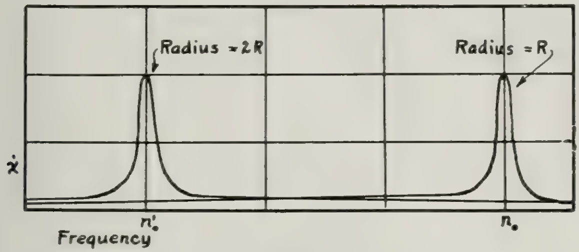

FIGURE 13

Effect of Change in Radius of Diafram on Resonance and Breadth of Tuning

is reduced one-half. The breadth of tuning is unaffected by the change in the diameter for, in this case, r is reduced to $\frac{r}{4}$, and m to $\frac{m}{4}$, thus leaving $(n_2 - n_1)$ unchanged (within our approximation). The sharpness of tuning, however, has decreased since in the expression for it, $\frac{n_o'}{n_2 - n_1}$, n_o' is four times as large as n_o.

The change of resonance velocity with change of r may be deduced from equation (3). If the maximum velocity at resonance for a diafram of diameter, D, is given by

$$\dot{x}_{max} = \frac{F}{r} \qquad \text{cm./sec.} \qquad (16)$$

then, for a diafram of diameter, $\frac{D}{2}$, elasticity remaining constant, this velocity would be given by

$$\dot{x}'_{max} = \frac{4F}{r} \qquad \text{cm./sec.} \qquad (17)$$

That is, when the diameter is reduced one-half, the resistance would be reduced one-fourth, and the velocity would be four

times as great; but, by equation (9), the change in diameter increases the elastic constant by 4 and thus reduces the resonance velocity to one-fourth. The combined effects of the resistance and elastic constants on the maximum resonance velocity of the diafram thus leave this velocity unchanged for all diameters of the diafram (within our approximation). This result indicates that diafram damping and elasticity offers a field for more work.

It should be pointed out that, besides the assumption of a constant mechanical resistance, r, for various thicknesses, t, of the diafram, some other contributing causes also render the curves of Figures 12 and 13 of value only as approximations. Among these are: the increasing membraneous quality in the diafram as it becomes thinner; the change in the magnetic distribution as the diafram becomes increasingly concave; and the resulting change in the air gap which produces corresponding changes in the eddy currents present in the diafram. In practice, these curves may be more nearly realized if the air gap between the diafram and pole pieces is kept constant for all diafram thicknesses by means of the proper adjustment of the distance between the pole pieces and the plane of the clamping ring upon which the diafram rests. Since a telephone diafram is at all times subjected to the pull of the magnets, it possesses more or less the characteristics of a stretched membrane, and these characteristics increase as the thickness of the diafram decreases. The fundamental frequency of a stretched membrane depends, among other things, upon the tension in its surface, while the corresponding determining factor of a diafram is its elasticity. It is thus quite evident that a complete change in the relations which determine the natural frequency and breadth of tuning may ensue as the diafram is made increasingly thinner and the natural period becomes a function of the size of the air gap and the strength of the magnetic pull. The curves of Figure 12 are more nearly true for diaframs having a minimum thickness of about 0.02 cms.

Sensitiveness

The sensitiveness of a telephone receiver has to do with the intensity of sound received for a given impressed vibratory voltage. The pull on the diafram for a given air gap is proportional to the ampere-turns and to the flux density in the air gap due to the permanent magnet, and inversely proportional to the magnetic reluctance. The variation of these quantities is

limited by considerations of space, weight, etc., but they indicate one line of attack (the electrical) in designing telephone receivers when sensitiveness requires attention.

The sound should reach the ear under conditions which will concentrate it with a minimum of loss. The intensity of sound will then vary approximately as the square of the amplitude of diafram vibration and as the area of the diafram.

If selectiveness of tuning is being carried out in the design, the acoustical resonance of the chamber between the ear and receiver cap may be advantageously considered.

The electrical, mechanical and acoustical lines of attack afford a considerable range of variation in the adjustment of the sensitiveness of the telephone receiver.

Audibility

A test of the sensitiveness of the telephone receiver is the audibility current at resonance. For the same size of diafram, length of air gap, and construction of cap, the intensity of sound received is proportional to the square of the amplitude of vibration of the diafram. Audibility in radio work, however, has come to be considered as proportional to the first power of the amplitude rather than to its square. For a given telephone receiver and at a given frequency, the amplitude of diafram vibration is proportional to the current in the coils.

The ear is not a trustworthy indicating instrument for measuring audibility but it is the one most used. This of course is a transient state of the art. When audibility measurements become vital, precision instruments, either null or deflection, will be devised from the means now waiting to be utilized.

Careful observations of the amplitude of vibration of the telephone diafram were made by Shaw[17] for various audibilities. While he found a difference in the sensitiveness of the two ears, the minimum audible amplitude at the center of a diafram on a clamping circle 5 centimeters (2 inches) in diameter may be taken as 7×10^{-8} cm. (3×10^{-8} inch). Shaw's table includes four degrees of audibility, namely: (1) minimum audibility, expectant ear; (2) minimum audibility, unexpectant ear; (3) loud; (4) overpowering; having the relation, 1 : 70 : 1400 : 7000.

Following is a table in which these measurements are applied to four telephone receivers, *A*, *B*, *C*, and *D*, which were studied by Kennelly and Affel[18] and a description of which appears in their paper.

[17] Bibliography, 8, pages 360–366.
[18] Bibliography, 4, page 449.

	A	B	C	D
Amplitude at Resonance	7.53×10^{-4} cm.	7.19×10^{-4} cm.	10.35×10^{-4} cm.	6.64×10^{-4}
Current at Resonance	0.00202 amps.	0.002 amps.	0.00204 amps.	0.00116 amps.
Audibility at Resonance	10760	10280	14780	9480
Min. Aud. Current, Expectant Ear	1.878×10^{-4} milliamps.	1.948×10^{-4} milliamps.	1.38×10^{-4} milliamps.	1.224×10^{-4} milliamps.
Min. Aud. Current, Unexpectant Ear	0.01342 milliamps.	0.01389 milliamps.	0.00985 milliamps.	0.00874 milliamps.
Current, Loud Sound	0.268 milliamps.	0.278 milliamps.	0.197 milliamps.	0.1749 milliamps.
Current, Overpowering Sound	1.342 milliamps.	1.389 milliamps.	0.985 milliamps.	0.874 milliamps.

Table of Audibility and Audibility Current of Four Telephone Receivers
Four grades of audibility are given

Receiver *D*, the most sensitive, was of the watch case type with a resistance of over 1,000 ohms, while the three others were the desk type, having a resistance of less than 100 ohms. The minimum audible amplitude for the expectant ear, 7×10^{-8}, would seem to be the logical starting point for a scale of audibility measurements.

Some Types of Receivers

The following four types of receivers give points of difference which are of interest:

(1) Monopolar	
(2) Bipolar	ferrotype diafram
(3) Monopolar, with boundary return	
(4) Quadrapolar	mica diafram

(1) In the monopolar receiver, the force is active at the center of the diafram, and the magnetic return is thru air. The equivalent mass of the diafram is thus relatively small (a good point), and the magnetic reluctance relatively large (a weak point).

(2) In the bipolar receiver, the force is active at points on the diafram removed from the center, and the air gaps in the magnetic path are small. The equivalent mass of the diafram is thus relatively large (a weakness), and the magnetic reluctance small (a good point).

(3) In the monopolar, with boundary return, the force is active at the center of the diafram, and the air gaps in the magnetic path are small. These points are both favorable for sensitiveness, but the return flux at the boundary is not utilized in force action as in the bipolar.

(4) In this type of receiver, the force is applied at the center of the diafram by means of a connecting rod from the vibrating armature which is suspended between the two sets of opposite magnetic poles, and the air gaps in the magnetic path are small, also the return flux is partially utilized in force action.[19]

In most telephones there is a magnetic amplifying action due to the dependence of the magnetic reluctance, in this case, upon the air gap. As the reluctance is a function, among

[19] Since writing this classification, a new telephone has appeared which constitutes the fifth class, the characteristic features being two poles of permanent magnets bridged by a soft iron solenoid core, the magnetic return path being thru a third pole which is an armature operating between the two poles. A rod connects the armature to the center of the diafram. The equivalent mass of the diafram is small, the return magnetic flux is in iron and the air gaps are small, and both air gaps are utilized in the force action. The electro-magnetic features are thus all good. The magnetic amplifying principle utilized in this telephone makes it a very sensitive receiver.

other things, of the plan of the magnetic path as well as of the precision realized in the mechanical construction of the parts, this amplification varies from one telephone to another.

This brief outline merely indicates a line of analysis which may be extended to include many other characteristics of telephone receivers.

Telephones are used for a variety of purposes, and a knowledge of the elements involved, electrical, mechanical and acoustical, make possible a design which will be the most suitable for a given purpose. In radio work, the telephone occupies a most important place, and the particular application requires a peculiar construction in which size, weight, selectivity, sensitiveness, etc., are determining factors.

SUMMARY: The motional-impedance circle of telephone receivers is explained, the experimental methods of determining it are given, and the equation of motion of the diafram is derived. The effect of imperfect clamping of the diafram is studied.

The influence of thickness, radius, and elasticity of the diafram on its natural period are considered.

The electrical, mechanical, and acoustic methods of improving receiver sensitiveness are given. The significance of audibility measurements is next treated; together with suggested improvements in the direction of quantitative measurement of receiver response.

The characteristic advantages and disadvantages of monopolar, bipolar, and quadrapolar receivers are given, followed by a partial bibliography of receiver investigations.

TABLE OF SYMBOLS

α	Phase angle	radians
D	Diameter of diafram	cms.
Δ	Logarithmic decrement per second	hyp. rad./sec.
ε	Naperian logarithmic base	numeric
F	Maximum pull on diafram	dynes
f	$(=F \sin \omega t)$ Instantaneous pull on diafram	dynes $\angle$
i	$(=\sqrt{-1})$	numeric
J_o	Bessel's function of zeroth order	numeric
k	Proportionality constant	numeric
k	Constant involving diafram dimensions	cms.$^{-1}$
λ	Constant satisfying boundary conditions	numeric
M	Diafram mass within clamping ring	gms.
m	Equivalent mass of diafram	gms.
n	Frequency of vibration	cycles/sec.
n_o	Resonance frequency	cycles/sec.
n_o'	Natural frequency (size of diafram varied)	cycles/sec.
n_1	Frequency of quadrantal point below res.	cycles/sec.
n_2	Frequency of quadrantal point above res.	cycles/sec.
n_r	Natural frequency for free oscillations	cycles/sec.
ω	$(=2\pi n)$	rad./sec.
ω_o	Resonance angular velocity	rad./sec.
ω_r	Resonance angular vel., free oscil.	rad./sec.
P	Constant of amplitude ratio	cms.
π	3.1416	numeric
r	Mechanical resistance constant of diafram	dynes/kine
r	Distance from center of diafram	cms.
ρ	Density of diafram material	gms./cm.^{3}
s	Elastic constant of diafram	dynes/cm.
T	Periodic time	secs.
t	Thickness of diafram	cms.
t	Elapsed time in vibratory motion	secs.
X	Initial displacement of diafram center	cms.
x	Instantaneous displacement diafram center	cms. $\angle$
x_o	Initial amplitude	cms.
x_m	Max. amplitude r cms. from center	cms.
x_r	Amplitude at r from center	cms.

$\dot{x}$	Velocity at center of diafram	cms./sec. ∠
$\dot{x}'_{max}$	Max. vel. at res. (size of diafram varied)	cms./sec.
$\dot{x}_{max}$	Maximum velocity at resonance	cms./sec.
$\dot{x}_o$	Initial max. vibrational velocity	cms./sec.
$\ddot{x}$	Acceleration at center of diafram	cms./sec.^{2} ∠
Y	Young's modulus	dynes/cm.^{2}
∽	Cycles per second	
∠	Indicates complex quantity	

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CONTENTS

The Officers and the Committees of the Institute for 1918 will appear in a forthcoming issue.

OSCILLATING AUDION CIRCUITS*

By

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1. Introduction and General Principles.—The purposes of this paper are to explain the general principles upon which depend the use of the audion for generating electrical oscillations and for amplifying externally impressed oscillations by "regenerative action," and to discuss in detail the action in certain circuits.[1] It will be shown that the criterion for the generation of an oscillation and for the measure of its intensity is directly determinable from the constants of the circuit and from the characteristics of the audion.

The term "audion," in accordance with common usage, is applied to an evacuated bulb having three electrodes: a *filament*, maintained at incandescence by a heating current; a *plate* or wing; and a *grid* interposed between the filament and the plate. These three electrodes are electrically connected to an external circuit, which includes a battery so arranged as to make the filament a cathode (negative) and the plate an anode (positive) for a thermionic discharge thru the bulb. The grid may be considered an auxiliary anode whose purpose is to control the main discharge to the plate by variations in its potential relative to the filament. The device may thus be properly described as a *thermionic relay*.

The physical action in the audion is briefly as follows: The incandescent filament gives off electrons (negatively charged particles) at a rate depending on its temperature. If no voltage

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[1]The author takes this opportunity to acknowledge his great indebtedness to Mr. E. H. Armstrong who has done the pioneer work with the oscillating audion. The fundamental principles and various circuits in which they are practically applied are given in Mr. Armstrong's article in the "Electrical World" for December, 1914 (volume 64, number 24, page 1149) and in his paper presented before The Institute of Radio Engineers in March, 1915 ("Proc. I. R. E.", volume 3, page 215).

[2]Langmuir, "Proc. I. R. E.", volume 3, page 261 (1915). This paper goes fully into the physical action of thermionic conduction and its application.

were present to draw away these electrons, they would be driven back to the filament by the negative potential which they produce in the surrounding space. However, the positive potential of the plate will attract the electrons and cause a continuous stream to flow from the filament to the plate, constituting a "thermionic current." Since the grid lies in the path of the electrons, its potential will have even more effect on the electron stream than will that of the plate, a positive grid potential increasing the plate current, a negative grid potential decreasing it. In bulbs having a high vacuum the relation of the plate current to the plate and grid potentials follows simple and definite laws;[2] but in ordinary bulbs, the residual gas molecules are ionized by the impact of the electrons and have an important effect on the current, causing a wide variation in the characteristics of bulbs of identical form but of accidental difference in the kind and amount of residual gas.

Independently of its use as a relay, the audion also serves as a *rectifier*, which is essential to its use as a radio detector. This rectifying property affects the oscillations only indirectly, and will not be particularly considered here.

The relay action of the audion is illustrated by its *characteristic curve*, *M N*, Figure 1, showing the relation between the plate current and the potential of the grid relative to the filament. An audion has one such characteristic curve for every combination of filament temperature and plate potential relative to the filament. The effectiveness of the audion as a relay depends primarily on the *slope* of the characteristic curve. This slope, being the quotient of a current by a voltage associated therewith, is of the dimensions of a conductance and may be called the *mutual conductance* of the grid toward the plate. For any small change in grid potential, the effective mutual conductance is the slope of the tangent to the characteristic curve at the point corresponding to the actual grid potential; while if the grid potential is varying periodically, the effective mutual conductance may be considered the slope of the secant line connecting the points corresponding to highest and lowest grid potential.[3] Evidently the mutual conductance has a maximum

[3]A more accurate expression for the effective mutual conductance in this case may be found by assuming the variations in grid potential to be sinusoidal and plotting the corresponding wave of plate current against time. The quotient of the fundamental harmonic component of the plate current by the grid potential is then the mutual conductance. This refinement hardly seems necessary in any practical case, especially as the "dynamic characteristic" of the audion may differ from the "static characteristic" and may be difficult to determine.

at the point of inflection P, Figure 1, and in general decreases with increasing amplitude of oscillation.

In the following discussion we will deal only with the *variations* in the plate current, grid potential, etc., from their mean values during the oscillation. In other words, the pulsating currents and voltages will each be resolved into a continuous

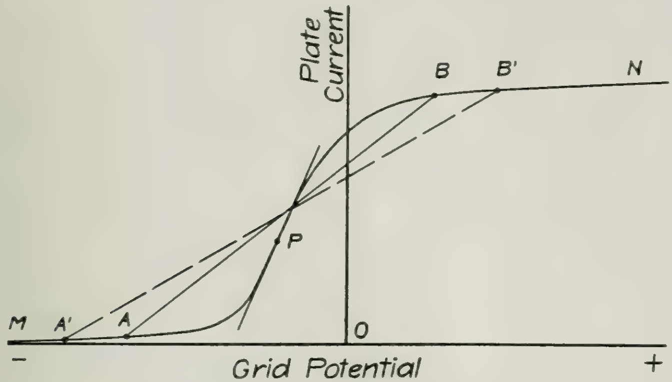


FIGURE 1

part and an alternating (or oscillating) part; and the latter will be treated by the usual methods of the alternating-current circuit. For simplicity, also, the typical circuit diagrams will show only the elements essential to the oscillating current, omitting the heating battery, plate-circuit battery, stopping condenser, telephone receivers, etc. The oscillating voltage tending to send current *from* the filament *to* the grid will, for brevity, be called the "grid voltage"; the oscillating voltage tending to send current from the plate to the filament will be called the "plate voltage"; and the oscillating current from the plate will be called the "plate current." These quantities, in their positive senses, are represented in Figure 2, their virtual (or r. m. s.) values being denoted by E_g, E_p and I_p respectively.

If at any instant of time the grid voltage is such as to make the grid negative with respect to the filament, current will tend to flow from the filament to the grid, as represented by the dotted arrow in Figure 2. But, by virtue of the relay action, current is caused to flow from the filament to the *plate*, as represented by the full arrow, and as given by the equation,

$$I_p = E_g g, \tag{1}$$

where g is the mutual conductance of the grid toward the plate. If at the chosen instant the plate is positive with respect to the filament, there will be an *output* of power from the audion, equal to

$$P = E_p I_p = E_p E_g g. \tag{2}$$

If the audion is to act as an oscillating-current generator, the required value of g is then given by the relation,

$$g = \frac{P}{E_g E_p}, \tag{3}$$

where P represents the oscillating-current power supplied by the audion. The criterion for the generation of oscillating currents thru the relay action of the audion is thus as follows:

The audion must be so connected to the oscillating-current

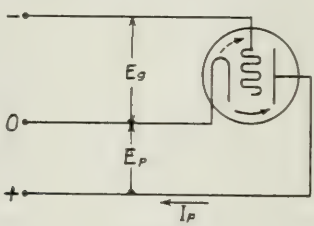


FIGURE 2

circuit that the grid and the plate are of opposite polarities with respect to the filament; and the quotient of the total oscillating-current power by the product of the grid voltage and the plate voltage (or, if these voltages are not in phase, the product of one by the in-phase component of the other) must be equal to a possible value of the mutual conductance of the audion. The smaller this quotient $\frac{P}{E_g E_p}$, the more easily will the audion oscillate, or—with a given setting of the audion—the stronger will be the oscillation[4]. If the quotient $\frac{P}{E_g E_p}$ is greater than the value of g obtainable from the audion, no free oscillation will occur; but

[4]An oscillation will be stable only if the characteristic curve is such that a greater range in grid potential corresponds to a smaller slope of the secant line (i. e., $A'B'$, Figure 1, has a smaller slope than AB). Otherwise the oscillation will increase until this condition obtains. This effect is analogous to the stability or instability of a self-excited generator, which also depends on the slope of a characteristic curve.

an externally impressed oscillation which causes the grid and plate to be of opposite polarities may be greatly amplified; this has been called by Armstrong "regenerative action."

If an externally impressed oscillation causes the grid and plate to be of the *same* polarity, its energy will be *absorbed* by the audion and it will be weakened or more rapidly damped out. The audion may thus be used to prevent or diminish undesirable oscillations.

The above discussion neglects two features which may affect the oscillations:

First, the plate current depends somewhat on the plate voltage as well as on the grid voltage, and so should be represented by $(I_p = E_g g - E_p g_p)$, instead of (1). This means that in the denominator of (3), and all equations derived therefrom, we should strictly introduce the factor $\left(1 - \frac{E_p g_p}{E_g g}\right)$; but as the ratio $\frac{g_p}{g}$ is small in most designs of audion[5], this factor may usually be neglected unless $\frac{E_p}{E_g}$ is large.

Second, there will be a current in the grid circuit, due to electrons or positive ions reaching the grid. This current is usually small, and may always be neglected in high-vacuum audions when the grid is negative. In audions having considerable positive ionization, the grid current may assist in producing oscillations[6].

Equation (3) shows the plate potential and grid potential to be interchangeable; so these electrodes may be interchanged in their connection to an oscillating-current circuit. Whether connections should be made so that E_g is greater than E_p or the reverse depends on the following considerations: The value of g for given adjustments of the audion depends on E_g, being smaller

[5]Langmuir, "Proc. I. R. E.", volume 3, page 279.

[6]This action is due to the fact that within a certain range, the grid current *decreases* with increasing grid potential. The grid circuit thus has a characteristic curve like that of the electric arc and, like the arc, may be used to produce oscillations. The following explanation for the decrease in grid current with increase in grid potential (this being negative) is offered: When the potential of the grid is started from zero and made more and more negative, it will at first cause more positive ions to flow to it and so increase the current; but as the flow of electrons from the filament is diminished by the negative potential of the grid, fewer gas molecules will be ionized and hence the number of positive ions reaching the grid will ultimately fall and decrease the grid current.

Positive ionization may also cause an oscillation in the plate circuit, due to alternate breakdown and recovery of the gas. This may occur in periods of several seconds or at audio frequencies.

for higher values of E_g. Hence if the highest possible value of E_g is desired, for a given energy of oscillation (as in radio receiving), E_g should be made large relative to E_p; while if the greatest possible energy of oscillation is desired (as in radio sending), E_g should be made small relative to E_p. A limitation in the latter case is imposed by the effect of E_p on the plate current, as explained above.

The derivation of the equations of oscillation in particular cases is given in the articles immediately following. In most radio-frequency oscillating-current circuits the resistances of the branches are so small in comparison with the reactances that only the latter need be considered in finding the current distribution. For a similar reason the plate current of the audion may be neglected in comparison with the main oscillating currents. With these assumptions, all currents will be in phase with each other and may be calculated as in a direct-current circuit. The current of each branch may then be squared and multiplied by the resistance of that branch to give the power loss; and the total power loss may be substituted in equation (3): this is here called the "loss method." In certain cases where the resistances are high or the coupling between circuits very loose, or where high accuracy is desired, both resistances and reactances must be considered together. This is best done by the use of complex quantities, and is here called the "complex method" (Article 4).

2. Simple Oscillating-Current Circuits Treated By the Loss Method.—An elementary oscillating-current circuit contains a single coil connected to a single condenser, as represented by (C, L), Figure 3a. Any accidental disturbance in this circuit will set up a free oscillation whose angular frequency ω is such as to make the joint reactance zero:

$$\omega L - \frac{1}{\omega C} = 0, \tag{4}$$

or

$$\frac{1}{\omega^2} = CL. \tag{5}$$

If the loss in the effective resistance r were not supplied in some manner, this oscillation would die out according to the familiar exponential law. However, the alternation of the current in L induces a voltage in the grid coil by virtue of the mutual inductance M_g. This causes an alternating current in the plate circuit which flows against the voltage induced in the plate coil (if this is properly connected) and so supplies power to

maintain the oscillation. This power is, of course, transferred to the coil L by the mutual inductance M_p.

With the notation indicated in Figure 3a, the grid voltage is

$$E_g = I\,\omega\,M_g; \tag{6}$$

and the plate voltage is

$$E_p = I\,\omega\,M_p. \tag{7}$$

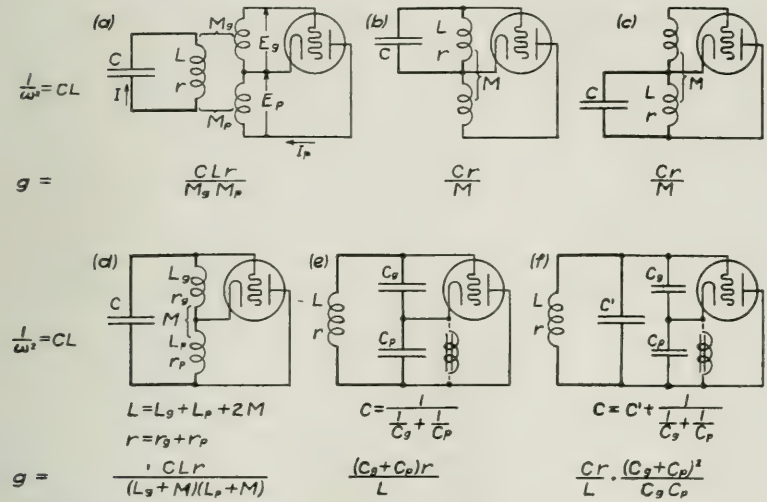


FIGURE 3

The loss in the oscillating-current circuit is

$$P = I^2 r. \tag{8}$$

Substituting in (3), the value of g necessary to maintain an oscillation is

$$g = \frac{P}{E_g E_p} = \frac{I^2 r}{I\,\omega\,M_g \cdot I\,\omega\,M_p} = \frac{r}{\omega^2 M_g M_p}. \tag{9}$$

Substituting the value of ω from (5), we have also

$$g = \frac{C L r}{M_g M_p}. \tag{10}$$

In the above equations r represents the effective resistance of the coil and condenser in series, and may be considered constant only when the main loss is due to the direct-current resistance of the coil. Instead of expressing the loss in terms of a

series resistance r, we may express it in terms of an effective shunt conductance g', related to r by the equation,

$$r = g' \omega^2 L^2 = \frac{g' L}{C}. \tag{11}$$

Substituting in (10),

$$g = \frac{g' L^2}{M_g M_p}. \tag{12}$$

The conductance g' may be considered constant when the main loss is due to true leakage in the condenser. It will also be constant when the main loss is due to eddy currents in the coil; for this eddy-current loss is proportional to the square of the frequency (for a given working current and negligible skin effect), and so makes r proportional to ω^2 and thence g' constant in (11).

When the main loss is in a dielectric, various experiments have shown the power factor to be approximately constant. The corresponding effective resistance is then

$$r = p \omega L = p \sqrt{\frac{L}{C}}, \tag{13}$$

where p is the power factor of the capacity. Substituting in (10),

$$g = \frac{p \sqrt{C L^3}}{M_g M_p}. \tag{14}$$

Suppose that the loss is made up of three parts, that due to direct-current resistance, that due to eddy currents, and the loss in the dielectric of the coil itself. The total loss may then be expressed as

$$P = I^2 r + I^2 g' \frac{L}{C} + I^2 \frac{C'}{C} p \sqrt{\frac{L}{C}}, \tag{15}$$

where g' represents the effective conductance due to eddy currents, and where C' represents that part of the total capacity associated with the dielectric of the coil and having the power factor p. Substituting in (3),

$$g = \frac{C L r + g' L^2 + p C' \sqrt{\frac{L^3}{C}}}{M_g M_p}. \tag{16}$$

If the frequency is varied by changing C, this expression has a minimum value when

$$C L r = \frac{1}{2} p C' \sqrt{\frac{L^3}{C}}, \tag{17}$$

or, in words, when the dielectric loss is twice that due to direct-current resistance. The author has found experimentally that there is a particular frequency at which the oscillation of such a coil is the strongest, verifying in a general way the above conclusion. Evidently the effect of eddy-current loss on the strength of an oscillation is the same at all frequencies.

The circuit of Figure 3a may be modified in various ways, as indicated in Figures 3b to 3f. The main oscillating-current circuit in each case is equivalent to a single coil connected to a single condenser; so the frequency is given by equation (5), in which C is the joint capacity of all condensers and L the joint self-inductance of all coils.

Figure 3b is derived from Figure 3a by combining the main coil and the grid coil into one. The value of g is then given by putting $(M_g = L)$ and $(M_p = M)$ in equation (10):

$$g = \frac{C\,r}{M}. \tag{18}$$

Similar substitutions lead to the same equation for Figure 3c.

In Figure 3d the effective mutual inductance M_g between the grid coil alone and the two coils together is $(L_g + M)$, and similarly for M_p. Making these substitutions in (10),

$$g = \frac{C\,L\,r}{(L_g + M)\,(L_p + M)}. \tag{19}$$

Evidently the mutual inductance between the coils may be omitted.

In Figure 3e or 3f the total voltage is $I\omega L$, which is divided between the grid and plate condensers in inverse proportion to their capacities—

$$E_g = I\,\omega L\,\frac{C_p}{C_g + C_p} \text{ and } E_p = I\,\omega L\,\frac{C_g}{C_g + C_p}. \tag{20}$$

Substituting these equations with (8) and (5) in (3),

$$g = \frac{r}{\omega^2 L^2 \dfrac{C_g C_p}{(C_g + C_p)^2}} = \frac{C\,r}{L} \cdot \frac{(C_g + C_p)^2}{C_g C_p}. \tag{21}$$

For Figure 3e, we may substitute for the joint capacity C in this equation the value $\dfrac{C_g C_p}{C_g + C_p}$, giving

$$g = \frac{(C_g + C_p)\,r}{L}. \tag{22}$$

In the circuits of Figures 3a to 3d, the two coils connected

to the filament may evidently be parts of a single coil having an intermediate tap. The oscillation will occur most easily when this tap is such as to make E_g and E_p approximately equal. This will not usually be the most desirable place, however, as explained in the next to last paragraph of the preceding article. Similar remarks apply to the connection between the condensers in Figures 3e and 3f. These circuits may be further extended by subdividing the coils and condensers. The essential action will remain the same as long as the main oscillating-current circuit is electrically equivalent to a single coil connected to a single condenser; for any such circuit is characterized by the fact that it permits of an oscillation at but a single frequency.

3. Coupled Circuits Treated By the Loss Method.—Two oscillating-current circuits (C_1, L_1) and (C_2, L_2) may be coupled by having mutual inductance M_{12} between their coils, as represented by the heavy lines in Figure 4. A free oscillation in the combined circuit will have such a frequency that the resultant voltage in each separate circuit is zero:

$$I_1\left(\omega L_1-\frac{1}{\omega C_1}\right)+I_2\,\omega\,M_{12}=0; \tag{23}$$

and

$$I_2\left(\omega L_2-\frac{1}{\omega C_2}\right)+I_1\,\omega\,M_{12}=0. \tag{24}$$

Combining these equations,

$$\left(\omega L_1-\frac{1}{\omega C_1}\right)\left(\omega L_2-\frac{1}{\omega C_2}\right)-(\omega\,M_{12})^2=0; \tag{25}$$

and solving for ω,

$$\frac{1}{\omega^2}=\frac{C_1L_1+C_2L_2}{2}\pm\sqrt{\left(\frac{C_1L_1-C_2L_2}{2}\right)^2+C_1C_2M_{12}^2}. \tag{26}$$

The two circuits are said to be "in tune with each other" when their separate natural frequencies are equal, or when

$$C_1L_1=C_2L_2. \tag{27}$$

For this condition of tuning, (26) becomes

$$\frac{1}{\omega^2}=C_1L_1\pm\sqrt{C_1C_2M_{12}^2}. \tag{28}$$

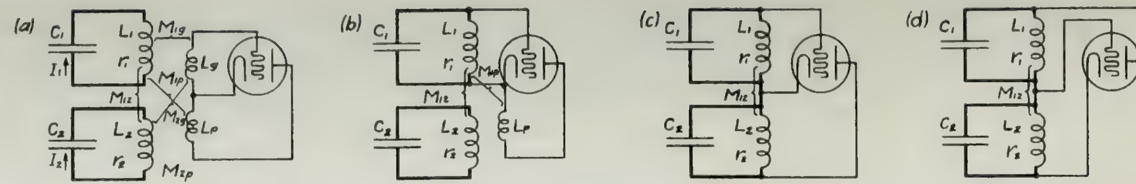
Substituting the "coefficient of coupling,"

$$k_{12}=\frac{M_{12}}{\sqrt{L_1L_2}}, \tag{29}$$

this becomes

$$\frac{1}{\omega^2}=C_1L_1\,(1\pm k_{12})=C_2L_2\,(1\pm k_{12}). \tag{30}$$

Fig. 4



$$\frac{1}{\omega^2} = C_1L_1(1 \pm k_{12}) = C_2L_2(1 \pm k_{12})$$

$$k_{12}^2 = \frac{M_{12}^2}{L_1L_2}$$

$g =$

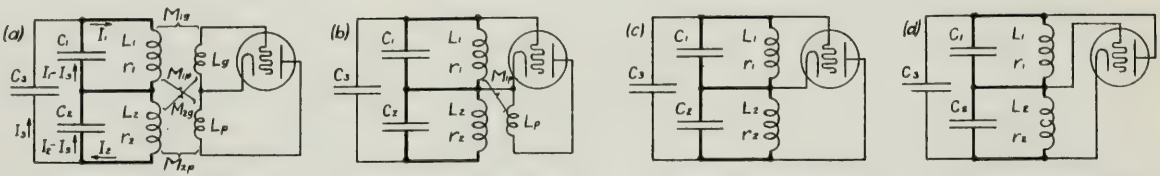
(a) $\dfrac{(C_1r_1+C_2r_2)(1 \pm k_{12})}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$

(b) $\dfrac{C_1r_1 + C_2r_2}{M_{1p}}$

(c) $\dfrac{C_1r_1 + C_2r_2}{\pm\sqrt{L_1L_2}\,(1 \pm k_{12})}$

(d) $\dfrac{C_1r_1 + C_2r_2}{(\mp\sqrt{L_1L_2} - L_2)(1 \pm k_{12})}$

Fig. 5



$$\frac{1}{\omega^2} = (C_1+C_3)L_1(1 \pm k) = (C_2+C_3)L_2(1 \pm k)$$

$$k^2 = \frac{C_3^2}{(C_1+C_3)(C_2+C_3)}$$

$g =$

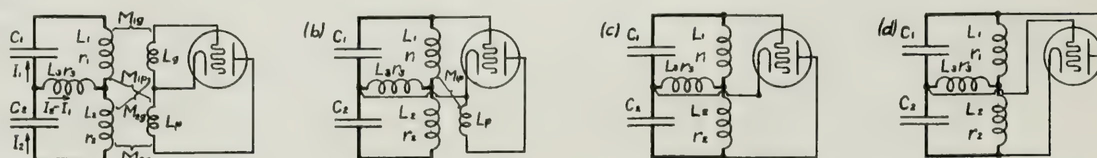
(a) $\dfrac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$

(b) $\dfrac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{M_{1p}}$

(c) $\dfrac{[(C_1+C_3)r_1+(C_2+C_3)r_2][1 \pm k]}{\pm\sqrt{L_1L_2}}$

(d) $\dfrac{[(C_1+C_3)r_1+(C_2+C_3)][1 \pm k]}{\mp\sqrt{L_1L_2} - L_2}$

Fig. 6 *



$$\frac{1}{\omega^2} = C_1(L_1+L_3)(1 \pm k) = C_2(L_2+L_3)(1 \pm k)$$

$$k^2 = \frac{L_3^2}{(L_1+L_3)(L_2+L_3)}$$

$g =$

(a) $\dfrac{[C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3][1 \pm k]}{\sqrt{L_gL_p}\,(k_{1g} \pm k_{2g})(k_{1p} \pm k_{2p})}$

(b) $\dfrac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{M_{1p}}$

(c) $\dfrac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{\mp\sqrt{(L_1+L_3)(L_2+L_3)}\,[1 \pm k]}$

(d) $\dfrac{C_1r_1+C_2r_2+(\sqrt{C_1} \pm \sqrt{C_2})^2 r_3}{[\pm\sqrt{(L_1+L_3)(L_2+L_3)} - (L_2+L_3)][1 \pm k]}$

* In this figure M_{1g} represents the mutual inductance between L_g and the combination of L_1 and L_3; and similarly for M_{2g}, M_{1p} and M_{2p}.

The ratio of currents is, by (23),

$$\frac{I_2}{I_1}=\frac{\omega L_1-\frac{1}{\omega C_1}}{-\omega M_{12}}=\frac{L_1\left(1-\frac{1}{\omega^2 C_1 L_1}\right)}{-M_{12}}; \tag{31a}$$

and by (29) and (30), for the condition of tuning,

$$\frac{I_2}{I_1}=\frac{\mp L_1 k_{12}}{-k_{12}\sqrt{L_1 L_2}}=\pm\sqrt{\frac{L_1}{L_2}}. \tag{31b}$$

The physical significance of the double sign ($\pm$) in the above equations is that a coupled circuit has *two* natural frequencies, and so permits of two independent oscillations. Whether both oscillations will actually occur and, if so, their relative magnitudes depend on the way in which oscillation is started or maintained. The oscillation at the lower frequency has currents flowing in such directions in the two coils as to add in their magnetic effect, giving the equivalent of an increased self-inductance. The oscillation at the higher frequency has currents flowing in such directions in the two coils as to oppose in their magnetic effect, giving the equivalent of a decreased self-inductance. When the circuits are in tune, one natural frequency is thus lower and the other higher than that of each separate circuit; and the difference between the frequencies is the greater, the closer the coupling. Equation (31b) shows that the energies of oscillation in the two circuits are equal $\left(\frac{1}{2}L_1 I_1^2=\frac{1}{2}L_2 I_2^2\right)$.

If an audion is coupled to the main oscillating-current circuit, as in Figure 4a, it will be able to maintain an oscillation if the constants of the circuit are chosen so as to satisfy equation (3). We will take first the general case where the grid and plate circuits are each coupled to both oscillating-current circuits. Assuming the circuits to be in tune, the grid and plate voltages are then respectively, by (31b).

$$E_g=I_1\,\omega\, M_{1g}+I_2\,\omega\, M_{2g}=I_1\,\omega\left(M_{1g}\pm M_{2g}\sqrt{\frac{L_1}{L_2}}\right); \tag{32}$$

and

$$E_p=I_1\,\omega\, M_{1p}+I_2\,\omega\, M_{2p}=I_1\,\omega\left(M_{1p}\pm M_{2p}\sqrt{\frac{L_1}{L_2}}\right). \tag{33}$$

The power loss in the main oscillating-current circuit is

$$P=I_1^2\, r_1+I_2^2\, r_2=I_1^2\left(r_1+r_2\frac{L_1}{L_2}\right). \tag{34}$$

Substituting in (3),

$$g=\frac{P}{E_gE_p}=\frac{r_1+r_2\frac{L_1}{L_2}}{\omega^2\left(M_{1g}\pm M_{2g}\sqrt{\frac{L_1}{L_2}}\right)\left(M_{1p}\pm M_{2p}\sqrt{\frac{L_1}{L_2}}\right)} \tag{35}$$

$$=\frac{\frac{r_1}{L_1}+\frac{r_2}{L_2}}{\omega^2\sqrt{L_gL_p}\,(k_{1g}\pm k_{2g})\,(k_{1p}\pm k_{2p})}, \tag{36}$$

where the k's are coefficients of coupling between the main coils and the audion coils, in fashion analogous to (29). By (30),

$$g=\frac{(C_1r_1+C_2r_2)\,(1\pm k_{12})}{\sqrt{L_gL_p}\,(k_{1g}\pm k_{2g})\,(k_{1p}\pm k_{2p})}. \tag{37}$$

The equation for g when the circuits are not in tune is more complex; it is derived in the same manner, except that equations (31a) and (26) are used instead of (31b) and (30) respectively, to give the ratio of currents and the frequency.

Special modifications of Figure 4a are shown in Figures 4b to 4d, where instead of inductive coupling the audion is sometimes directly connected to the main circuit, giving the equivalent of unit coefficient of coupling. For Figure 4b, we may put in (37)

$$k_{1g}=1,\ \ k_{2g}=k_{12},\ \ k_{2p}=0,\ \text{ and }\ L_g=L_1, \tag{38}$$

giving

$$g=\frac{C_1r_1+C_2r_2}{\sqrt{L_1L_p}\cdot k_{1p}}=\frac{C_1r_1+C_2r_2}{M_{1p}}. \tag{39}$$

This shows (since there is no "$\pm$" sign) that the two possible oscillations occur *with equal ease*, provided the circuits are in tune as assumed above. When the circuits are not in tune, the values of g differ for the two oscillations and only that oscillation actually occurs which corresponds to the lower value of g. The expression for g in this general case may be derived in the same way as above; but it is more instructive to view the matter physically. If the circuit (C_2, L_2) of Figure 4b is far out of tune with (C_1, L_1) and the coupling is loose, its energy of oscillation will be small, since this is all supplied from (C_1, L_1) and not directly from the audion. The oscillation then occurs approximately as if (C_2, L_2) were absent, leaving the simple circuit of Figure 3b. This oscillation is strong, since the loss in (C_2, L_2) is small. If (C_2, L_2) has a low natural frequency, its current will flow as in a short-circuited coil and will oppose

the current of (C_1, L_1), thus decreasing the effective self-inductance and raising the frequency. On the other hand, if (C_2, L_2) has a high natural frequency, its current will flow as in (C_1, L_1), thus increasing the effective self-inductance and lowering the frequency. Hence if the natural frequency of (C_2, L_2) is gradually changed from a low to a high value, the strength of the oscillation will fall as the condition of tuning is approached, after which it will rise again; and, at the instant of tuning, the oscillation will suddenly change from the higher to the lower natural frequency. In practice it is found that an oscillation once started tends to maintain itself with unchanged frequency, especially if it is strong; so the change from one natural frequency to the other does not take place until after the tuning point has been passed. There is then a rapid increase in the strength of the oscillation. During the transition period both oscillations exist and produce beats. If a "stopping condenser" is placed in series with the grid circuit and the proper adjustments are made, these beats may be produced continuously if the circuits are exactly or very nearly in tune.[7]

For Figure 4c, we may put in (37)

$$k_{1g}=1,\ k_{2g}=k_{1p}=k_{12},\ k_{2p}=1,\ L_g=L_1 \text{ and } L_p=L_2, \tag{40}$$

giving

$$g=\pm\frac{C_1 r_1+C_2 r_2}{(1\pm k_{12})\sqrt{L_1 L_2}}. \tag{41}$$

This shows that only one of the two possible oscillations can be maintained by the audion, with a given sense of the coupling, since the other would correspond to a negative value of g. Re-

[7]This continuous production of beats may be explained briefly as follows: Suppose an audion having a stopping condenser in series with the grid is suddenly started oscillating. The oscillations will cause the grid to build up a negative charge, by the same principle of unilateral conductivity that is employed in the simple audion detector. The grid will soon become so negative that the slope of the characteristic (i. e., the value of g) will be insufficient to maintain an oscillation. The energy of the oscillation, however, prevents it from immediately ceasing; so the grid becomes even more negative while the oscillation is dying out. When the oscillation has ceased, the negative charge gradually leaks off the grid, until a point on the characteristic is reached where an oscillation can again start, after which the phenomenon is repeated. We then have an *intermittent oscillation*, the "group frequency" of which depends on the intensity with which the audion tends to oscillate and on the rate at which the negative charge can leak off the stopping condenser. Now if adjustments are made so that the group frequency is near the beat frequency of the coupled circuit, it will assume that frequency and cause continuous beats. This action has been observed at audible oscillation frequencies by the author, as well as at radio frequencies. That the intermittent oscillation and the continuous production of beats occur under similar conditions has been observed by Armstrong ("Proc. I. R. E.," volume 3, page 227); but he does not seem to have noted that one is essentially a consequence of the other.

versing the coupling changes the oscillation from one of the natural frequencies to the other. Let us call that sense of coupling "normal" which would cause the grid and plate to have opposite potentials when current flows in one coil alone [k_{12} positive in (41)]. Then we may say that normal coupling gives the stronger oscillation and the lower frequency, and as the coupling is made closer the oscillation is strengthened and the frequency falls. With "reversed coupling," on the other hand, the oscillation becomes weaker and the frequency rises as the coupling is made closer. In either case, of course, the oscillation is weakened when the coupling is made very loose, on account of the effects of the resistance on the current ratio (see the following article). If the two circuits are not in tune (as assumed above), the oscillation will be weaker. This is in contrast with Figure 4b, where the weakest oscillation occurs with the circuits in tune. The equations for this general case show, as above, that only one oscillation can occur with a given sense of coupling.

For Figure 4d, we may put in (37)

$$k_{1g}=k_{12},\ k_{2g}=1,\ L_g=L_2,\ L_p=L_1+L_2+2\,M_{12},$$

$$k_{1p}=-\frac{L_1+M_{12}}{\sqrt{L_1\,(L_1+L_2+2\,M_{12})}},\ \text{and}\ k_{2p}=-\frac{L_2+M_{12}}{\sqrt{L_2(L_1+L_2+2M_{12})}},\tag{42}$$

giving

$$g=\frac{C_1r_1+C_2r_2}{-L_2-M_{12}\mp\sqrt{\frac{L_2}{L_1}}\,(L_1+M_{12})}=\frac{C_1r_1+C_2r_2}{(1\pm k_{12})\,(\mp\sqrt{L_1L_2}-L_2)}.\tag{43}$$

This shows that to make g positive and thus have the possibility of an oscillation, L_1 must exceed L_2,[8], and only one oscillation can occur with a given sense of the coupling. Unlike Figure 4c, however, the oscillation will have the *higher* natural frequency with *normal* coupling, and the lower frequency with reversed coupling—"normal coupling" here indicating that a current in one coil alone would give opposite potentials to the plate and the filament. A study of this circuit will show that the possibility of an oscillation requires the voltage across L_2 to be less than that across L_1 and to *subtract* from it, the difference being

[8]Of course the same result may be attained by connecting the plate to a coil L_p coupled with L_1, instead of directly to L_1, and using such mutual inductance between them that the voltage across this coil L_p exceeds that across L_2.

the plate voltage. This differential effect causes the oscillation to occur with greater difficulty than in the preceding cases, but tends to prevent oscillation at other frequencies, which would not give the proper polarities to the plate and grid (see Article 5).

Instead of coupling two oscillating-current circuits thru the mutual inductance between their coils, they may be coupled by having capacity or self-inductance in common, as shown in Figures 5 and 6 respectively. The equations for these cases, as given on the figures for the condition of tuning, are derived similarly to those of Figure 4. The general properties of these circuits depend mainly on the mode of connecting the audion and so are essentially the same as for the corresponding circuits of Figure 4. There is this difference, however: with mutually inductive coupling, the sense of the coupling can be reversed; but not with capacity or self-inductive coupling. Thus in Figure 5c, we have the equivalent only of *normal* coupling, and consequently the lower natural frequency; while in Figure 6c, we have the equivalent only of *reversed* coupling, and consequently the higher natural frequency. Sometimes mutual inductive coupling can advantageously be combined with capacity coupling: for example, where the inherent capacity C_3 gives more coupling than desired, it may be counteracted by a reversed mutual inductance.

It may be noted that if the values of L_1 and L_2 are equal in Figure 5 (or in Figure 6), the higher-frequency oscillation will have a frequency independent of the coupling, for no current then passes thru the coupling condenser (or coil).

In Figure 5 one of the main condensers (C_1 or C_2) may be open-circuited; and in Figure 6 one of the main coils (L_1 or L_2) may be short-circuited. The main circuits of the two figures then become identical, except for notation, and consist of a series group (of coil and condenser) connected to a parallel group. With the two groups interconnected in this way, the significance of their being "in tune" is indefinite; for the equations for ω and k on Figures 5 and 6 are not the same. The coupling will be loose when the series self-inductance is high in comparison with the shunt self-inductance; and for this condition the two sets of equations are nearly in agreement.

Capacity coupling may also be accomplished by connecting a condenser in the place of the coil L_3, Figure 6. This would require a large capacity to give loose coupling and so would be less convenient than the arrangement of Figure 5. By analogy, self-inductive coupling may also be accomplished by connecting

a coil in the place of the condenser, C_3, Figure 5. This would require a high self-inductance to give loose coupling, and so would be less convenient than the arrangement of Figure 6. Besides, at radio frequencies, the inherent capacity of such a high-inductance coil would usually be considerable.

If a high non-inductive resistance is substituted for the coupling condenser C_3 of Figure 5, or a low non-inductive resistance for the coupling coil L_3 of Figure 6, we have what may be called "resistance coupling." This gives a single frequency of oscillation when the circuits are in tune, which is independent of the coupling. It evidently results in greater losses and does not seem to have sufficient redeeming features.

4. Certain Circuits Treated by the Complex Method.—For the exact determination of the conditions for oscillation in an audion circuit, we may express the various independent currents as complex quantities and write a number of equations each expressing the fact that the total voltage around a closed cycle is zero. One additional equation is given by (1), Article 1, the plate current being expressed in terms of the grid voltage, which in turn may be expressed in terms of the currents and the constants of the branches connected between the grid and the filament. These equations may be combined so as to eliminate all currents, resulting in a single complex equation which may be solved for ω and g.

As a simple example, we have in Figure 3b:

$$I\left(r+j\,\omega L+\frac{1}{j\,\omega C}\right)+I_p\cdot j\,\omega M=0; \qquad (44a)$$

and

$$I_p=-g\frac{I}{j\,\omega C}, \qquad (44b)$$

where the I's represent the currents in complex form. Substituting (44b) in (44a) and dividing thru by I, we eliminate the currents and obtain

$$r+j\,\omega L+\frac{1}{j\,\omega C}-\frac{j\,\omega Mg}{j\,\omega C}=0;$$

or

$$1-\omega^2 CL+j\,\omega Cr-j\,\omega Mg=0. \qquad (45)$$

Separating real and imaginary parts,

$$1-\omega^2 CL=0 \qquad \text{or} \quad \frac{1}{\omega^2}=CL; \qquad (46a)$$

and

$$j\,\omega Cr-j\,\omega Mg=0 \qquad \text{or} \quad g=\frac{Cr}{M}. \qquad (46b)$$

These equations are identical with those given on Figure 3b, showing that in this case the loss method was exact.

Now going to coupled circuits, we have similarly in Figure 4b:

$$I_1\left(r_1+j\,\omega L_1+\frac{1}{j\,\omega C_1}\right)+I_2\cdot j\,\omega\,M_{12}+I_p\cdot j\,\omega\,M_{1p}=0; \qquad (47a)$$

$$I_2\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)+I_1\cdot j\,\omega\,M_{12}=0; \qquad (47b)$$

and

$$I_p=-g\frac{I_1}{j\,\omega C_1}. \qquad (47c)$$

Combining,

$$\left(r_1+j\omega L_1+\frac{1}{j\,\omega C_1}-\frac{j\,\omega\,M_{1p}\,g}{j\,\omega C_1}\right)\left(r_2+j\,\omega L_2+\frac{1}{j\,C_2\,\omega}\right)-(j\omega M_{12})^2=0.$$

Clearing of fractions and separating real and imaginary parts,

$$(1-\omega^2C_1L_1)\,(1-\omega^2C_2L_2)-(\omega C_1r_1-\omega M_{1p}g)\,\omega C_2r_2 - \omega^4C_1C_2M_{12}^2=0; \qquad (48a)$$

and $(1-\omega^2C_1L_1)C_2r_2+(1-\omega^2C_2L_2)\,(C_1r_1-M_{1p}g)=0.$ (48b)

If $M_{1p}g$ is eliminated, these equations lead to a cubic for ω^2, showing three possible oscillations, which in general occur with unequal ease. If the circuits are *tuned*, however, the equations are simplified. Putting

$$C_1L_1=C_2L_2 \qquad (49)$$

in (48b) we have either

$$g=\frac{C_1r_1+C_2r_2}{M_{1p}}; \qquad (50)$$

or else

$$\frac{1}{\omega^2}=C_1L_1=C_2L_2. \qquad (51)$$

Substituting these values successively in (48a), we have

either $\quad (1-\omega^2C_2L_2)^2+\omega^2C_2^2r_2^2-\omega^4C_1C_2M_{12}^2=0;$ (52)

or else

$$g=\frac{\omega^2C_1M_{12}^2}{M_{1p}r_2}+\frac{C_1r_1}{M_{1p}}=\frac{M_{12}^2}{L_1M_{1p}r_2}+\frac{C_1r_1}{M_{1p}}. \qquad (53)$$

Solving (52) for ω,

$$\frac{1}{\omega^2}=C_2L_2-\frac{C_2^2r_2^2}{2}\pm\sqrt{-C_2L_2\cdot C_2^2r_2^2+\left(\frac{C_2^2r_2^2}{2}\right)^2+C_1C_2M_{12}^2}. \qquad (54)$$

Substituting the coefficient of coupling,

$$k_{12}=\frac{M_{12}}{\sqrt{L_1L_2}}, \qquad (55)$$

and the power factor,[9]

$$p_2 = r_2 \sqrt{\frac{C_2}{L_2}}, \tag{56}$$

the above equations reduce to the following:

$$\frac{1}{\omega^2} = C_2 L_2 \left(1 - \frac{p_2^2}{2} \pm \sqrt{k_{12}^2 - p_2^2 + \frac{p_2^4}{4}}\right) \text{ or } C_2 L_2; \tag{57}$$

and

$$g = \frac{C_1 r_1 + C_2 r_2}{M_{1p}} \quad \text{or} \quad \frac{C_1 r_1 + C_2 r_2 \dfrac{k_{12}^2}{p_2^2}}{M_{1p}}. \tag{58}$$

These equations show that when $\left(k_{12} < \sqrt{p_2^2 - \frac{p_2^4}{4}}\right)$ only one oscillation is possible, since two values of ω^2 in (57) are then imaginary. This oscillation has the natural frequency of the separate circuits and is the stronger the weaker the coupling. When k_{12} lies between $\sqrt{p_2^2 - \frac{p_2^4}{4}}$ and the very nearly equal value p_2, three oscillations are possible; but only that at the natural frequency of the separate circuits occurs, for this gives the lowest value of g. On the other hand, when $(k_{12} > p_2)$, this oscillation gives the highest value of g; so either of the other oscillations can occur, both giving the same value for g, which is independent of the coupling. There is thus a certain critical coupling,

$$k_{12} = p_2, \tag{59}$$

below which only one oscillation can occur and above which two oscillations can occur with equal ease. The frequencies for this critical coupling are given by

$$\frac{1}{\omega^2} = C_2 L_2 \text{ or } C_2 L_2 (1 - p_2^2.) \tag{60}$$

The difference between these two frequencies is the lowest possible beat frequency.

The conclusions of the preceding paragraph were verified experimentally by the author two years ago, the experiments[10] being first made at audible frequencies so that the change in frequency and the accompanying beats could be made directly evident in a telephone receiver. Many tests at radio frequencies

[9]See footnote on "Notation" page.

[10]These experiments were actually made with a circuit differing from Figure 4b by having the grid and the plate interchanged. As explained previously, such interchange always leads to an exactly similar set of equations.

have since confirmed these conclusions, the beat frequency then being audible and showing the change from one oscillation to another. With the values of p_2 found in well designed circuits, the minimum beat frequency in long-wave circuits is below audibility, but in short-wave circuits will often lie within the audible range.

The above results show that the equations derived by the loss method, as given on Figure 4b, are not exact for this circuit, but are approximately correct as long as the coupling is considerably closer than the critical coupling.

The complex method applied similarly to the circuit of Figure 4c gives for the condition of tuning the following exact equations:

$$\frac{1}{\omega^2}=C_1L_1\left[1+\frac{p_1\,p_2}{2}\pm\sqrt{k_{12}^2+p_1\,p_2+\frac{p_1^2\,p_2^2}{4}}\right]; \tag{61}$$

and

$$g=\frac{C_1r_1+C_2r_2}{k_{12}\sqrt{L_1L_2}(1-k_{12}^2)}\left[-k_{12}^2-\frac{p_1\,p_2}{2}\pm\sqrt{k_{12}^2+p_1\,p_2+\frac{p_1^2\,p_2^2}{4}}\right]. \tag{62}$$

These equations show that, as long as k_{12} is large compared with $\sqrt{p_1\,p_2}$, the resistances do not appreciably affect the frequency or the current ratio; so the simpler equations given on Figure 4c apply with sufficient accuracy. When the coupling is loosened, so that k_{12} is comparable with $\sqrt{p_1\,p_2}$, the oscillation rapidly becomes weaker. With reversed coupling there is therefore a particular value of k_{12} giving the strongest oscillations, as has been found experimentally.

5. Regenerative and Absorbing Actions.—As mentioned in Article 1, an audion may either be connected so as to supply power tending to maintain an oscillation or so as to absorb power from the oscillation. The first action is equivalent to the insertion of a *negative* resistance in some part of the circuit; the second is equivalent to the insertion of a positive resistance. The equivalent positive resistance r_{an} added to any branch (n) may be expressed as $\left(-\frac{P}{I_n^2}\right)$, where $(-P)$ is the power absorbed by the audion (or $(+P)$ is the power supplied by the audion) and I_n is the current of the branch. Hence by equation (2), Article 1,

$$r_{an}=-\frac{P}{I_n^2}=-\frac{E_gE_p\,g}{I_n^2}. \tag{63}$$

Consider the circuit of Figure 4b and suppose an oscillation to be produced by a source in series with C_2. We then have

$$E_g = \frac{I_1}{\omega C_1}; \tag{64}$$

and

$$E_p = I_1 \, \omega \, M_{1p}. \tag{65}$$

Substituting in (63), the equivalent resistance added in series with L_1 is

$$r_{a1} = -\frac{E_g E_p g}{I_1^2} = -\frac{M_{1p} g}{C_1}. \tag{66}$$

Since this is negative (with normal coupling) and constant for all impressed frequencies, the audion gives constant regenerative amplification for all oscillations impressed in series with C_2. This constitutes the one serious objection to the use of this circuit in radio receiving, C_2 being the antenna capacity, for the interference is severe, especially from strays. The tendency of this circuit to oscillate equally well at its two natural frequencies is evidently a special case of the constant regenerative amplification. With reversed coupling, r_{a_1} is positive and constant for all frequencies, the audion thus opposing all oscillations.

Consider now the circuit of Figure 4c and suppose an oscillation to be produced by a source in series with C_2, as above. We then have

$$E_g = \frac{I_1}{\omega C_1}; \tag{67}$$

$$E_p = I_2 \, \omega L_2 + I_1 \, \omega \, M_{12}; \tag{68}$$

and

$$I_1 \left(\omega L_1 - \frac{1}{\omega C_1} \right) + I_2 \, \omega \, M_{12} = 0. \tag{69}$$

Substituting in (63), the equivalent resistance in series with L_1 is

$$r_{a_1} = -\frac{E_g E_p g}{I_1^2} = -\frac{g}{\omega C_1} \left[\frac{-\omega L_2}{\omega M_{12}} \left(\omega L_1 - \frac{1}{\omega C_1} \right) + \omega M_{12} \right] \tag{70}$$

$$= -\frac{g L_2}{\omega^2 C_1^2 M_{12}} \left[1 - \omega^2 C_1 L_1 + \omega^2 C_1 \frac{M_{12}^2}{L_2} \right] \tag{71}$$

$$= -\frac{g \sqrt{L_1 L_2}}{k_{12} C_1} \left[\frac{1}{\omega^2 C_1 L_1} - 1 + k_{12}^2 \right]. \tag{72}$$

If k_{12} is positive, r_{a_1} will be negative for all frequencies lower than that given by

$$\frac{1}{\omega^2} = C_1 L_1 (1 - k_{12}^2) \tag{73}$$

and will be positive for all higher frequencies; the reverse is true if k_{12} is negative. We thus have regenerative action on one side, and absorbing action on the other side of a certain critical frequency, according to the sense of the coupling. The tendency of this circuit to oscillate at one of its two natural frequencies with normal coupling and at the other natural frequency with reversed coupling is in agreement with the above result. This circuit may be employed in radio receiving to reduce interference from frequencies either higher or lower than that to which it is tuned, but not from both at the same time.

Consider finally the circuit of Figure 4d and again suppose an oscillation to be produced by a source in series with C_2. We then have

$$E_g = -I_2\,\omega L_2 - I_1\,\omega\,M_{12}; \tag{74}$$

$$E_p = \frac{I_1}{\omega C_1} + I_2\,\omega L_2 + I_1\,\omega\,M_{12}; \tag{75}$$

and

$$I_1\left(\omega L_1 - \frac{1}{\omega C_1}\right) + I_2\,\omega\,M_{12} = 0. \tag{76}$$

Substituting in (63), the equivalent resistance in series with L_1 is

$$\begin{aligned} r_{a_1} = -\frac{E_g E_p g}{I_1^2} &= g\left[-\frac{\omega L_2}{\omega M_{12}}\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \omega\,M_{12}\right]\times \\ &\qquad \left[\frac{1}{\omega C_1} - \frac{\omega L_2}{\omega M_{12}}\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \omega\,M_{12}\right] \end{aligned} \tag{77}$$

$$\begin{aligned} &= \frac{g L_2}{\omega^2 C_1^2 M_{12}^2}\left[1 - \omega^2 C_1 L_1 + \omega^2 C_1 \frac{M_{12}^2}{L_2}\right]\times \\ &\qquad \left[L_2 + M_{12} - \omega^2 C_1 L_1 L_2 + \omega^2 C_1 M_{12}^2\right] \end{aligned} \tag{78}$$

$$\begin{aligned} &= \frac{g L_2}{k_{12}^2 C_1}\left[\frac{1}{\omega^2 C_1 L_1} - 1 + k_{12}^2\right]\times \\ &\qquad \left[1 + k_{12}\sqrt{\frac{L_1}{L_2}} - \omega^2 C_1 L_1\,(1 - k_{12}^2)\right]. \end{aligned} \tag{79}$$

This will be positive for all frequencies except those between the following limits:

$$\frac{1}{\omega^2} = C_1 L_1\,(1 - k_{12}^2); \tag{80}$$

and

$$\frac{1}{\omega^2} = \frac{C_1 L_1 (1 - k_{12}^2)}{1 + k_{12}\sqrt{\frac{L_1}{L_2}}}. \tag{81}$$

The audion thus has an absorbing effect for all frequencies outside of this narrow range. By using a high-power bulb and making

suitable adjustments it is possible to increase greatly the effective resistance of the circuit for all interfering frequencies and at the same time to give regenerative action at the frequency for which it is tuned.

When the audion is oscillating, or on the verge of oscillating, we may substitute in the above equations the values of g given on Figure 4. After reduction, we then obtain for the ratio of the equivalent added resistance to the equivalent original resistance of the whole circuit referred to the coil (1):
for Figure 4b, by (66),

$$\frac{r_{a_1}}{r_1+C_2 r_2 / C_1}=-1; \tag{82}$$

for Figure 4c, by (72),

$$\frac{r_{a_1}}{r_1+C_2 r_2 / C_1}=-\frac{1}{k_{12}}\left[\frac{1}{\omega^2 C_1 L_1 (1+k_{12})}-1+k_{12}\right] \tag{83}$$

$$=-\frac{1}{k_{12}}\left(\frac{\omega_n^2}{\omega^2}-1\right)-1; \tag{84}$$

and for Figure 4d, by (79),

$$\frac{r_{a_1}}{r_1+C_2 r_2 / C_1}=\frac{1}{k_{12}}\left[\frac{1}{\omega^2 C_1 L_1 (1-k_{12})}-1-k_{12}\right]\times$$

$$\left[1+\frac{L_2}{\sqrt{L_1 L_2}-L_2}\cdot\frac{1+k_{12}}{k_{12}}\left(1-\omega^2 C_1 L_1 [1-k_{12}]\right)\right] \tag{85}$$

$$=\left[\frac{1}{k_{12}}\left(\frac{\omega_n^2}{\omega^2}-1\right)-1\right]\left[1+\frac{L_2}{\sqrt{L_1 L_2}-L_2}\cdot\frac{1+k_{12}}{k_{12}}\left(1-\frac{\omega^2}{\omega_n^2}\right)\right]. \tag{86}$$

In these equations ω_n represents that natural frequency which the audion tends to maintain; in (84) it is given by

$$\frac{1}{\omega_n^2}=C_1 L_1 (1+k_{12}); \tag{87}$$

and in (86) it is given by

$$\frac{1}{\omega_n^2}=C_1 L_1 (1-k_{12}). \tag{88}$$

It will be seen that the resistance ratio reduces to (-1) at this natural frequency in (84) and (86), and has this value at all frequencies in (82). This means that the audion then adds a negative resistance equal to the original positive resistance; so the total effective resistance is zero, which is simply another way of expressing the necessary condition for a sustained oscillation.

The above three special cases of coupled circuits differ so from one another in regard to their regenerative or absorbing

action at various frequencies, that it seems desirable to investigate the general circuit of Figure 4a. If the separate circuits are in tune and an oscillation is produced by a source in (C_2, L_2), the values of E_g, E_p, and I_2/I_1, are given respectively by equations (32), (33) and (31a). Substituting in (63), the equivalent resistance in series with L_1 is

$$r_{a_1} = -\frac{E_g E_p g}{I_1^2} = -\omega^2 g \left[M_{1g} - M_{2g}\frac{L_1}{M_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)\right] \times \left[M_{1p} - M_{2p}\frac{L_1}{M_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)\right]$$

$$= -\omega^2 g L_1 \sqrt{L_g L_p}\left[k_{1g} - \frac{k_{2g}}{k_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)\right] \times \left[k_{1p} - \frac{k_{2p}}{k_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)\right]. \tag{89}$$

When the circuit is oscillating, or on the verge of oscillating, we may substitute the value of g from (37), giving for the ratio of the added resistance to the original resistance

$$\frac{r_{a1}}{r_1 + C_2 r_2 / C_1} = -\omega^2 C_1 L_1 [1 \pm k_{12}] \frac{k_{1g} - \frac{k_{2g}}{k_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)}{k_{1g} \pm k_{2g}} \times \frac{k_{1p} - \frac{k_{2p}}{k_{12}}\left(1 - \frac{1}{\omega^2 C_1 L_1}\right)}{k_{1p} \pm k_{2p}}. \tag{90}$$

If this resistance ratio is plotted against $(1/\omega^2)$, the resulting curve will in general be a hyperbola having the resistance axis as one asymptote and crossing the frequency axis at the points,

$$\frac{1}{\omega^2} = C_1 L_1 \left(1 - \frac{k_{12} k_{1g}}{k_{2g}}\right) \text{ and } \frac{1}{\omega^2} = C_1 L_1 \left(1 - \frac{k_{12} k_{1p}}{k_{2p}}\right). \tag{91}$$

As these expressions are significant only when positive, we may have zero, one, or two frequencies giving zero added resistance. The three special cases treated above exemplify these three possible conditions. In each of these cases one of the natural frequencies was found to be within the range of negative added resistance, giving a possible free oscillation; but the general case, as shown by studying (89), permits the range of negative r_{a_1} to lie between, above, or below the two natural frequencies. This means that the audion may have regenerative action over a range of frequency and still be unable to maintain a free oscillation, no matter how low the resistances.

6. Application of Oscillating Audion Circuits.—The

following record of the various oscillating audion circuits that have been developed is probably incomplete and is given with the hope that it will be supplemented in the discussion and that attention will be called to any inadvertent errors.

The single oscillating-current circuits of Figure 3 are mainly employed as a basis for the more useful coupled circuits. They simply afford means for giving the grid and the plate opposite polarities for any oscillation that may be produced in the main circuit; and the choice among them is ordinarily only a matter of convenience. When the main circuit of Figure 3b is coupled to a local tuned circuit, the arrangements of Figure 4b, 5b or 6b result, according to the method of coupling; but we might equally well have used any of the other circuits of Figure 3 to couple with the local circuit, giving the same properties that have been described for Figure 4b.

Figure 3b is the basis of one of Armstrong's methods[11] for producing oscillations that has been widely used—viz., mutually inductive coupling between the plate circuit and a tuned grid circuit. What may be called the "conjugate" of this circuit, the grid and the plate being interchanged, Figure 3c, was devised by the author about two years ago and successfully used for radio receiving. It is, however, less convenient than Figure 3b, because, to give a high grid voltage and at the same time not have too strong an oscillation, the capacity C must be so high (of the order of 50 milli-microfarads) as to preclude the use of an ordinary variable air condenser. Figure 3d, employing a single tapped coil, is commonly known as the "Western Electric" circuit; without the mutual inductance between the coils, it is included among those originally given by Armstrong.[12] The circuit of Figure 3e is used in commercial radio receivers and for other purposes. Figure 3f might not be recognized as the basis of Armstrong's circuit employing an audio-frequency self-inductance (such as that of telephone receivers) in the lead from the filament to the oscillating-current circuit.[13] Here C_p represents the capacity (added or inherent) shunting the audio-frequency self-inductance; and C_g represents the inherent capacity between the grid, with the apparatus connected thereto, and the filament, with its connected apparatus. Armstrong considers the audio-frequency self-inductance in circuits of this form to be essential to the action. This, however, is not the case; for

[11]"Proc. I. R. E.", volume 3, page 219, Figure 8.

[12]Loc. cit., page 222, Figure 13.

[13]Loc. cit., Figure 12. As has been pointed out by Armstrong, this arrangement is identical with the "ultraudion" connection of de Forest.

experiments made by the author show that a high non-inductive resistance will answer the same purpose, tho requiring a higher battery voltage to give the same direct current. The real purpose of the high self-inductance is simply (as in many other audion connections) to provide a path for the direct battery current from the filament to the plate, without short-circuiting the capacity C_p which is essential to the oscillation.

Figures 4b, 5b, and 6b give examples of a class of coupled circuits in which one of the component circuits is not coupled, or only slightly coupled, with the audion. Such circuits are in general characterized by the following properties: (a) if the coupling between the component circuits is not very loose, the audion tends to produce oscillations equally well (or almost equally well) at the two natural frequencies of the combination; (b) oscillations of all frequencies impressed in the local circuit (C_2, L_2) are then amplified by regenerative action, or absorbed, according to the polarity of the connections to the audion; and (c) with very loose coupling of the local circuit (C_2, L_2), the main oscillating-current circuit (C_1, L_1) oscillates readily at its own natural frequency. The property (a) permits an audion to be used as a generator of "beats," as has been done by Armstrong.[14] If for the local circuit (C_2, L_2) is used an antenna and tuning coil, this property also makes the circuit particularly applicable for the reception of undamped waves by the "self-heterodyne" principle: for the audion may be allowed to oscillate at one natural frequency while the other natural frequency is brought to coincidence with that of the signal to be received. By making adjustments carefully, the audion may be brought to the verge of changing from one frequency to the other, under which condition enormous amplification is possible.[15] Unfortunately, this

[14]Loc. cit., Figure 17 used for this purpose is identical with Figure 6b of this paper.

[15]This method of signal amplification does not appear to have previously been brought to the attention of the Institute, tho it has been described by the author on several occasions. It should be noted that to attain the result described, the coupling between the oscillating-current circuits must be carefully adjusted so that the two possible free oscillations will give a beat note of the desired pitch. For long waves, this means a coupling coefficient between 1 and 5 per cent. Dr. Austin has used ("Proc. I. R. E.", volume 4, page 252) a so-called "sensitizing circuit" which could operate as described here; but he explains that its purpose is merely to weaken the oscillation produced by the audion. His results, of comparatively slight and constant increase in amplification, bear out his explanation for the arrangement as he used it. Armstrong, however, in the discussion of Dr. Austin's paper offers the explanation that the sensitizing circuit gives the combined circuit two natural frequencies and so permits it to be exactly in tune with the incoming oscillation while it is oscillating at another frequency, thus eliminating the reactance offered to the incoming oscillation when the circuit is slightly

condition is very critical; so a strong signal or accidental disturbance may suddenly change over the frequency of oscillation to that of the signal, which immediately disappears. Moreover, any impulse due to strays will set up simultaneous oscillations at the two natural frequencies, giving beats between them which are audible. As the audion tends to maintain both of these oscillations, a musical note results which may last a large fraction of a second and sounds like the clang of a bell. With the circuit in exact tune with the signal, this bell-like noise due to strays has just the same pitch as the signal, which it thus completely masks. In the presence of strays it is therefore necessary to operate the circuit somewhat out of tune, thus sacrificing the particular advantage of circuits of this class. The property (b) of these circuits (the tendency to amplify at *all* frequencies) is of course a disadvantage in radio receiving, as it reduces selectivity.

Circuits for heterodyne reception may be arranged so that the audion tends to produce oscillations at two frequencies, one of which oscillations does not enter the antenna circuit and so is not set up by impulses therein. This obviates the serious objection to the circuit discussed in the preceding paragraph, while retaining its advantages. However, better results are obtained when one audion is used for producing oscillations and a second audion for amplifying the signal oscillation by regenerative action.[16] The reason is as follows: When the audion is oscillating, the value of g (or mean slope of the characteristic curve, Figure 1) is such as to make the total effective resistance of the circuit equal to zero *for that oscillation*. When a second oscillation is superposed, the range of variation in grid voltage and plate current will be extended over a section of the characteristic curve the slope of which is less steep; the value of g for the second oscillation is therefore lower, and the total effective resistance for this oscillation will not be zero. If the original oscillation is very weak, so that only the practically straight portion of the characteristic curve is included, then the value of g for the second oscillation is nearly the same as for the first, and better amplification is obtained, as has been found in practice. Such a weak oscillation, however, requires very critical adjustments, and is likely to cease suddenly. When two audions are used, one may be on the verge of oscillating at the signal frequency, and will then require only a weak impressed voltage in the

out of tune. Nothing, however, was said about the tendency of the audion to oscillate at the incoming frequency.

[16]Hogan, "Proc. I. R. E.", volume 3, page 256.

antenna to produce relatively strong oscillations, which then give beats with the permanent oscillation of the other audion.

The circuit of Figure 4b, with its modifications, has been used by the author in the measurement[17] of effective resistance of coils at various frequencies. The test coil is connected to a tuning condenser, constituting the local circuit (C_2, L_2). The two component circuits are brought into tune at the desired frequency and the coupling between them is adjusted to the critical value [Article 4, equation (59)], as shown by the behavior of an ammeter[18] in the plate circuit when the tuning is varied. The local circuit is then removed and a resistance is inserted in series with L_1 until the oscillation has the same strength as before. This resistance multiplied by the ratio of self-inductances gives the effective resistance of the local circuit [compare equations (18) and (58)]. This method permits the measurement of the effective resistance (as well as the capacity) of a coil at its own natural frequency, with as much facility as at lower frequencies.

Figures 4c, 5c, and 6c give examples of a class of coupled circuits in which one component circuit is connected (or coupled) to the grid of the audion and the other to the plate. Such circuits are in general characterized by the following properties: (a) the audion tends to produce an oscillation at only one of the natural frequencies; (b) the audion gives regenerative action on one side, and absorbing action on the other side, of a certain frequency near the natural frequencies; and (c) very loose coupling may be employed before the oscillation is greatly weakened.

Figure 4c is the basis of a circuit devised by the author in the fall of 1915 for receiving damped waves, C_2 being the antenna capacity.[19] About the same time this circuit, with a slightly different arrangement of the telephone receivers and battery, was brought out by Mr. F. B. Chambers and is commonly known as the "Chambers circuit." In using this circuit the author has found that the capacity of the audion frequently gives too close coupling (on the principle of Figure 5c), and this

[17]Described in a paper presented before the Radio Club of America in February, 1917, and published in "QST" for April, 1917.

[18]When the coupling is closer than the critical value, the ammeter shows a sudden drop when the oscillation changes from one of the natural frequencies to the other. When the coupling is loosened so that this drop just ceases to occur, the maximum ammeter reading is used as an index of the strength of the oscillations when the circuits are in tune.

[19]This circuit was given in a discussion before the Radio Club of America in June, 1916, and was published in "QST" for September, 1916.

may to advantage be partially neutralized by reversed coupling of the coils. The circuit of Figure 4c is not particularly suited to sustained wave reception (altho it is sometimes so used), because it must either be slightly out of tune for the received oscillation or else the received energy will tend to be absorbed by the audion.

Figure 5c is the basis of one of Armstrong's methods of producing oscillations—viz., the use of self-inductance in the plate circuit.[20] C_3 here represents the capacity between the grid and the plate of the audion, with their connected apparatus; and C_2 represents the inherent or added capacity in parallel with the plate inductance. It may be noted that, if C_2 is zero, the two component circuits may still be tuned by adjusting C_3 or L_2.

So far as the author is aware, the principle of Figure 4d, 5d, and 6d is new. If C_2 is made the antenna capacity of a radio receiving set, the audion will have a strong tendency to reduce interference, since it absorbs energy from oscillations at other frequencies than that to which it is tuned. The general circuits of Figures 4a, 5a, and 6a can give even better results in the reduction of interference from strays; for they can be arranged so as to give the best regenerative action at a frequency differing slightly from the natural frequency and can then have an absorbing action for the oscillations set up by strays at the natural frequency. Of course, the set then being slightly out of tune with the signal, some reactance for the signal oscillation will be introduced. The audion used in this way for combined regenerative and absorbing action should not also be used as a detector; for its grid is connected (or closely coupled) to the antenna circuit (C_2, L_2). A second audion (or other detector) should be connected in the usual way to the circuit (C_1, L_1), where interference is minimized in the usual manner by the loose coupling of the two tuned circuits.

The foregoing discussion makes no pretense of including all possible or useful oscillating audion circuits, even of the simpler sort. Many others have been studied by the author which exhibit different and interesting properties; but the limits of space do not allow their inclusion. It is hoped, however, that the

[20] "Proc. I. R. E.", volume 3, page 220, Figure 9, and following.

methods of treatment here described and illustrated will be of service to those working with the oscillating audion and will lead to the further development of that quite wonderful device. This is the real object of the paper.

SUMMARY: A general method is presented for investigating theoretically the conditions of oscillation in circuits supplied by an audion. This method is based on the relay action of the audion, as represented by its characteristic curve, and has for its fundamental idea the use of the slope of this curve as a physical constant of the audion, called the "mutual conductance." Two modes of applying this method are given: the simple, but approximate, "loss method" and the exact "complex method." Various particular circuits are discussed and formulas given for the natural frequency and the required value of mutual conductance to maintain an oscillation. These circuits include the elements of all the common oscillating audion circuits depending on the relay action; so it is seen that the actions in these various circuits are not distinct and independent, but are here brought under a single set of physical ideas.

The circuits chosen for detailed discussion are those which most simply illustrate the method of treatment and which exhibit the various useful properties that may be obtained with different connections of the audion. It would be impossible to include in any paper of reasonable length a discussion of, or even a reference to, all the possible oscillating-current circuits that may have practical application.

The behavior of certain circuits toward impressed oscillations (as from an antenna) at various frequencies is considered. And it is found that the audion under certain conditions tends to maintain such oscillations (regenerative action), and under other conditions tends to absorb their energies. It is also found that with certain arrangements the audion will have regenerative action only over a very short range in frequency, absorbing energy from oscillations at all other frequencies. This result affords a means of reducing interference, especially from unsustained oscillations such as are caused by strays.

The paper concludes with a brief record of oscillating audion circuits as developed by various investigators, and their practical applications.

In two appendices the author discusses respectively: the family of characteristics obtained for different plate voltages, and derives therefrom data on oscillation characteristics; and the interchangeability of the grid and plate circuits so far as oscillation production is concerned.

The following units belong to a consistent system based on the milli-ampere, the volt, and the micro-second. They give much more convenient numerical values in radio work than the ordinary units based on the ampere, volt, and second; and many of them are for this reason already in common use. Of course, all equations not involving numerical coefficients apply equally to this system or to the ordinary system.

SYMBOL	NAME	UNIT
C_n	Capacity in branch (n)	Milli-microfarad
E_n	Alternating voltage across branch (n)	Volt
E_g	Grid voltage, alternating part [Article 1]	Volt
E_p	Plate voltage, alternating part [Article 1]	Volt
g	Mutual conductance of grid toward plate [Article 1]	Milli-mho
g_p	Self-conductance of plate [Article 1]	Milli-mho
g'	Conductance of a branch	Milli-mho
I_n	Alternating-current in branch (n)	Milli-ampere
I_p	Plate current, alternating part [Article 1]	Milli-ampere
j	Imaginary unit, $\sqrt{-1}$	Numeric
k_{mn}	Coefficient of mutually inductive coupling, $\sqrt{\frac{M_{mn}}{L_m L_n}}$	Numeric
k	Coefficient of self-inductive or capacity coupling [Figures 5 and 6]	Numeric
L_n	Self-inductance of branch (n)	Milli-henry
M_{mn}	Mutual inductance between branches (m) and (n)	Milli-henry
ω	Angular frequency	Radian per microsecond
P	Power	Milli-watt
p_n	Power factor[21] of branch (n), $r_n \sqrt{\frac{C_n}{L_n}}$	Numeric
r_n	Resistance of branch (n)	Kil-ohm
r_{an}	Equivalent "absorbing" resistance of the audion referred to branch (n) [Article 5]	Kil-ohm

[21]This is strictly the ratio of resistance to reactance at the natural frequency, and not the ratio of resistance to impedance, which actually is the power factor. The numerical difference in any radio circuit is insignificant; and for the sake of vividness it is convenient to employ the common term "power factor."

APPENDIX I*

In Article 1 of the paper it was mentioned that the plate potential affects the oscillation, and a method of taking this into account was mentioned—viz., by means of a constant g_p expressing the quotient of a variation in the plate current by the corresponding variation in the plate potential. A more general method of treating the effect of the plate potential is given below.

In Figure 7 the full lines constitute a family of characteristic curves, such as that of Figure 1, each curve taken for a different constant plate potential, as may readily be traced experimentally by employing adjustable batteries in both grid and plate cir-

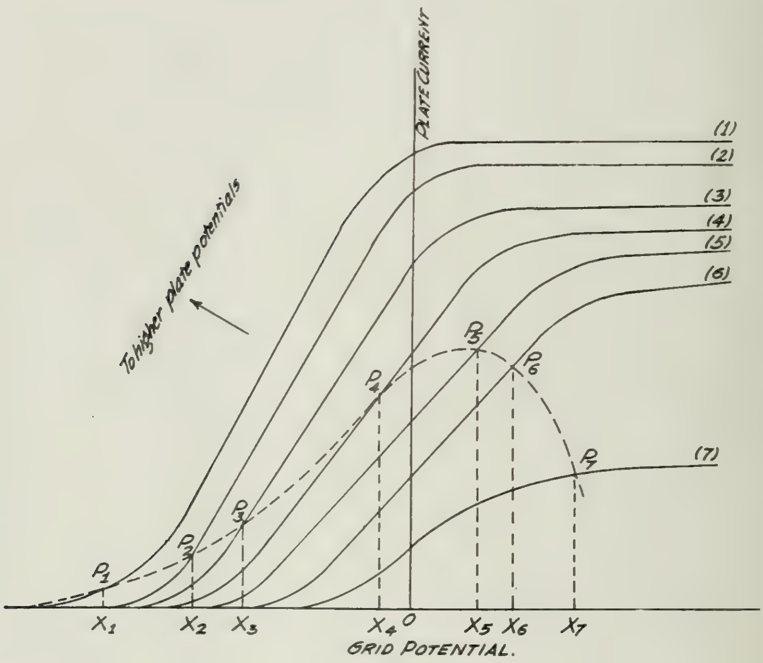


FIGURE 7

cuits. Now in any oscillating audion circuit let the ratio of the alternating plate voltage E_p to the alternating grid voltage E_g be

$$n = \frac{E_p}{E_g}.$$

---

* The subject matter of this appendix was presented orally at the reading of the paper itself, September 5, 1917.

These voltages being ordinarily in phase, their instantaneous values, and therefore the variations in plate and grid potentials, have the same ratio n. Hence if we start with the plate and grid potentials corresponding to the point P_1, for example, and let the plate become more negative by such an amount ΔE_p as to correspond to the curve (2), then the grid will become more positive by the amount,

$$\Delta E_g = \frac{E_p}{n}$$

which may be laid off along the axis as $X_1 X_2$ to locate the point P_2. Further points P_3, and so on, may be similarly located on successive curves, and these determine the dotted "derived characteristic" representing conditions during an oscillation. It is interesting to notice that the derived characteristics may exhibit a maximum point even where the original characteristics do not.

The derived characteristic may be used (in the same way as Figure 1) to give the value of the effective mutual conductance g under working conditions, thus including completely in the equations of the paper the effect of variations in the plate potential.

Probably the most useful application of derived characteristics is in determining the relation between the coil inductances, or between the condenser capacities, in the plate and grid circuits that will result in the greatest power output from the audion as an oscillating-current generator. Derived characteristics are drawn for various assumed values of n (and, if desired, for various value of grid battery voltage), and the output for each is roughly estimated as one-eighth the product of the range in plate current by the range in plate voltage.

APPENDIX 2

In the paper itself, it is stated that the grid and plate of an audion may always be interchanged in their connection to an oscillating-current circuit, and that the circuit equations will then remain of exactly the same form. As the only proof given for this statement was the symmetry of E_g and E_p in equation (3), it is thought desirable to append the following complete proof.

Resolving any oscillating-current network supplied by an audion into the appropriate number of independent circuits and equating to zero the sum of the voltages around each circuit, we have equations of the usual form:

$$
\left.\begin{aligned}
&I_1Z_{11}+I_2Z_{12}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{1n}+I_pZ_{1p}=0;\\
&I_1Z_{21}+I_2Z_{22}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{2n}+I_pZ_{2p}=0;\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\text{and}\quad &I_1Z_{n1}+I_2Z_{n2}+\ldots\ldots\ldots\ldots\ldots+I_nZ_{nn}+I_pZ_{np}=0.
\end{aligned}\right\}\quad(1)
$$

By the definition of the mutual conductance g, we have also

$$
\frac{I_p}{g}=E_g=I_1Z_{g1}+I_2Z_{g2}+\ldots\ldots\ldots\ldots+I_nZ_{gn}+I_pZ_{gp};\quad(2)
$$

$$
\text{or}\quad I_1Z_{g1}+I_2Z_{g2}+\ldots\ldots\ldots\ldots+I_nZ_{gn}+I_p\left(Z_{gp}-\frac{1}{g}\right)=0.\quad(3)
$$

In these equations, the general coefficient,

$$
Z_{rs}=Z_{sr},\quad(4)
$$

represents the impedance in common between the circuits r and s (conductive or mutually inductive); Z_{rr} represents the negative sum,

$$
Z_{rr}=-Z_{1r}-Z_{2r}-\ldots\ldots\ldots\ldots\ldots\ldots\ldots-Z_{nr}-Z_{pr};\quad(5)
$$

the I's represent the independent circuit currents; and the subscripts g and p refer to the grid and plate circuits respectively.

Eliminating the I's from the simultaneous equations (1) and (3), there results an equation between their coefficients which is most simply expressed by the determinant:

$$
\begin{vmatrix}
Z_{11} & Z_{12}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{1n} & Z_{1p}\\
Z_{21} & Z_{22}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{2n} & Z_{2p}\\
\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots & \ldots\\
Z_{n1} & Z_{n2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{nn} & Z_{np}\\
Z_{g1} & Z_{g2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{gn} & Z_{gp}-\frac{1}{g}
\end{vmatrix}=0.\quad(6)
$$

Interchanging the two subscripts of each Z, in accordance with (4), and interchanging columns with rows, as is permissible in a determinant, we obtain

$$
\begin{vmatrix}
Z_{11} & Z_{12}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{1n} & Z_{1g}\\
Z_{21} & Z_{22}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{2n} & Z_{2g}\\
\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots & \ldots\\
Z_{n1} & Z_{n2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{nn} & Z_{ng}\\
Z_{p1} & Z_{p2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots & Z_{pn} & Z_{pg}-\frac{1}{g}
\end{vmatrix}=0.\quad(7)
$$

This equation differs from (6) only in having the g and p subscripts interchanged. Hence, in general, *interchanging the connections of the grid and the plate of the audion to the oscillating-current circuit does not alter the circuit equations.* It is understood, of course, that in making this interchange, the apparatus, nat-

urally pertaining to the plate, as the plate battery, telephone receivers, and so on, is transferred with the plate, and similarly for the grid apparatus. Altho the general properties of an oscillating-current circuit are not affected by interchanging the grid and the plate, yet the strength of the oscillation may be affected; for the effective value of g will in general be changed because of the effect of the plate potential on the oscillation, as explained in Appendix 1.

Equation (6), or (7), embodies the general solution of an oscillating-current circuit supplied by the audion; and may be directly employed instead of the step-by-step procedure given in Article 4 of the paper. Take, for example, the circuit of Figure 4b, and let the circuits be chosen as represented by the dotted lines in Figure 8. Equation (6) for this case becomes.

$$\begin{vmatrix} -\left(r_1+j\,\omega L_1+\dfrac{1}{j\,\omega C_1}\right) & j\,\omega M_{12} & j\,\omega M_{1p} \\ j\,\omega M_{12} & -\left(r_2+j\,\omega L_2+\dfrac{1}{j\,\omega C_2}\right) & 0 \\ \dfrac{1}{j\,\omega C_1} & 0 & -\dfrac{1}{g} \end{vmatrix} = 0. \quad (8)$$

Expanding,

$$\left(r_1+j\,\omega L_1+\frac{1}{j\,\omega C_1}\right)\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)\left(-\frac{1}{g}\right) + \frac{j\,\omega M_{1p}}{j\,\omega C_1}\left(r_2+j\,\omega L_2+\frac{1}{j\,\omega C_2}\right)+\frac{(j\,\omega M_{12})^2}{g}=0. \quad (9)$$

This immediately reduces to (48) of the paper.

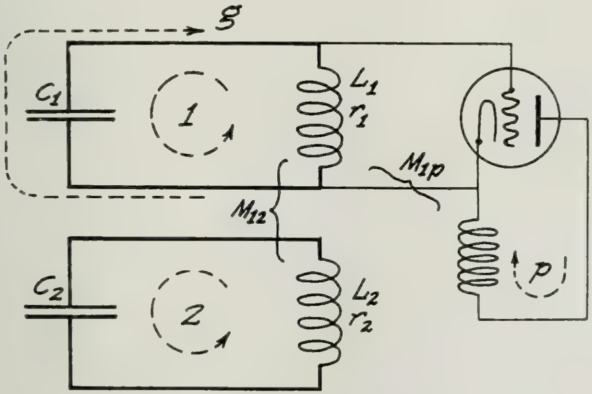


FIGURE 8

L. A. Hazeltine: Dr. de Forest calls attention to certain early discoveries of the oscillating property of the audion which do not come within the rules laid down in the paper, in that no radio-frequency apparatus was interposed between the plate and the filament. Such oscillations are due to positive ionization and not to relay action, and were not considered as coming within the scope of the paper; they were referred to, however, in Article 1 (see especially footnote 6).

Since the paper was written certain applications of oscillating-audion circuits have come to the writer's attention. A particularly interesting case is that of Meissner, who, working independently of Armstrong and along different lines, invented oscillating audion circuits at practically the same time (1913). One of Meissner's circuits is that of Figure 3a of the paper, and will be found in a letter from Meissner in the "Electrician" (London), volume 73, page 702, 1914.

A collection of oscillating audion circuits of various inventors is given by Goldsmith in his serial articles on "Radio Telephony," "Wireless Age," June and July, 1917, where further references will be found*. Arrangements especially suitable for extreme conditions and employing the pliotron oscillator, which is also a thermionic relay, have been given by White in the "General Electric Review" (for very high or very low frequencies, in the September, 1916, number; and for large currents or high voltages, in the August, 1917, number).

Logwood has recently patented a circuit for transmitting (number 1,218,195, abstracted in the "Electrical World," April, 21, 1917) which serves to confirm certain statements of the paper. This circuit is essentially that of Figure 3f; but the audio-frequency inductance is connected between the *grid* and the filament, with a shunt capacity C_g, while the capacity C_p is simply the inherent capacity of the plate. As C_p is thus smaller than C_g, the plate voltage will exceed the grid voltage, giving an increased output for transmitting, as explained in Article 1.

If proper regard is given in certain cases to the inherent capacity of the audion and other apparatus, all the various oscillating audion circuits referred to above will, in the writer's opinion, be found to be included among those considered in the paper, or to be directly derivable therefrom.

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*A considerably more complete treatment of oscillating audion circuits will be found in a recent volume: Goldsmith, "Radio Telephony" (Wireless Press, 25 Elm Street, New York).

THE DETERMINATION OF THE AUDIBILITY CURRENT OF A TELEPHONE RECEIVER WITH THE AID OF THE WHEATSTONE BRIDGE*

By

Edward W. Washburn

(Professor of Ceramic Chemistry in the University of Illinois)

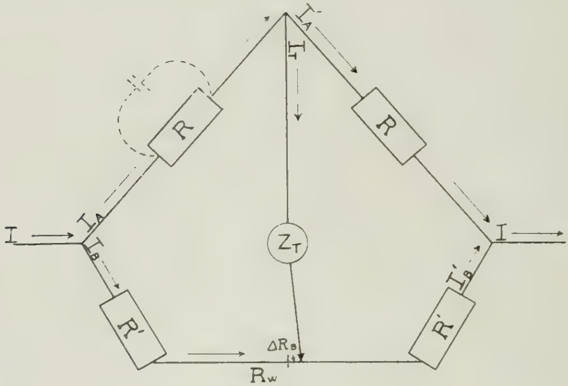
The use of the telephone receiver as a detector for small alternating currents in radio telegraphy, in many electrical measurements with the alternating current bridge[1], and for various other purposes makes it desirable to have a simple and reliable method for determining the audibility current of a telephone. By audibility current is meant that current which just suffices to produce a barely audible sound in the telephone. The curve connecting the audibility current of a telephone with the frequency employed is a valuable indication of the field of usefulness of the instrument and of its availability for the above purposes.

The usual method of determining the current-sensitivity curve of a telephone for various frequencies is to base it upon the volt-sensitivity curve of the telephone. The volt-sensitivity is usually determined with the aid of a suitable slide wire connected in shunt across a non-inductive branch of an alternating current circuit which is provided with a suitable hot-wire ammeter. [Cf. Wien, *Ann. Phys.*, **4,** 456 (1901)]. Having measured the volt-sensitivity in this way, the audibility current is then obtained by dividing the audibility voltage at each frequency by the impedance of the telephone for that frequency. Since, however, both the effective resistance and the inductive reactance of a telephone depend upon the frequency, it becomes necessary in this method to measure each of these quantities for each frequency employed. Moreover, as has been pointed out by Austin [*N. B. S. Bull.*, **5,** 155 (1908)], it is doubtful whether the values thus obtained are really applicable to the

* Received by the Editor, January 24, 1917.

[1] The importance of knowing the absolute value of the audibility current of a telephone receiver for use with an apparatus for measuring the electrical conductivity of liquids has been discussed by the writer in another place. ["Jour. Amer. Chem. Soc.," **38,** 2431 (1916).]

purpose in hand, since they are measured with the aid of currents which are very much larger than the audibility current of the telephone. In other words, both the effective resistance and the inductive reactance of the telephone are functions not only of the frequency of the current but also of its magnitude. This source of uncertainity can be avoided and the whole determination much simplified by means of a method based upon the equation of the Wheatstone bridge. While this method has



been devised by the writer primarily for the purpose of determining the audibility current of the low resistance types of telephones which are used as indicating instruments in alternating-current bridge measurements, it seemed nevertheless that it also might possibly have some advantages over the methods now in use in determining the audibility current of the high resistance types of telephone receivers used in radio work.

The Wheatstone bridge set-up is shown in the accompanying figure. The resistances R and R' must be substantially free from both inductance and capacity. For work with low resistance telephones, the Curtis type of resistance coil serves very well for this purpose. For high resistance telephones, especially for measurements at the higher frequencies, the type of film resistance devised by the writer [*Journ. Amer. Chem. Soc.*, **35,** 179 (1913)] would be preferable since even the largest units are practically free from any inductance or capacity as well as from any skin effect, even for the highest frequencies at which a telephone could be employed.

The two resistances marked R' are connected by a stretched wire of suitable resistance (R_W), provided with a scale and a

sliding contact. The telephone is connected to this sliding contact and to a point between the two resistances R as indicated. The four resistances R and R' should be as compactly and symmetrically assembled as possible and the telephone cord should be surrounded by a flexible metal sheath which is earthed. A small variable air condenser is connected in shunt across whatever resistance requires it in order to complete the exact balancing of reactances, as it is very necessary, especially for high resistances and high frequencies, to obtain perfect electrical symmetry in the set-up.

The resistance of that portion of the slide wire which lies between its central point, and the position occupied by the sliding contact when the audibility current I_T is passing through the telephone, will be denoted by ΔR_B. The current I comes from a high audio frequency generator whose frequency can be controlled and kept constant at any desired value and whose wave form is practically a sine curve. The current enters the bridge network at the left and divides as indicated. In series with the bridge is placed a suitable high frequency ammeter (not shown in the figure) for indicating the magnitude of the current I. For very small currents, the most convenient ammeter to employ is a vacuum thermocouple connected to a sensitive millivoltmeter, while for larger currents a sensitive hot-wire ammeter can be employed. The lead wires which carry the current I to the bridge network should be twisted or twin wires and should be enclosed in a grounded metal sheath.

The procedure consists in adjusting the current I until the range of silence on the slide wire is found to have a convenient measurable value, say about two or three centimeters. If the telephone is being tested to determine its suitability for use as an indicating instrument with the alternating current bridge, the determination of the range of silence should be carried out in the usual manner in which the minimum is determined when working with an alternating current bridge. If, however, the telephone is one which is to be used for radio work, the dot-and-dash method would naturally be employed in determining the range of silence on the slide wire. The range of silence is obviously equal to $2\Delta R_B$ ohms. Having determined the value of $2\Delta R_B$, the value of I as indicated by the high frequency ammeter is then recorded and the calculation of I_T is carried out by means of the following equation, which can be readily derived with the aid of Kirchhoff's laws [Maxwell, "*Electricity and Magnetism,*" I, 477, 3rd Edition].

$$I_T = \frac{2\,\Delta\, R_B\, R\, I}{B\,(B+2\,R_T - 2j\,x_T)} \tag{1}$$

where B is written in place of $R+R'+\frac{1}{2}R_W$. This equation may be put in the form

$$I_T = \frac{2\,\Delta\, R_B\, R\, I}{B} \times \frac{1}{(B+2\,R_T) - 2j\,x_T} \tag{2}$$

or numerically:

$$I_T = \frac{2\,\Delta\, R_B\, R\, I}{B\sqrt{(B+2\,R_T)^2 + 4\,x_T^2}} \tag{3}$$

where x_T, the inductive reactance, and R_T the effective resistance, are the values which, for the frequency in question, correspond to the audibility current I_T. To distinguish these values from the values which are directly measured using currents of much greater magnitude, we will designate the former values as the *audibility reactance* and the *audibility resistance* respectively.

Now if we were to employ in this equation values for R_T and x_T determined in the usual manner; that is, for currents much greater than I_T, then each of the values so obtained would differ from the true or audibility values by some fractional amount p, and the quantity under the radical in the above equation should, therefore, be written

$$[B+2\,R_T\,(1\pm p_R)]^2 + 4\,x_T^2\,(1\pm p_x)^2 \tag{4}$$

Now it is evident that the fractional error in I_T which would result from the fractional errors p_R and p_x in the quantities R_T and x_T, respectively, can be made smaller than any assigned value if the quantity B is taken sufficiently large. The problem, therefore, reduces itself to the calculation of the minimum value which B may have without producing an appreciable error in the value of I_T.

Now determinations of the appearance of an audible sound in a telephone are not sufficiently reproducible, even for a given ear, to justify an attempt to measure the corresponding current with a greater degree of accuracy than, say ten per cent. There will obviously, therefore, be no object in employing a larger value for B than that which is necessary in order to insure the attainment of this degree of accuracy. The condition, therefore, which determines the minimum allowable value for B will be mathematically expressed by the relation

$$\frac{4\,p\,[B\,R_T + Z_T\,(2+p)]}{(B+2\,R_T)^2 + 4\,x_T^2} \not> 0.2 \tag{5}$$

where $p_R = p_x = p$. From this expression we find the desired condition, namely that

$$B \nless 2\left[-R_T(1-5p)+\sqrt{Z_T^2[5p(2+p)-1]+R_T^2(1-5p)^2}\right] \tag{6}$$

where R_T and Z_T are the *approximate* values of the effective resistance and impedance and p is the fractional amount by which each of these values differs from the true or audibility value.

To illustrate the use of this equation, let us consider a specific case. Suppose it is desired to determine the audibility current, at 1000 cycles, of a telephone receiver the effective resistance of which is approximately 8,000 ohms and the effective impedance of which is approximately 10,000 ohms at this frequency. Let us suppose further that there is the possibility that each of these values might differ from the true or audibility value by as much as *fifty per cent.* In other words it is desired to choose a value for B such that an error of fifty per cent. in *each* of the values R_T and Z_T shall be without appreciable influence upon the value found for I_T. Putting $p=0.5$ in equation (6) and solving, we find that

$$B \nless 76{,}000 \text{ ohms.} \tag{7}$$

If, therefore, we choose R and R' in Figure 1 each equal to say 40,000 ohms, B would then be equal to 80,000 ohms and would obviously be well above the minimum value just calculated.

A little consideration will show that this method of determining the audibility current of a telephone not only eliminates any error arising from uncertainty as to the true values of the effective resistance and the inductive reactance, but that it can even be so employed as to make it unnecessary to determine the values of these two quantities at all. That is, knowing the direct current value of the resistance of our telephone, we may then estimate or guess at the approximate values for its impedance and its effective resistance and may then take such a value for p that we can feel sure that our estimated values do not differ from the actual values by more than $\frac{p}{100}$ per cent. We then solve equation (6) for B.

Furthermore, if the condition

$$B \nless 8R_T + 4\sqrt{(Z_T^2 + 4R_T^2)} \tag{8}$$

is fulfilled, equation (3) reduces to the simple form

$$I_T = \frac{2\Delta R_B R I}{B^2} \tag{9}$$

That is, the terms containing R_T and Z_T are entirely negligible. Thus, for example, if one used a bridge in which a value 10,000 ohms was taken for B, this bridge could then be used to determine the audibility currents of all the ordinary types of low resistance telephones without paying any attention to either the resistance or the inductance of the telephone. That is, equation (9) could be applied directly.

As an example of the determination of the audibility current of a low resistance telephone receiver by this method the following values may be cited. The high audio frequency generator was operated at 990 cycles per second. The telephone the audibility current of which was to be measured had a direct current resistance of 170 ohms and its effective resistance at 1,000 cycles was about 220 ohms, its impedance at this frequency being about 290 ohms. The value chosen for B was 5,000 ohms and it will be noticed that this fulfills the condition laid down by equation (8) so that the value of I_T may be directly calculated from equation (9). The range of silence on the slide wire was found to be $2\Delta R_B = 0.05$ ohm and the vacuum thermocouple in series with the bridge indicated a current, $I = 0.4$ milliampere. Substituting these values in equation (9), we find the audibility current of the telephone to be 0.002 microampere at 990 cycles.

In conclusion, it may be pointed out that by combining the audibility current of a telephone, obtained in this way, with the audibility voltage obtained by the shunt method, the true or audibility impedance of the telephone can be calculated.

Department of Ceramic Engineering,
University of Illinois,
January 20, 1917.

SUMMARY: A method of determining the audibility *current* of a telephone receiver, using a Wheatstone bridge with the telephone as indicator is described practically and theoretically. The method differs from the usual shunted telephone method which determines an audibility *voltage*.

A method is given whereby a desired percentage accuracy can be secured even if the telephone resistance and impedance are only very approximately known.

DISCUSSION

Charles S. Ballantine (by letter): I have read Professor Washburn's paper with very great interest, and believe that radio engineers are greatly indebted to him for his valuable contribution to a very indefinite subject. It is very evident upon examination, for instance, of the experimental data given by such observers as Austin, Fuller, and others that a certain amount of variability in results exists which cannot be wholly accounted for on the basis of the influence of the "personal equation" of the observers. This regrettable lack of agreement is probably due to some extent to a lack of precision in the definition of the fundamental unit upon which the measurement of signal strength is based, i. e., the audibility current of the telephone receiver. For this reason the method of Professor Washburn becomes of value in making such comparisons and deriving theoretical values for the currents from the shunt readings. There are several points in connection with the method described which may be mentioned here. It has been stated in certain foreign publications that as the wave form of the measuring current departs from the pure harmonic form a corresponding loss of sensitiveness in the receiver results. I have never discovered any exact quantitative data accruing from any investigation of a receiver accompanying such statements, and it is difficult to decide at once whether the results obtained by the above method would be applicable in actual practical conditions. If such is the case, it is evident that the method using the Wheatsone bridge on any current form and comparing the results with shunted telephone readings obtained with actual signaling currents of another wave form would have very little engineering value. Another point of interest is the influence of the mechanical movement of the receiver diafram upon the currents in the windings. This motion would obviously result in a reactive E. M. F. which would oscillate in several periods giving rise to transients and further complicating the functional relation between the impedance and frequency. The proposed method provides for the elimination of the variable inductance error due to the saturation of the iron with different currents but fails to take into consideration the effect of the resonance of the diafram when comparison is to be made between audibility current and shunt readings.

In order to obtain a quantitative estimate of the actual influence of these factors, an experimental investigation was made on a typical telephone receiver using Professor Washburn's

method. The first group of experiments was made to determine the influence of wave form on the measurements, and for this purpose three different conditions were imposed as shown in Figure 1. Type *A* current was supplied from an alternator giving

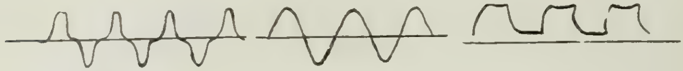


FIGURE 1

currents up to 240 cycles frequency. The oscillogram taken across the bridge showed nearly 40 per cent. of third harmonic current which produced a rather sharply peaked wave form as a limiting condition. The usual harmonic current shown as type *B* was supplied from a small 50-watt laboratory harmonic set which gave currents of any frequency up to 1,200 cycles per second. The other extreme condition was supplied by a broken direct current giving an extremely flat topped wave form showing the usual exponential rise and decay. It is believed that the variation in form factor of the three types employed was sufficient to indicate any error if it existed. Unfortunately, owing to the limited range of the peaked wave alternator, investigation could not be made with this current in the neighborhood of the resonance point of the diafram but it is believed from the close agreement of the three types up to the limit of this machine at 240 cycles that extrapolation on this basis is permissible. The integrated value of the current in each case was given by a Leeds-Northrup galvanometer of the reflecting type shunted across a Siemens-Halske vacuum thermo-element.

The telephone receiver used in the investigation was one of the Western Electric Company's special receivers and showed a measured resistance to direct current of 543 ohms. The impedance at 1,000 cycles with currents of the order of 0.015 ampere was found to be about 1,054 ohms. The principal results are represented in the table and plotted graphically in Figure 2. The heavy line curve was obtained under the usual operating conditions and the depression in the curve at the point of resonance 535 cycles is very marked. The dotted curve which does not evidence the slightest trace of this effect was obtained with a paper washer placed between the core and the diafram and the cap screwed on very lightly. The effect of damping and air gap

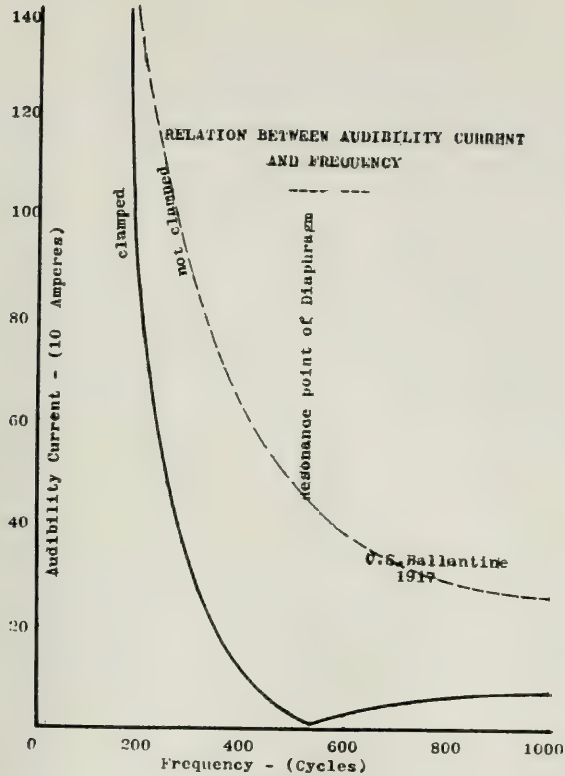


FIGURE 2

is very clearly shown by this curve. Figure 3 shows a series of values taken near the point of resonance and the slope of this curve would tend to indicate the advisability of taking ample precaution to see that the frequency of the current used in determining the audibility current by this method and that of the received signals upon which shunt readings have been obtained correspond, or the accuracy in determining the actual values of the signals in amperes is considerably lessened. With these points in mind, it seems possible by means of Professor Washburn's valuable solution of the problem to evaluate a shunted telephone reading in micro-amperes with a degree of accuracy approaching scientific precision, permitting the use of this method where the use of thermo-element and galvanometer would not be practical.

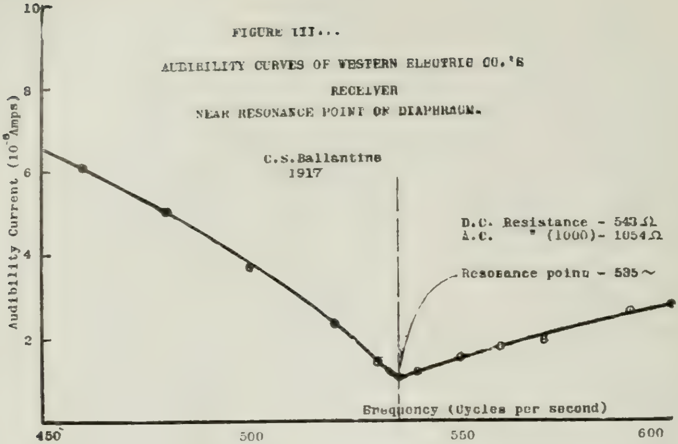


FIGURE 3

Table 1

Frequency	Current (x10)
178	99.0
200	80.1
300	30.2
410	10.1
500	3.75
520	2.32
535	1.10
600	2.75
800	6.20
1,000	7.50
1,030	7.60

H. A. Frederick: In the same way in which it appears obvious that by increasing the resistance of the bridge arms relative to the impedance of the telephone receiver, the system may be made a constant current system to any desired degree of precision and hence the audibility current be determined with any desired accuracy. So also by decreasing the resistance of the bridge arms relative to the impedance of the receiver we may obtain with any desired precision a constant voltage system. From

a determination made with such a bridge may be obtained simply the audibility voltage of the telephone. The quotient of this audibility voltage by the audibility current gives the modulus of the impedance at this value of current.

In the determination of the audibility voltage there be placed in series with the telephone receiver a variable condenser which is then adjusted so as to give the minimum audibility voltage a value of audibility voltage E' is obtained which if divided by the audibility current will give the effective resistance of the telephone receiver at this value of current, thus giving the argument of the impedance. The audibility power of course follows simply as the product of the square of the audibility current into the effective resistance just obtained. This last quantity is, of course, the quantity of real significance in the comparative determination of the efficiency of various telephone receivers.

March 13, 1917.

ADDITIONAL NOTE ON "THE COUPLED CIRCUIT BY THE METHOD OF GENERALISED ANGULAR VELOCITIES"

By

V. Bush

(Consulting Engineer, American Radio and Research Corporation, Medford, Massachusetts)

In the December, 1917, issue of the "Proceedings of The Institute of Radio Engineers," Dr. Carson gives a very interesting criticism of my summary of Wagner's proof of Heaviside's formula which appeared in the October issue. While this proof is not essential to the paper, since we have to thank Dr. Carson himself for an excellent proof of a formula which includes Heaviside's as a special case, it seems to me that Wagner's method gives promise of further application which makes discussion well worth while.

Dr. Carson bases his criticism upon the statement that the path of integration of the infinite integral (1) passes through a pole of the function. It will be noted, however, from Figures 3 and 4 of my paper that this pole is avoided by surrounding it with a small semicircle, as is, of course, the usual method of avoiding this difficulty. Perhaps I should have been more explicit in abstracting. The integral taken over the path as I showed it properly defines the function with which we have to deal.

This same remark applies to the contour integrals (2) and (4). It is unnecessary to become involved in the difficulties of evaluating an integral the path of integration of which cuts a pole of the function since we can easily avoid the pole in the manner indicated, and base the entire proof upon integrals thus defined.

It was not my intention to attempt rigor in a short abstract, for such questions can better be referred to the original paper. I believed, however, that Wagner's method would be of aid in forming a concrete conception of the quantity I have termed at the suggestion of Dr. Kennelly the *threshold impedance* of an oscillating circuit.

The use of the idea of the threshold impedance of an oscillating-current circuit will, I believe, be of considerable aid to practical men in the application to specific problems of Heaviside's formula. In much the same way in the early days of alternating currents the concept of the impedance of a circuit aided greatly in the practical application of a theory which was at that time in a somewhat similarly involved condition.

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CONTENTS

The Institute of Radio Engineers announces with regret the death of

Jesse Edgar Baker.

Mr. Baker was born at Springfield, Illinois, on February 14, 1888. He was educated at the School of Arts and Crafts, at Berkeley, California, and then entered in June, 1913, the McCloud School of Art and Design in Los Angeles, California.

Having been a student of electricity for some time, he enlisted on January 1st, 1917, in the United States Naval Reserve Force, and, because of special qualifications, was sent to one of the Naval Radio Schools. He was thereafter assigned as a Radio Electrician to several of the United States superdreadnaughts.

While in service he contracted scarlet fever, and died on March 6th, 1918. He was accorded a military burial at Inglewood Park Cemetery, Los Angeles, on March 20th. A delegation of sailors from San Pedro acted as pallbearers.

Mr. Baker was an Associate member of The Institute of Radio Engineers since September 9th, 1917.

SOME ASPECTS OF RADIO TELEPHONY IN JAPAN*

By

Eitaro Yokoyama

(Engineer of the Ministry of Communications, Tokio, Japan)

Outline of the Evolution of Radio Telephony in Japan

The investigation of radio telegraphy in Japan has been carried on for some twenty years, since the year following the basic invention of Marconi. Great advances both in theory and practice have been made. However, systematic research in radio telephony was not begun until 1906 at the Electrotechnical Laboratory of the Ministry of Communications, under the direction of Professor Dr. Osuke Asano, Ex-Director of the Laboratory. Under his direction, and more lately under the direction of Dr. Morisaburo Tonegawa, present Director, Dr. Wichi Torikata, Chief of the Radio Section, and his staff have devoted themselves to exhaustive researches continuing up to the present.

The primary object of the investigations carried on in the Laboratory at first was to obtain steady continuous electrical oscillations by any simple means, and various detailed researches were initiated. Nearly all the devices already described in publications, (including the Poulsen arc, and the Lepel and new Telefunken gaps) were tried; but they led to no useful results in the Laboratory and had no practical success. Good articulation was not obtained nor that simplicity and compactness of the apparatus which are of vital importance in devices intended for public use. The mercury vapor gap, and revolving gaps of special design, were also tried, but in vain. In 1912, after the lapse of six years, the staff of the Laboratory finally devised a new and special kind of oscillation gap which turned out to be excellent for the purpose and was, therefore, patented in Japan and several other countries. The title "T-Y-K" was given to the system of radiotelephony involving the use of these special oscillation gaps as its main feature, the initials

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of the three inventors' names being thereby represented.†
The system has been of practical utility in Japan.

Research in radio telephony is still being continued in the Laboratory in the search for perfection. There was recently invented a kind of rarefied gas discharger, which was developed thru the co-operation of Mr. Noboru Marumo and the writer. This will be described at some length below.

Meanwhile, Mr. Tsunetaro Kujirai, Assistant-Professor in the Engineering College, Tokio Imperial University, made some valuable contributions to the field. He developed a kind of arc generator in 1910, and a static frequency transformer in 1915, both of which are suitable for radio telephonic purpose and, therefore, were patented in Japan. These devices will also be considered below.

In addition to the above-mentioned gentlemen, Mr. Hidetsugu Yagi, Professor in the College of Engineering, Tohoku Imperial University, Sendai, Mr. Mitsuru Sayeki, Radio Engineer of the Ministry of Communications, and other radio engineers in the Japanese Navy and Army have also rendered much service in furthering progress of the work.

"T-Y-K" System of Radio Telephony

This is a spark system of radio telephony, and intended especially for short distance communication service, the essential features being compactness, simplicity, and cheapness of the apparatus.

The utility of this radiophone system has been sufficiently proved by its practical employment on both land and sea. The system is now in daily commercial use at the three land stations in the Bay of Ise, which is located in the central portion of the coast of Japan. Tho the distance between the farthest two of the above stations is merely eight miles (13 km.), experimental communication was established over a distance of more than 70 miles (110 km.) between shore and ship.

A wall-set type of the apparatus is shown in Figure 1. It was evolved by the joint effort of Messrs. W. Torikata, Masajiro Kitamura and the writer, and is now used in Japan.

Many publications have already been made relative to the system, certain features of this and a further description will not be necessary in this paper.

†(The inventors being Messrs. Wichi Torikata, Eitaro Yokoyama, and Masajiro Kitamura.—Editor.)

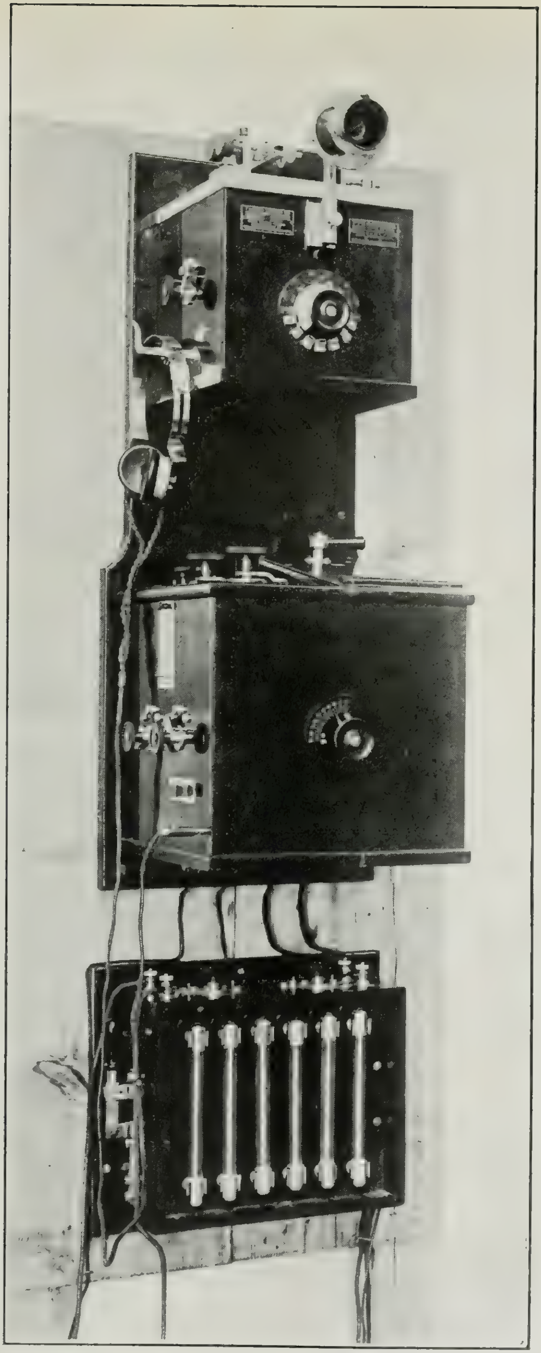


FIGURE 1—A Wall-set Type of "T-Y-K" Radio Telephone Apparatus

A Rarefied Gas Discharger—Evolution of the Discharger

The investigation was begun with a glass bulb discharger containing rarefied air as shown in Figure 2, which was inserted in the position of discharger in an ordinary oscillation producing

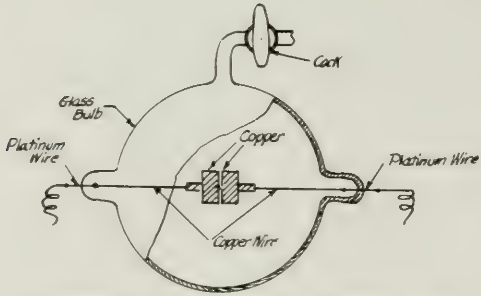


Figure 2—First Form of the Rarefied Air Discharger

circuit as shown in Figure 3. In Figure 2, the glass bulb was of spherical form with a diameter of about 10 cm. (4 inches), and the electrodes made of copper, flat cylindrical in shape, 12 mm. (0.3 inch) in diameter, facing each other wth a clearance as small as a fraction of a millimeter. In Figure 3, G is a direct

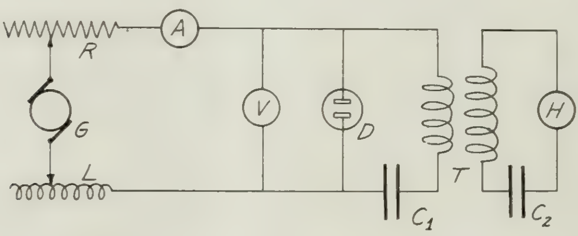


Figure 3—Oscillation Producer with Rarefied Air Discharger

current generator of 500 volts, R a resistance, L an inductance, A a milliammeter, V a static voltmeter, D a rarefied air discharger, C_1 a condenser in the primary oscillatory circuit, C_2 the same in the secondary, T an oscillation transformer and H an ammeter for radio frequency current. When the system was

in adjustment and the degree of rarefaction of the bulb properly adjusted, the generation of oscillation current in the secondary was indicated by the deflection of ammeter H; but it was noticed that there was objectionable irregularity in the discharge thru the gap and consequently in the oscillation current in the secondary, and, furthermore, the oscillations died out after a couple of seconds. When the test was repeated with an aluminum electrode in place of one of the copper ones, the other parts of the bulb remaining the same, it was found that the discharge was not only very steady but also that the oscillations lasted much longer. As the discharge was smoother than any obtained previously in a usual spark transmitter, the research was pressed in many details for the purpose of utilizing the new discharger as a source for radio telephone transmission or radio frequency measurements.

It was, however, noticed that the oscillations would not last more than a few minutes with a bulb of the above construction. In the test a copper electrode was used as anode and aluminum as cathode. If the polarity were changed, it could be seen distinctly that the discharge became irregular and the oscillations faded away very quickly.

It was found that when the discharge was continued until the oscillations in the secondary died away entirely and the bulb was then allowed to cool for some time, (the power source being disconnected) and then the oscillations were started again, strong oscillations took place in the secondary as steadily and of the same power as the first time. This led the investigators to the theory that the temperature rise in the electrodes might play an important part in causing the decay of oscillating current. Another bulb with large metallic stems attached to the electrodes was then made as shown in Figure 4. Besides this, the bulb was so constructed that the electrodes could be easily replaced which enabled the investigators to make tests with several different forms and materials of electrodes. In tests with this bulb connected in the circuit of Figure 3, there were obtained not merely a steady discharge in the gap and smooth oscillations in the secondary, but also a remarkable increase in the duration of the oscillations, for example, as much as thirty or forty minutes. After further investigations dealing with the construction of the bulb and the shape of the electrodes, the investigators developed a rarefied air tube of the form shown in Figure 5, with which they succeeded in producing exceedingly steady currents in the secondary for hours continuously.

The tube of Figure 5 is suitable only for small powers, for example, for 100 watts. There was, therefore, developed another form of discharger suitable for fairly large power, which latter form is shown in Figure 6. The latter has a thick-walled metallic case in place of the glass bulb in the former, the interior construction remaining almost the same.

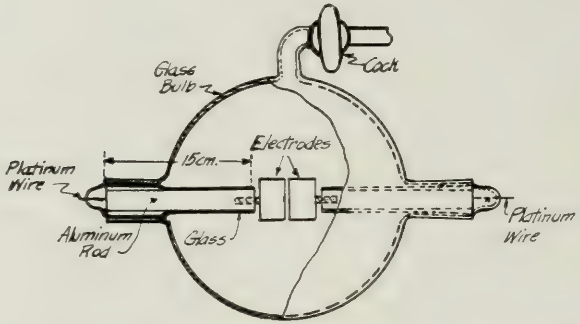


FIGURE 4—Second Form of Rarefied Air Discharger

The dischargers, as finally constructed, were used in a radio telephone transmitter circuit which was the same as the circuit of Figure 3 except that the secondary closed circuit was replaced by a real antenna, and a very steady and pure wave, suitable

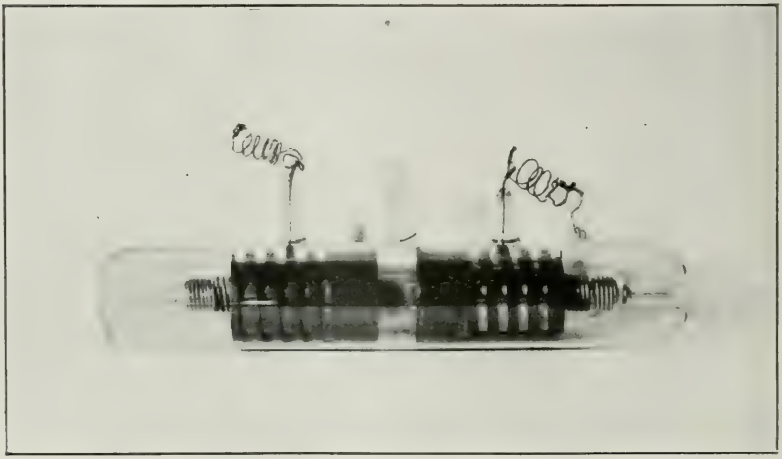


FIGURE 5—Latest Form of Rarefied Air Discharger for Small Power

for radiophone use was obtained in the antenna circuit with a very close coupling such as about 60 per cent. in the oscillation transformer. This indicated a good quenching action in the discharger.

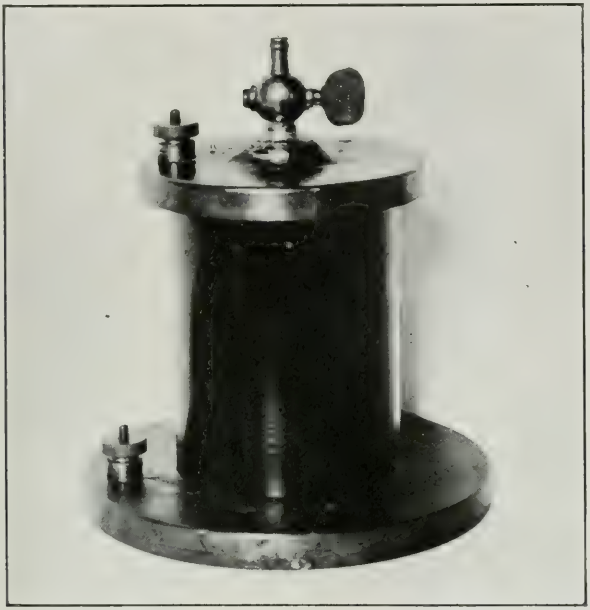


FIGURE 6—A Form of Rarefied Air Discharger for Larger Powers

The above descriptions apply to a discharger having an atmosphere of rarefied air, but the kind of atmosphere used has a perceptible influence on its operation and life. For instance, the experiments indicated that ammonia gas was very effective, carbonic acid gas fairly good, while alcohol, ether, and benzene in the state of vapor were all useless.

It is very interesting to note that Professor R. A. Fessenden[1] and Professor Max Wien[2] made similar investigations with vacuum dischargers.

ARRANGEMENTS FOR THE TESTS

The writer desires to describe briefly the results of the tests dealing with the influence of air pressure in the discharger, length

[1] United Kingdom Patent, Number 28,647, 1907.
[2] "Jahr. der draht. Telegraphie u. Telephonie," volume 4, page 135.

of the discharge gap, dimensions, shape and materials of the electrodes, and supply voltage to discharger, and the effect of these on its life. Thruout all the tests, there was almost always used the arrangement shown in Figure 3.

To exhaust the dischargers a Gaede rotary pump was used as shown in Figure 7, which pump is claimed to reduce 6 liters of air from atmospheric pressure to 0.006 mm. of mercury in 15 minutes. For a vacuum gauge a manometer was used as shown in Figure 8 which enabled measuring roughly any pressure between a little below 1 mm. and 220 mm. of mercury.

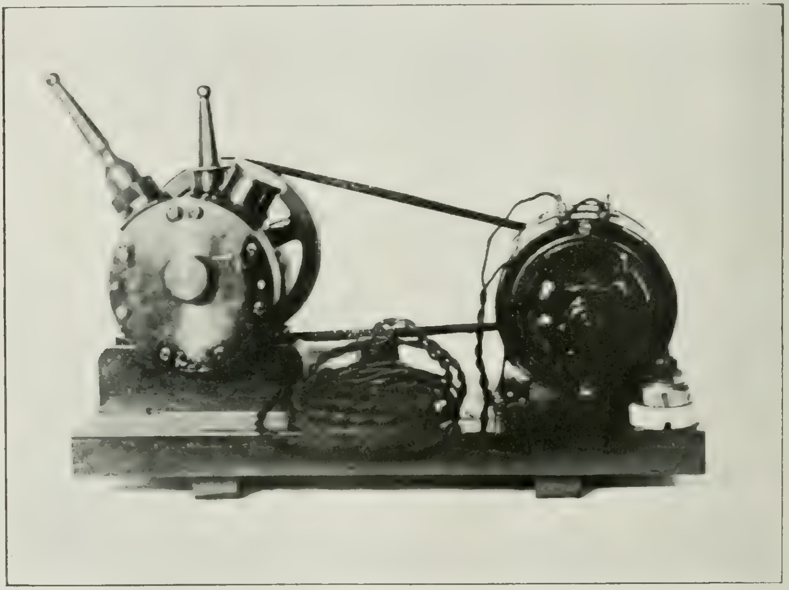


FIGURE 7—Gaede Rotary Pump Used in the Tests

In the tests the wave length was adjusted at about 2,000 meters thruout, with a condenser of capacity of about 0.065 microfarad in the primary and about 0.02 microfarad in the secondary. As a final test, however, the result was confirmed with a circuit of some 500 meters wave length, using a condenser of rather small capacity in the primary and a real antenna in the secondary.

Influence of Air Pressure on the Discharge

The discharge conditions in the rarefied air discharger largely depend on the air pressure. A change in the type of discharge was observed with variation of pressure. Copper was used as the positive electrode of the discharger and aluminum as the negative, the clearance between being about half a millimeter

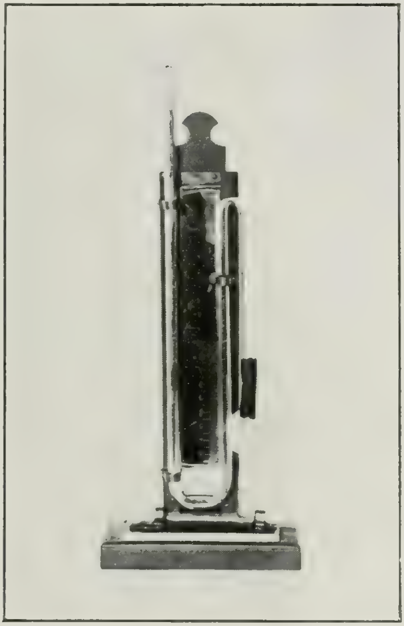


Figure 8 Vacuum Gauge Used in the Tests

(0.02 inch). At the center of discharge surface of the latter a pin-hole was bored, which made the discharge steadier. The voltage of the direct current power source was kept at nearly 440 thru the test.

The observations under these condition are summarized as follows:—

(1) On reducing the air pressure gradually, beginning with ordinary atmospheric pressure, it was noticed that the discharge suddenly bridged over between the electrode surfaces at a pressure somewhere near 70 mm. of mercury.

(2) Within the range from about 70 mm. down to about 40 mm.,

there will be quiet discharge in a straight line between the centers of the electrode surfaces. The discharge is not very steady tho the oscillations produced thereby in the secondary are apparently quite smooth.

(3) On further reducing the pressure to below 40 mm., the discharge comes to have an entirely different appearance. The discharge, which before was in the form of a single straight line, splits into many lines terminating over the whole surfaces of the electrodes like a brush, while sometimes it changes into the form of a glow discharge, these changes depending on the conditions in the oscillation circuit. In this case, the terminal voltage at the gap has its smallest value, and the oscillations in the secondary are very weak and irregular.

(4) At pressures between about 10 mm. and 2 mm., the discharge is again gathered into a single straight line, just as in case (2). In this case, the discharge is, however, remarkably steadier and accordingly the oscillations are also steadier and smoother.

(5) Below the above limit of pressure, the discharge tends to change into a diffused glow, and the oscillations rapidly die away.

It was noticed that good oscillations are produced only when the discharge is in stages (2) and (4), and that stage (4) is especially excellent for utilization as a power source in radio telephony.

Influence of Gap Clearance upon Discharge

Descriptions have been given above of the discharge phenomena, and the range of best pressure for the production of steady oscillations has been considered, but only in the case of a certain definite gap clearance namely, 0.5 mm. To investigate the relation between the gap clearance and the corresponding most desirable pressure, a series of tests were made, the gap clearance being varied from 2 cm. (0.8 inch) down to 0.1 mm. (0.04 inch), The results of these tests are shown in Figure 9. This figure shows that, in the case of short gap clearances, the useful range of air pressure is rather wide and extends to low vacua, while for longer gap clearances, it is closely limited to higher vacua. Furthermore, there is no abrupt change between stages (2)

and (4) for clearances longer than about 2 mm. (0.08 inch), stage (3) disappearing entirely.

To obtain data on the best operating conditions, the primary supply current, the terminal voltage at the gap, and the secondary oscillation current were simultaneously measured for various gap clearances with the corresponding most suitable air pressure, the supply voltage being kept constant. The results of

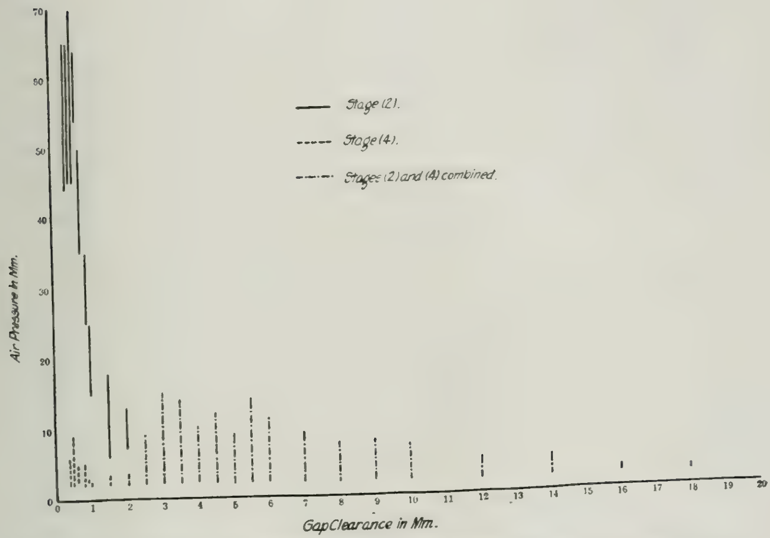


FIGURE 9—Relation Between Gap Clearance and Most Suitable Pressure

the tests in stages (2) and (4) are shown respectively in Figures 10 and 11. The data of the measurements for gap clearances longer than about 2 mm. (0.08 inch), (where the stages (2) and (4) cannot be differentiated), are all included in the curves of Figure 11. These two series of curves show that the increase in gap clearance decreases both the primary supply current and the secondary oscillation current, and increases the gap terminal voltage. The conclusion is, therefore, that it is most advantageous to use as short as possible a gap clearance. However, it must be remembered that there will be a lower limit to the shortest practical gap clearance because of other causes, such as heating of the electrodes which may melt them together in a very short gap.

Influence of Dimensions of Electrodes on Discharge

As stated above, the time of continuance of discharge in the gap (and accordingly the duration of the secondary oscillation current), depends largely on the dimensions of the discharger electrodes. The dischargers of Figures 2, 4, and 5 were compared in this connection, and the results are shown in Figure 12.

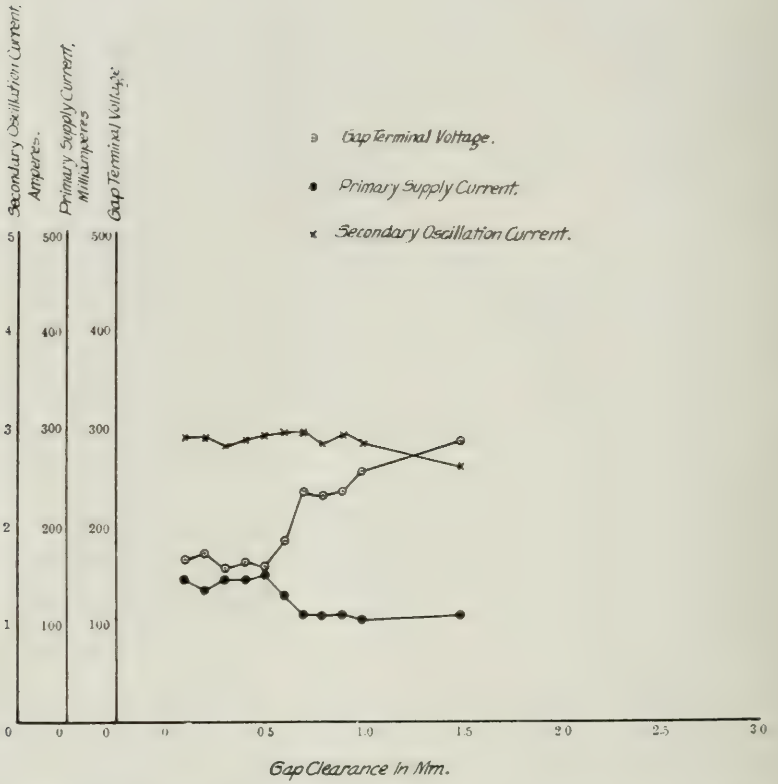


Figure 10—Relation Between Gap Clearance and Supply Current, Gap Terminal Voltage, and Secondary Oscillation Current. Stage (2)

With the discharger of Figure 2, the discharge conditions in the gap became highly unfavorable and the secondary oscillations remarkably feeble after only some thirty seconds of operation. Using the discharger of Figure 4, the time of continuance of the discharge and of the secondary oscillations was, on the contrary, prolonged as long as some thirty minutes. With the

discharge tube in Figure 5, the operating condition of the discharger was still very good after two hour's continuous use.

As the supply voltage, the supply current, and the capacity of condensers, etc., were not exactly the same in each case shown by the curves of Figure 12, the magnitude of the secondary oscillation current cannot be fairly compared from these curves.

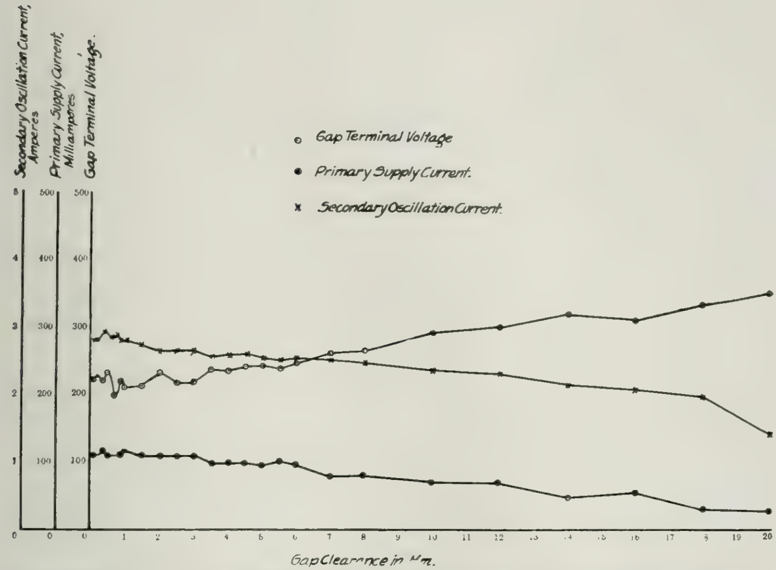


Figure 11—Relation Between Gap Clearance and Supply Current, Gap Terminal Voltage, and Secondary Oscillation Current. Stage (4)

In addition, since sufficient precautions were not taken as regards the perfect sealing of the dischargers (there being made for temporary experimental use), the air pressure within probably changed in the course of the test. Otherwise the tube of Figure 5 should have lasted far longer.

Influence of Shape of Electrodes on Discharge

At first, there were used electrodes of the form of Figure 13 (a). It was found that the discharge was not very regular and accordingly the oscillation produced not very steady. On attempting to use the electrodes having the form of Figure 13 (b), which were exactly the same except with a pin-hole at the center of their respective discharging surface, it was noticed

that discharge and oscillation were remarkably improved. Comparing these two kinds of dischargers as the oscillation generator for radiophone work, the latter was found very satisfactory, while the former gave objectionable noise in the receiving telephone because of the irregularity of the discharge. It being

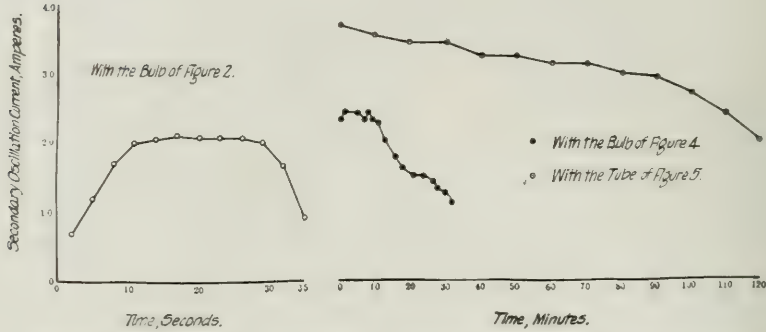


FIGURE 12—Effect of Dimensions of Electrodes on Time of Continuance of Discharge

evident that the shape of the electrodes had some influence on the nature and duration of the discharge, it was attempted to make a comparison using the four shapes of electrodes shown in Figure 13, all possible combinations of the four kinds being tried. In the test copper was used as anode and aluminum as cathode, the clearance between the electrodes being kept constant at 0.5 mm. (0.02 inch).

The best result was obtained by using the electrodes shown in Figure 13 (b) for both terminals. The discharge was irregular with the electrodes Figures 13 (a) or (d), as either one of the electrodes; this being due to the wondering of the discharge over the electrode surfaces. A discharger with (c) as either one of the electrodes also gave bad results in the long run, probably owing to excessive heating of the points.

Influence of Electrode Materials on Discharge

A discharger with copper electrodes as shown in Figure 2 was used at first. The secondary oscillation current produced in this case was not only very unsteady, but lasted only a few seconds. By replacing the copper anode by an aluminum one, the secondary oscillations were greatly improved as mentioned above. It is interesting to note that, in the ordinary atmosphere,

a discharger with copper—copper electrodes gives stronger secondary oscillations than one with copper—aluminum electrodes tho there is a little difficulty in starting the discharge in the former case.

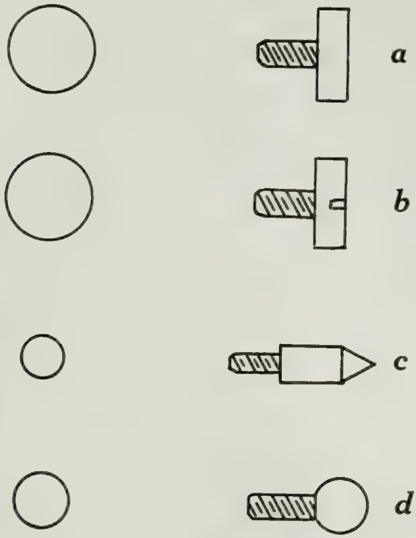


FIGURE 13—Various Forms of Electrodes Tested

A series of tests was made, several different metals being tried as electrodes, and it was finally found that aluminum was the best material for the negative electrode. With aluminum as a negative electrode and various metals as the positive electrode, the measurement of secondary oscillation current and corresponding primary supply current was made, other circuit condition remaining the same. The results of these measurements are plotted in the curves of Figure 14. It is noticeable in the curves that the combination of aluminum-aluminum electrodes gives the poorest result. This is probably due to the effect of polarity, dissimilar electrodes giving better result.

A series of experiments was also made with several different crystals (artificial and natural) such as silicon, carborundum, magnetite, zincite, etc., as electrodes. It is very interesting to find that there were several combinations of electrodes which gave very good results without the use of aluminum as the

negative electrode. Generally speaking, the results with crystals were nearly the same as those with metals. However, it is very difficult to shape crystals into suitable forms; and since there is no necessity for the use of crystals as electrodes, the investigations were carried no further in this connection.

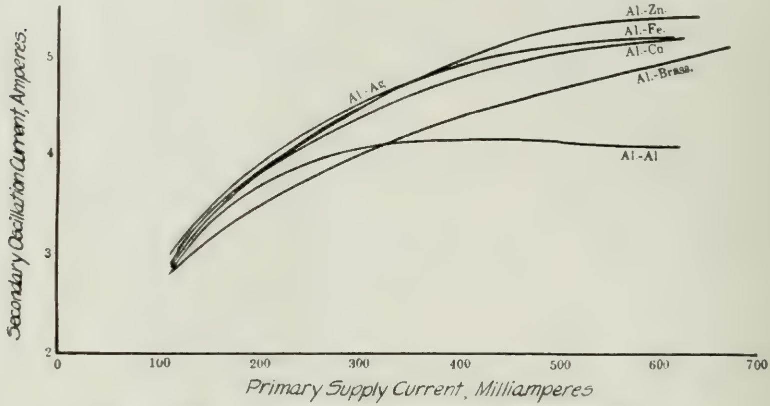


FIGURE 14—Oscillation Current Produced with Dischargers Having Combinations of Various Electrodes

Influence of Supply Voltage on Operation of Discharger

In the ordinary atmosphere a small clearance between electrodes is necessary to obtain continuously a discharge with a voltage less than 500. The use of so small a gap is likely to give rise to frequent short-circuits. In the case of the use of such a gap in air, it is necessary to insert a certain amount of resistance in series in the primary supply circuit in order to prevent short-circuits which might injure some parts of the apparatus because of the passage of abnormally large currents. As described above, the rarefied air discharger works well with a fairly wide clearance of electrodes in comparison with one at atmospheric pressure, tho the shorter the clearance the better operation can be obtained. As the operating conditions prevents short-circuiting, and the lack of air in the discharger doubtless greatly assists regular working, series resistance can be easily dispensed with, and a much lower voltage for the power source is sufficient for perfect functioning of the discharger. The experiment being made of varying the supply voltage from about 320 to 580, it was confirmed that equally good results were obtained at any point in this range of the supply voltage, the gap terminal

voltage remaining nearly constant and in the vicinity of from 230 to 240.

Some Considerations Relative to the Life of Discharger

Descriptions have already been given of the proper construction for the rarefied air discharger. In the long run, deviations from good adjustment will occur even in a well constructed discharger, and the secondary oscillation current will gradually fall off. Some consideration will be given here to the causes of this effect and their remedies.

The life of the discharger depends largely upon its working in either stage (2) or (4). It has already been mentioned that stage (4) is much more suitable than stage (2) in a discharger with rarefied air as atmosphere, but the latter stage can be made equally as suitable by introducing a certain kind of gas in the discharger instead of air. Since the discharger working in stage (2) can be, moreover, operated thru a much wider range of air pressure than when in stage (4), satisfactory operation in stage (2) is less effected by variation of pressure. A discharger in stage (2) with a special kind of gas inside is, therefore, as good as, and has a longer life than a discharger in stage (4) with air.

Supposing a discharger to have been well constructed, and with the precautions observed which have been considered under the several headings above, it will still finally reach the end of its life because of the pressure variation of the contained air arising from the following well known causes: (a) imperfect elimination of occluded gas from the metallic bodies, and of water vapor from the surfaces of these metallic bodies and the glass wall; (b) disappearance of gas due to discharge; (c) changes in electrode surfaces due to discharge.

The defect (a) can be gotten rid of, to a certain extent, by submitting the glass and the metallic bodies of the discharger to high temperature when constructed. As regards the defect (b), there seems to be no means whereby it can be perfectly cured, tho it can be regulated by a method similar to that used for the adjustment of the "hardness" of X-ray tubes. As for the defect (c), the use of electrodes constructed as in Figure 13 (b) makes the discharge steady for some time from the beginning, but the irregularity gradually increases in the long run, the smooth straight line discharge altering into a poor discharge in the form of a brush. The discharge could be improved by covering the surface of the electrodes with a thin film of glass or enamel except at the center of discharge surfaces.

Rotary Gap for Radiophone Transmitter[3]

Mr. T. Kujirai has invented a kind of nearly sustained oscillation producer which is suitable for radio telephone transmitters.

His first device was made in public in Japan as early as 1910. The principal part of the apparatus consisted of a metallic or carbon rotary disc, directly driven by an electric motor, as one electrode, this being in light contact with a metallic or carbon brush, as the other electrode; direct current being used as the power source. His sustained oscillation producer was a combination of the rotary gap and an ordinary oscillation circuit in shunt.

The apparatus has been successively improved, until he finally modified it in 1912 producing a form which is more suitable for radio telephone purposes. The latest arrangement consists of a rotary brass disc and an aluminum point, which is shown in Figure 15. The circuit arrangements used by him in connection with his gap are shown in Figure 16.

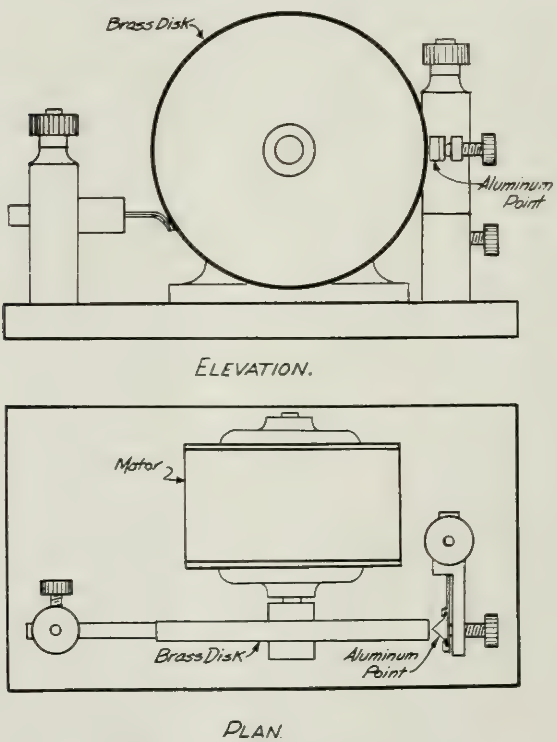


Figure 15—T. Kujirai's Rotary Gap

[3] For much of the information here given the writer is indebted to the inventor.

The power is supplied from a 500-volt direct current generator thru a resistance and an inductance to a gap not greater than 0.5 mm. (0.02 inch) in length. The supply current used in his arrangements varied 0.2 to 1.0 ampere, depending on the capacity in the oscillation circuit.

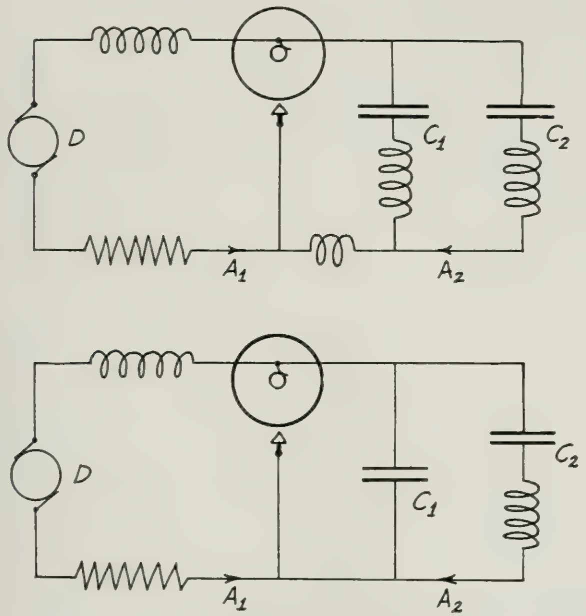


FIGURE 16—Circuit Arrangements for the Rotary Gap

The frequency of oscillation may be varied thru a wide range without affecting the stability of the discharge by varying one of the capacities in the oscillation circuit.

Figures 17 (a) and (b) are sample curves showing the variation of the oscillation current with the supply current.

Using this arrangement, he is said to have succeeded in communicating articulate speech more than 20 miles (32 km.).

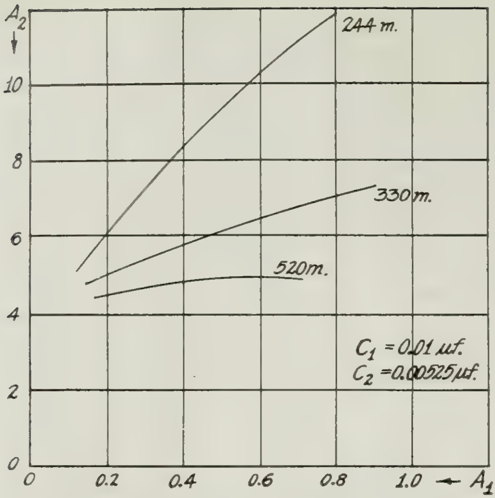


FIGURE 17 (a)—Variation of Oscillation Current with Supply Current

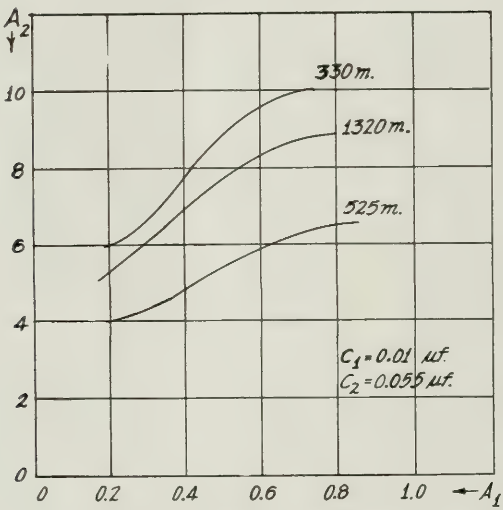


FIGURE 17 (b)—Variation of Oscillation Current with Supply Current

STATIC FREQUENCY TRANSFORMER[4]

Among others[5], Mr. T. Kujirai has invented a method of tripling the frequency of an alternating current in 1915, which

[4] For much of the information here given the writer is indebted to the inventor.

[5] See the paper "Radio Frequency Changers," by A. N. Goldsmith, "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," March, 1915.

method was found to be very useful, not only in the common electrical engineering field, but also in radio telephony.

This static frequency transformer consists of three elements, two of which have their cores oppositely polarized by direct current thru an inductance X and windings D_A, D_B, Figure 18, while the third element is non-polarized.

The primary (P_A, P_C, P_B) and secondary (S_A, S_C, S_B) windings are respectively connected in series, but the secondary (S_C) of the non-polarized transformer element is connected in opposition to those (S_A, S_B) of the other two polarized elements.

The function of this arrangement is that the induced electromotive forces E_A and E_B (Figure 19) in the secondary windings of the polarized transformer elements are asymmetrically distorted owing to the magnetic saturation of their iron cores, while the induced e. m. f. E_C in the non-polarized transformer element remains entirely symmetrical but remarkably peaked

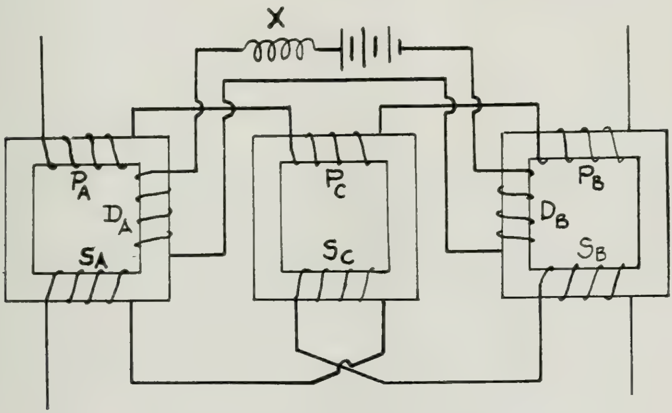


FIGURE 18—Arrangement of the Static Frequency Transformer of T. Kujirai

owing to the low magnetic density in its iron core. The distorted e. m. f.'s being superposed in opposition to the symmetrical one, the resultant of these e. m. f.'s will have such a form as E_z (Figure 19) which has a weak component of the fundamental frequency and a strong third harmonic, as well as higher harmonics.

Figures 20 (a), (b), and (c) are oscillograms showing the

constitution of the secondary e. m. f. (a) is an induced e. m. f. in the non-polarised core transformer, (b) the resultant e. m. f. of the two polarised core transformers, and (c) the resultant of the e. m. f.'s of the three transformers. The oscillograms were taken by the inventor at University College, London.

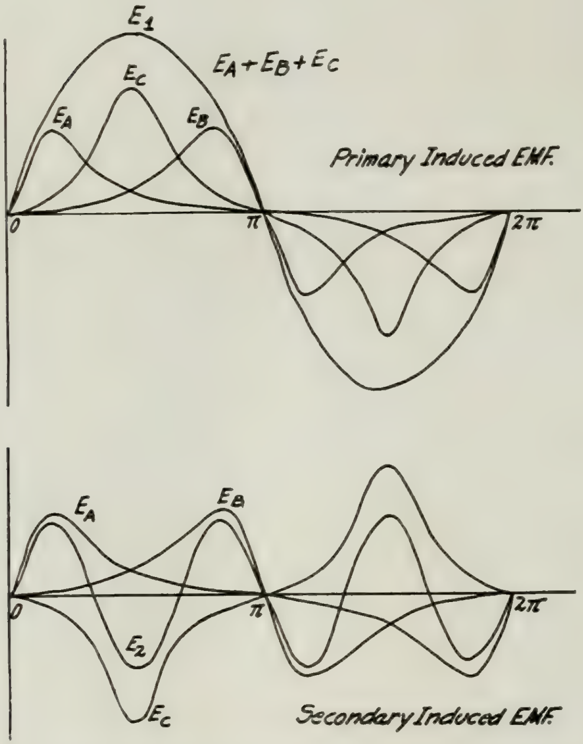


FIGURE 19—Diagram Showing the Principle of Kujirai's Transformer for Tripling Frequency

Figure 21 shows an example displaying the construction of a static frequency transformer which is now being used for experiments in radio telephony in connection with a radio frequency current alternator of the Alexanderson type. in the Electrical Engineering Laboratory of Tokio Imperial University. The principal electrical data of the transformer are as follows:

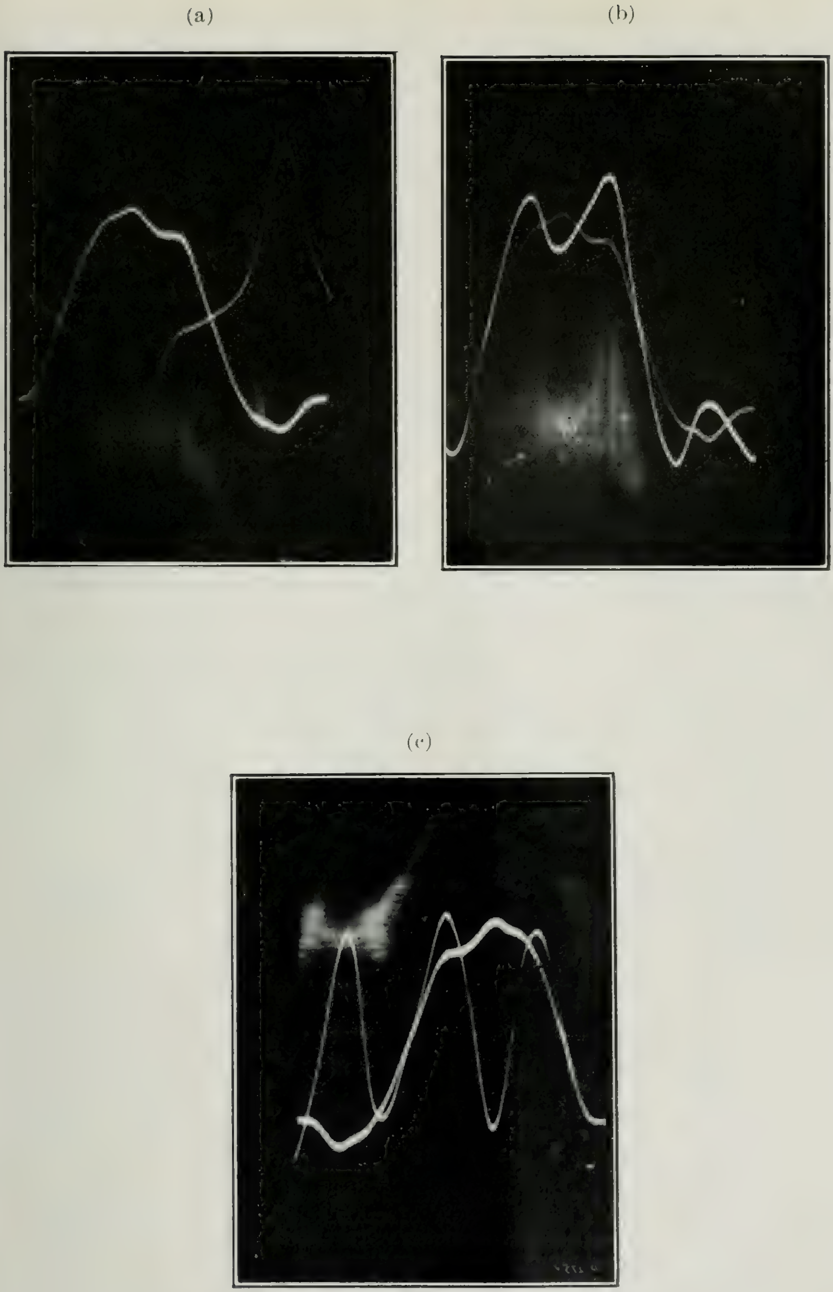


FIGURE 20—Oscillograms Showing the Constitution of Secondary E.M.F. in the Static Frequency Transformer of Kujirai

Primary capacity.......	1.3	k. v. a.
Primary frequency......	40,000	cycles
Secondary frequency....	120,000	"
Primary voltage........	260	volts
Secondary voltage......	120	"
Primary current........	5	amperes
Secondary current......	3	"

New York, August, 1917.

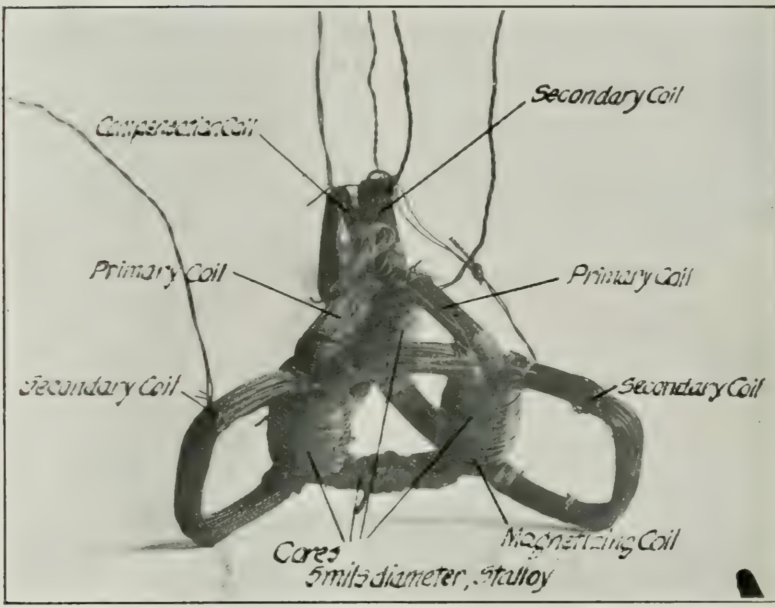


FIGURE 21—Static Transformer, with Case Removed

SUMMARY: After outlining the experiments in radio telephony carried out in Japan since 1906, the "T. Y. K." system is cited. A newer system using, in the most favorable case, copper and aluminum electrodes about 0.2 inch (0.5 mm.) apart in rarefied air or other gas at a pressure of about 40 to 70 mm. as the discharger is then described in great detail. The influence of electrode design, gas pressure, electrode material, and other factors is considered. Means of securing steady and continued operation are given.

Mr. T. Kujirai's earlier system of radio telephony is then described. This involves a direct current, moderate voltage discharger having a rotating brass disc and a fixed aluminum point as electrodes.

A ferromagnetic frequency tripler due to Mr. T. Kujirai is discussed and operating data for its use in conjunction with an Alexanderson radio frequency alternator are given.

A DYNAMIC METHOD FOR DETERMINING THE CHARACTERISTICS OF THREE-ELECTRODE VACUUM TUBES*

By

JOHN M. MILLER

(BUREAU OF STANDARDS, WASHINGTON, D. C.)

In three-electrode vacuum tubes, such as the audion or pliotron, we are concerned with two circuits, that between the grid and filament or input circuit and that between the plate and filament or output circuit. The current which flows in the grid circuit is of importance in determining the power input and the detecting action of the tube, but in the use of the tube as a relay or amplifier it is usually negligible and will not be considered in the following.

CHARACTERISTIC SURFACE AND CURVES OF THE PLATE CURRENT

In tubes, with a high vacuum, the value of the plate current, at a given instant, is a function of the values of the plate and grid voltages at the same instant. A surface, called the characteristic surface, is required to represent this function. Langmuir[1] has given the equation

$$i_p = A\ (v_p + k\ v_g)^{3/2}$$

as representing the equation of the characteristic surface. In this equation i_p is the plate current, v_p the plate voltage, v_g the grid voltage, and k is a constant for a given tube construction and is a relative measure of the effects of grid and plate voltages upon the plate current.

In investigating the functioning of the tubes experimentally, it is customary to determine the static characteristic curves of the plate current. These curves are the intersections of the characteristic surface with the plane surfaces v_p = constant, or v_g = constant. The first of these represents the variation of the plate current with the grid voltage when the plate voltage is

* Received by the Editor, May 9, 1918.

[1] I. Langmuir, "PROC. INST. RADIO ENGRS.," 3; page 261, 1915.

constant, the second, the variation of plate current with plate voltage with the grid voltage held constant. These characteristic curves are usually obtained by varying in steps the battery voltages applied between the elements of the tube and reading the plate current corresponding to the applied voltages. The slopes of these curves, being the ratio of current to voltage, have the dimensions of a conductance. This method is very slow and inaccurate.

In many cases the operation of a tube takes place about a point in the characteristic surface where the surface is nearly plane. Also when the variations are confined to a small area about the operating point and we are not concerned with distortion, the characteristic surface may be considered to be a plane. Thus the equation of the characteristic surface about an operating point (i_p, v_g, v_p) may be written, as given by Vallauri[2] in the form

$$i_p = a\,v_g + b\,v_p + c$$

where a is the slope of the plate-current grid-voltage characteristic curve passing thru the operating point while b is similarly the slope of the plate-current plate-voltage characteristic. The quantities a and b are fundamental in determining the behavior of a tube as an amplifier and oscillator as has been shown by Vallauri.

Amplification Constant and Internal Resistance

It is, however, more convenient to deal with the quantities $\frac{a}{b}$ and $\frac{1}{b}$. The first of these is the same as k in Langmuir's equation; and, as mentioned before, is a constant for a given tube. It is called by H. J. van der Bijl[3] the amplification constant. The quantity $\frac{1}{b}$ is the internal a. c. resistance of the tube in the plate circuit and will be designated R_i. Its value depends upon the plate and grid voltages and to some extent upon filament temperature.

Assume that the plate circuit is closed thru an impedance Z, which may be the impedance of a pair of telephones in the case of ordinary use or may take the form of a pure resistance in the case of a resistance-coupled amplifier or the primary of a transformer in the case of a transformer-coupled amplifier.

[2] G. Vallauri; "L'Elettrotecnica," IV, 3, page 43, 1917.
[3] Unpublished paper of September 17, 1917.

Assume also that an alternating e. m.f. e_g is impressed between the grid and filament. It then follows that the alternating current in the plate circuit will be the same as that which would flow in a simple a. c. circuit in which the impressed e. m. f. is $k\,e_g$ and which contains a resistance R_i in series with the inserted impedance.[4]

To prove this theorem let us introduce the additional notation

I_p = d. c. component of the plate current.
i = a. c. component of the plate current.
E_g = voltage of grid battery.
e_g = a. c. impressed e. m. f. on grid.
E_p = voltage of plate battery.

and assume an impedance $z=(x+j\,y)$ inserted in the plate circuit which has a d. c. resistance x'.

Then $i_p=I_p+i$

$v_g=E_g+e_g$

$v_p=E_p-I_p x'-i\,(x+j\,y).$

By Vallauri's equation we have

$$i_p=a\,v_g+b\,v_p+c$$

and substituting in this the values of i_p, v_g, and v_p,

$$I_p+i=a\,(E_g+e_g)+b\,[E_p-I_p x'-i\,(x+j\,y)]+c.$$

But the steady current

$$I_p=a E_g+b\,(E_p-I_p x')+c,$$

hence

$$i=a\,e_g-b\,i\,(x+j\,y)$$

$$\frac{a}{b}e_g=i\,\frac{1}{b}+i\,(x+j\,y)$$

or

$$k\;e_g=i\;R_i+\;i\;(x+j\;y)$$

This latter is the equation of e.m.f.'s for an impressed e.m.f $k\,e_g$ in a circuit with a resistance R_i in series with an impedance $(x+j\,y)$.

As an example take the case of a resistance-coupled amplifier, in which case an alternating e.m.f. e_g is impressed between the grid and filament of one tube and the e. m. f. across a resistance R in the plate circuit of that tube is introduced into the grid circuit of the next tube. The voltage amplification is the

[4] The author is indebted to Mr. H. H. Beltz of the Bureau of Standards for suggesting this theorem.

ratio of the e. m. f. handed on to the next tube, iR, to the impressed e. m. f., e_g.

We have

$$k e_g = i R_i + i R,$$

hence

$$\frac{i R}{e_g} = k \frac{R}{R_i + R}$$

This shows that the voltage amplification increases as the coupling resistance R is increased, approaching as a maximum the value of the amplification constant k. This assumes that R_i and, therefore, the actual voltage between plate and filament, remains unchanged. Other amplifier problems and the conditions for oscillation of numerous circuits may be readily worked out by the use of this simplifying theorem.

Experimental Method for Determining the Amplification Constant and Internal A. C. Resistance of Three-Electrode Vacuum Tubes

Instead of deriving the values of the amplification constants and internal resistance indirectly from the static characteristic curves, they may be determined directly and rapidly in a very simple manner using alternating current of audio frequency. The circuit is that shown in Figure 1, in which $c d$ is a slide wire supplied with a small audio frequency current from an alternator thru a step-down transformer or by coupling to a tube source. A variable ground connection not shown in the figure is likewise put on a rheostat which is in parallel with the slide wire in order to obtain a better minimum in the phones. R is a dial resistance box going up to about ten thousand ohms which may be connected in circuit by means of the switch S.

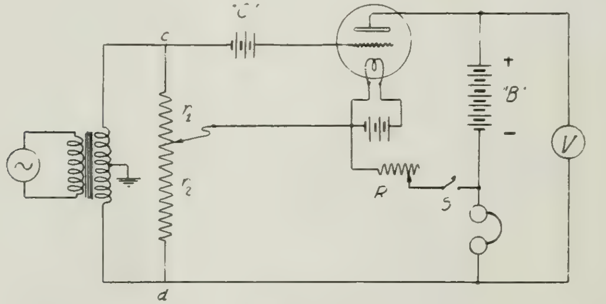


Figure 1

First, the amplification constant k is determined with the switch S open, by adjusting the slider until the telephones are silent when

$$k = \frac{r_2}{r_1}.$$

To determine the internal resistance R_i, the switch S is then closed and the slider set at, say, the middle point and silence is again obtained by varying the resistance R. Then

$$R_i = (k-1)\,R$$

Any other definite ratio of $\frac{r_2}{r_1}$ (less than k) can be used in which case

$$R_i = \left(\frac{r_1}{r_2}k - 1\right)R$$

The above expressions are readily proven. In the first determination an e. m. f. e_g which is proportional to r_1 is introduced between the grid and filament. As pointed out before, this has the effect of impressing an e. m. f. $k\,e_g$ in the plate circuit. The e. m. f. across r_2 which is $\frac{r_2}{r_1}\,e_g$ is 180° out of phase with the e. m. f. $k\,e_g$ and will balance it when $\frac{r_2}{r_1}\,e_g = k\,e_g$. Hence

$$k = \frac{r_2}{r_1}.$$

In the second case an e. m. f. e_g proportional to r_i is likewise impressed between grid and filament, which will produce an alternating current $\frac{k\,e_g}{R_i + R}$ in the plate circuit provided no current flows thru the phones. The e. m. f. across the resistance R is $\frac{k\,e_g\,R}{R_i + R}$. This is balanced by the e. m. f. $\frac{r_2}{r_1}e_g$ across r_2 when

$$\frac{r_2}{r_1}e_g = \frac{k\,e_g\,R}{R_i + R} \quad \text{or} \quad R_i = \left(\frac{r_1}{r_2}k - 1\right)R,$$

and in the case when the slider is set at the middle point $(r_1 = r_2)$,

$$R_i = (k-1)\,R.$$

As noted before, the amplification constant is very nearly a constant for a given tube, but the internal resistance R_i varies with plate and grid voltages and to some extent with filament current. Curves may be obtained for R_i as a function of these variables obtaining directly and accurately all of the data furnished indirectly and inaccurately by the static characteristic curves excepting the grid current characteristics.

In the actual set-up, the slider cd of Figure 1 was seven ohms in resistance and consisted of ten turns of resistance wire inductively wound on a marble cylinder, each turn corresponding to one hundred divisions on the scale. It would be preferable to use a straight wire in order to reduce the inductance. This can be marked to read amplification constant directly. The current in the slide wire and hence the voltages acting on the tube should be kept so small that the operation of the tube takes place over a portion of the characteristic so limited that it is practically a straight line. In the measurements described herein, a current of 50 milliamperes or less was used. This current was supplied by either an alternator or tube source and up to 2,000 cycles per second, and for the tubes investigated no change due to frequency was observed. A buzzer source of interrupted current will suffice in many cases.

The dial resistance box R consisted of non-inductively wound coils and, as noted before, went up to ten thousand ohms. In some cases the internal resistance of the tube may be so high that it becomes necessary to use a ratio of $\frac{r_1}{r_2}$ greater than unity in order that a balance can be obtained with this box.

In obtaining values of the internal resistance for given plate voltages, it must be remembered that since the direct current flowing thru the tube must flow thru the telephones and R in parallel, the actual voltage applied to the tube will be somewhat less than that of the plate battery. In order to measure the actual voltage on the tube, the voltmeter should be connected across the battery and telephone receivers as shown in Figure 1, and the measurement made with the voltmeter key closed. It is desirable to use a high resistance voltmeter and telephones of moderately low resistance, since then the applied voltage when the measurement of k is made, with the switch S open, will differ only slightly from that acting when R_i is measured.

In Figures 2 and 3 are shown curves of the internal resistance with varying plate voltages for two types of tubes, each intended for use as amplifiers or detectors. Tube 1 has an amplification constant of 14.5 while Tube 2 has only 7.5. The resistance of the former is, however, very much greater than that of the latter even when the plate voltages are, respectively, 100 and 20. This shows that, for a given type of tube, the associated apparatus should be designed so as to fit the tube characteristics or, with given apparatus, a tube should be chosen which most nearly fits the apparatus.

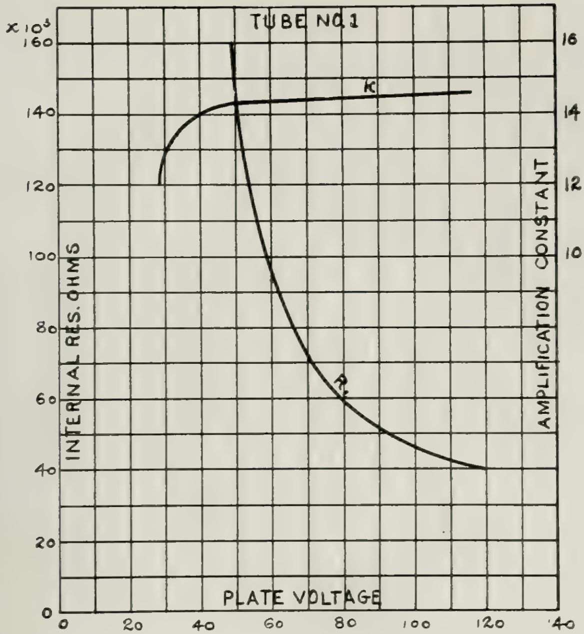


FIGURE 2

The above method has been applied for audio frequencies and to tubes of sufficiently high vacuum and operated at such

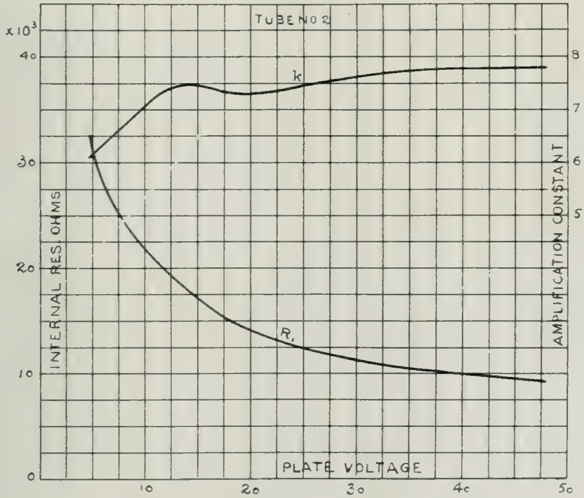


FIGURE 3

voltages that there was no lag of the plate current with respect to the applied grid and plate voltages. The method can, without doubt, be modified so as to measure the phase angle of the lag if such exists and to determine the dynamic values of the tube constants at radio frequencies.

Washington, D. C.
May 3, 1918.

SUMMARY: After considering the characteristics and "amplification constants" of three-electrode vacuum tubes, the author describes the theory and practice of a new method for determining the amplification constant and internal resistance of such tubes directly and rapidly. Examples of results thus obtained are given.

EDISON STORAGE BATTERIES FOR ELECTRON RELAYS*

By

Miller Reese Hutchison, E.E., Ph.D.

It is a well-known fact that the "wing" circuit of an electron relay must be energized by a source of electrical energy entirely free from pulsations of electromotive force.

Notwithstanding the splendid work which has been done in "ironing out" the commutator ripples of dynamo electric machines, there are frequent periods when, owing to any one of a number of causes, non-periodic pulsations result, which seriously affect the operation of the relay.

It is for this reason that batteries, both primary and secondary, have been found the most satisfactory sources of electrical energy for the wing circuit.

Until recently, batteries of miniature dry cells have been employed and have proven fairly satisfactory when absolutely new ones could be readily obtained from the factories; but such cells have a comparatively short period of usefulness, produce a "frying" sound in the receiver when polarization of the elements occurs, and are relatively expensive because of the necessity of frequent substitution by new ones, etc. These disadvantages are pronounced on shipboard, where the dampness makes the life of such a battery particularly short and uncertain, where the unreliability of any piece of apparatus is emphasized, and where a reserve stock of dry cells cannot be depended upon because of their rapid deterioration at sea. On long cruises this uncertainty is of considerable moment.

About a year ago, radio engineers and those upon whom devolves the responsibility of maintaining radio apparatus at remote land stations and aboard ship, cast about for a more dependable and more economical battery for this service.

My attention was first called to this demand by Professor Alfred Goldsmith, who, having used Edison storage batteries in the Radio Telegraphic and Telephonic Laboratory of the College of the City of New York, was familiar with their ruggedness and dependability.

* Received by the Editor, September 7, 1917.

The Edison type "WI-T" cell was thereupon developed and several batteries of these cells were sent to Professor Goldsmith for an extensive series of tests. After due time and a few minor changes, the commercially available battery appeared, incorporated with a standard Edison storage battery for heating the filament. Both have proven highly satisfactory in practical service and have been adopted as the standard for electron relay service by at least one Government.

Because of the prominent position which the new battery occupies in radio engineering, I have been invited to prepare an illustrated description for the radio profession.

A battery of *any* kind, for use in radio work, must, above all things, be *dependable,* even when subjected to the greatest of all abuse—neglect. Of course, when under the eye of a trained battery expert, almost any kind of a storage battery will give good service, if the demand upon it is such as does not necessitate ruggedness; but small units, widely scattered and in the hands of many who may be entirely unskilled in storage battery practice, seldom receive more attention than an occasional charge (which may be a prolonged overcharge at excessively high rates), and the replenishment of the solution with distilled water from time to time. In very small cells, the total liquid content is not sufficient to fill the smallest hydrometer; therefore the specific gravity of such a cell cannot well be ascertained. To remove all the electrolyte from *any* storage battery cell, when the cell is in a charged condition, will positively injure it; so that hydrometer readings of the electrolyte which are so necessary to keep some types of storage battery in condition are a practical impossibility in miniature cell operation. It is therefore requisite that a type of battery which requires no hydrometer readings should be used, and that such a battery should also incorporate the virtue of not being injured by oft-repeated overcharging, or by standing idle for protracted periods in a charged, semi-charged or, in some cases, totally discharged condition. Also if, by chance, the battery be charged in a reverse direction—a frequent error in small battery practice—the battery should not be injured.

When used on an aeroplane, it is of great importance that vibration and concussion should not injure the battery. It should also continue to operate for a short period, at least, even when completely inverted, and should not lose enough electrolyte by such temporary inversion as to affect its operation when right side up.

It should also be of as light weight as is consistent with rugged constructions.

All these and many more characteristics, so necessary for absolute dependability, are possessed by the Edison storage battery.

As is well known, the Edison storage cell consists of a plurality of inter-connected positive tubes and a plurality of inter-connected negative pockets, immersed in a solution of caustic potash. The positive tubes are of perforated, nickel-plated sheet steel. These are loaded with alternate layers of nickel hydrate and pure nickel flake. The negative pockets are likewise made up of perforated nickel-plated sheet steel, and are loaded with iron oxide. For a description of the very interesting processes of manufacture and of the chemical phenomena which take place in the Edison cell, the reader is referred to the various engineering bulletins and published articles on the subject.

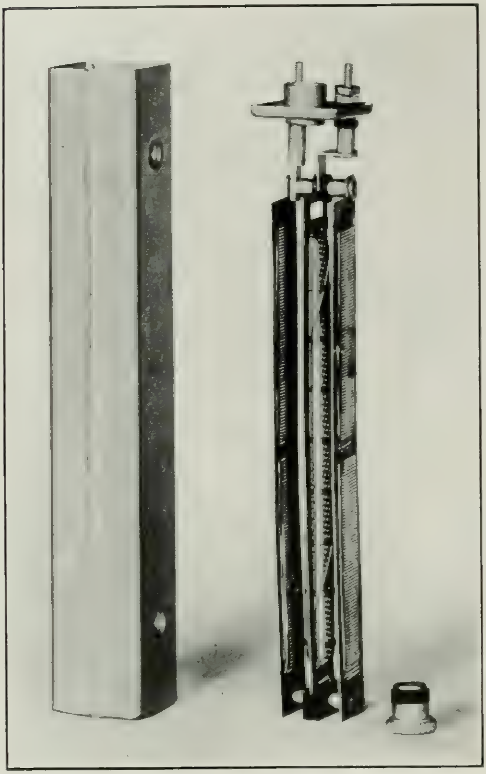
The number of tubes and pockets that make up the elements of a cell depends on the capacity required. In the wing circuit of ordinary electron relays the current rarely exceeds a few milli-amperes, and a single positive tube with four negative pockets provide ample capacity. The manner of uniting these parts to form the cell elements is illustrated in Figure 1, which also shows the steel container. Figure 2 shows the completed cell. All steel parts, it should be noted, are nickel-plated, the plating being *welded on*, in accordance with Edison standard practice.

Two of these cells are combined to form a "twin" cell by connecting their containers together, and grounding the inner positive of one cell and the inner negative of the other to their respective cans. The completed twin is known as type "WI-T."

At the low discharge rates used in electron relay work, the average voltage during discharge of a twin cell is 2.56, and the final voltage, when re-charging becomes necessary, is 2.4. Hence, 16 twin cells are employed for a 40-volt battery and 42 for a 110-volt battery.

The ampere-hour capacity of one of these cells is 1.25, and the normal charging rate 0.25 amperes. Hence, under normal conditions, the time of charge is five hours; but in an emergency, the normal charging rate can be greatly exceeded, without injuring the cell. The low discharge rate in the wing circuit enables a battery to run from several hundred to several thousand hours continuously, on one charge, depending upon the characteristics of the relay bulb.

To correspond with the different sizes of electron relay which have been standardized, two sizes of Edison storage battery units are furnished. For ship and shore work the unit is known as type "42-WI-T—5-B-4." It consists of 42 cells of



Metal Container Elements

FIGURE 1—Edison "WI" Cell Parts

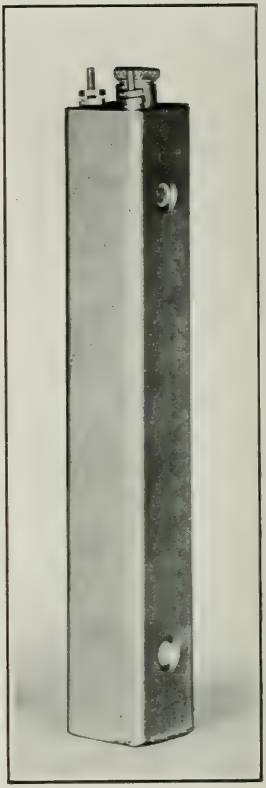


FIGURE 2—Edison Type "WI" Cell, Complete

the twin type described above (type "WI-T"), contained in the same case with five cells (6 volts) of the "B-4" type for heating the filament. The average voltage of the 42 twin cells at the low rate of discharge is 107.5, which is suitable for the wing circuit of the ship and shore relays.

For aeroplane service, the unit is known as Type "16-WI-T—5-M-20," and consists of 16 twin cells in the same container with five cells of type "M-20" for furnishing the filament current.

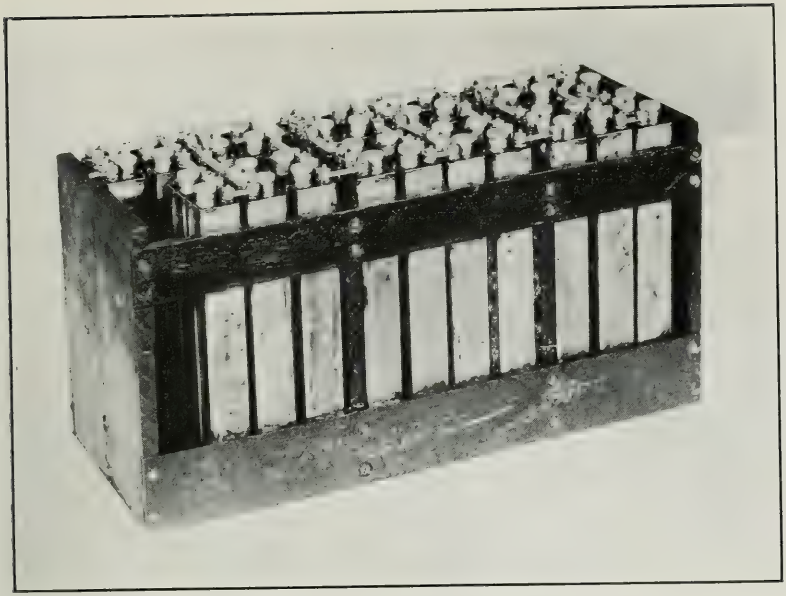


FIGURE 3—A Tray of 21 Twin Cells

The average voltage of the 16 twins is 40, as required for aeroplane relays.

The more important specifications of these two units are given in the following tables:

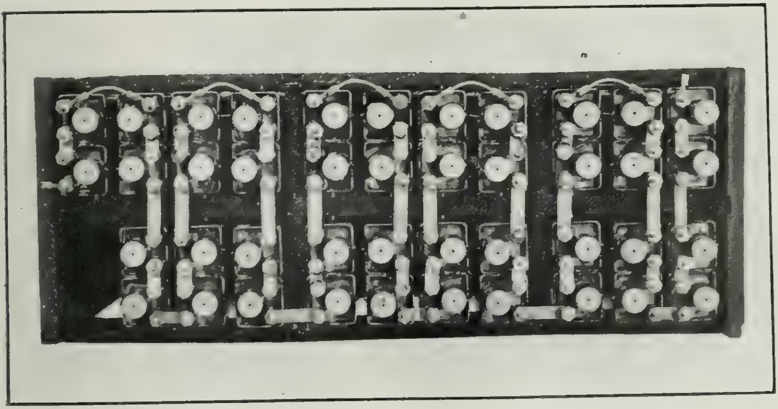


FIGURE 4—Plan View of a Tray of 21 Twin Cells

TABLE I*

Edison Storage Battery Unit

Type "42-WI-T-5-B-4." (Figure 5.)

For Electron Relays for Ship and Shore Stations.

"42-WI-T" Battery:

Ampere-hour capacity	1.25
Watt-hour capacity	135.
Discharge rate in wing circuit of electron relay, milliamperes	0.2 to 5.
Average voltage on discharge at this rate to 100 volts	107.5
Normal charging rate, amperes	02.5

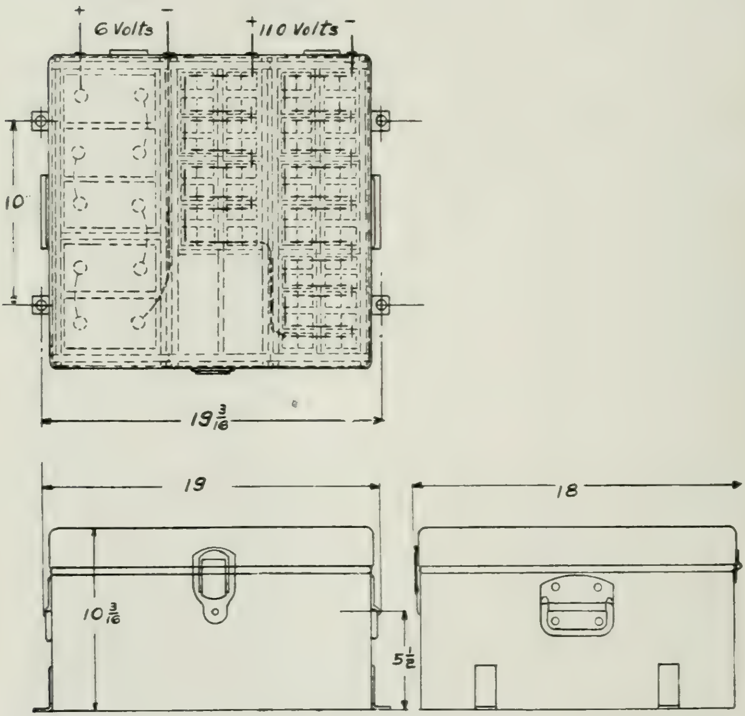


FIGURE 5—42 Type "WI-T" (110 Volts) and 5 Type "B-4" (6 Volts) Edison Cells Mounted in Steel Container

* Further information relative to these cells and their distribution may be obtained from Miller Reese Hutchison (Inc.), Orange, New Jersey.

"5-B-4" Battery:

Ampere-hour capacity	75.
Watt-hour capacity	450.
Normal charging rate, amperes	15.
Normal discharging rate, amperes	15.
Average voltage on discharge at normal rate to 5 volts	6.

Weight of unit complete, contained in heavily-japanned weather-proof steel box, equipped with carrying handles and securing means..........100 lbs. (45 kg.)

Over-all dimensions of steel container, inches, $18 \times 19 \times 10\frac{3}{16}$. ($45.7 \times 48.3 \times 25.9$ cm.)

Edison Storage Battery Unit

Type "16-WI-T–5-M-20" (Figure 6)

For Electron Relays for Aeroplane Service.

"16-WI-T" Battery:

Ampere-hour capacity	1.25
Watt-hour capacity	51.
Discharge rate in wing circuit of electron relay, milliamperes	0.2 to 3.
Average voltage on discharge at this rate, to 38 volts	41.
Normal charging rate, amperes	0.25

"5-M-20" Battery:

Ampere-hour capacity	12.
Watt-hour capacity	72.
Normal charging rate, amperes	2.5
Normal discharging rate, amperes	2.5
Average voltage at normal discharge rate to 5 volts	6.

Weight of unit complete, contained in heavily-japanned weather-proof steel box, equipped with carrying handles and securing means..............35 lbs. (16 kg.)

Over-all dimensions of steel container, inches $9\frac{5}{8} \times 11\frac{13}{16} \times 8\frac{1}{2}$ ($24.5 \times 30 \times 21.6$ cm.)

The lay-out of the ship and shore station unit is shown in Figure 5, and of the aeroplane unit in Figure 6.

These batteries for electron relays are also furnished in individual boxes, where desired. Figure 7 shows the lay-out of type "42-WI-T" battery and container, for the wing circuit of

ship and shore electron relays; Figure 8, the lay-out of type "5-B-4" battery for the filament circuit of these relays; Figure 9, the "16-WI-T" battery for wing circuit of the aeroplane set; and Figure 10 the "5-M-20" battery for the filament circuit of the same.

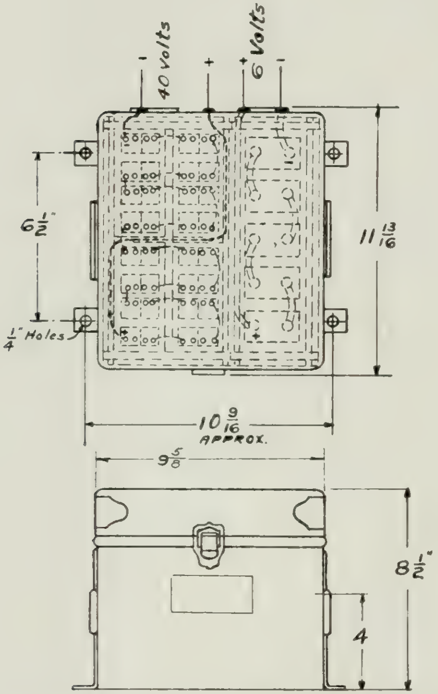


FIGURE 6—16 Type "WI-T" (40 Volts) and 5 Type "M-20" (6 Volts) Cells Mounted in Steel Container

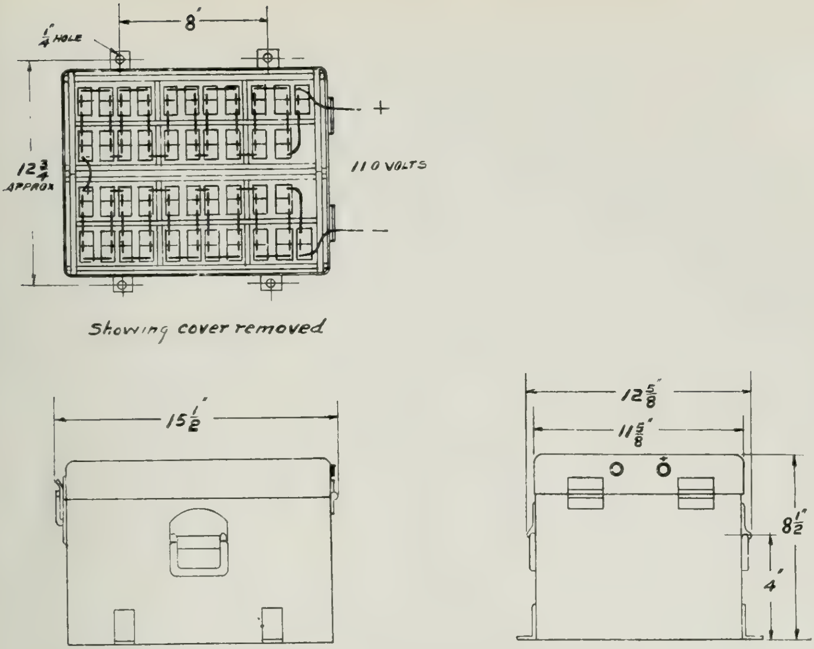


FIGURE 7—42 Type "WI-T" (110 Volts) Edison Cells Mounted in Container

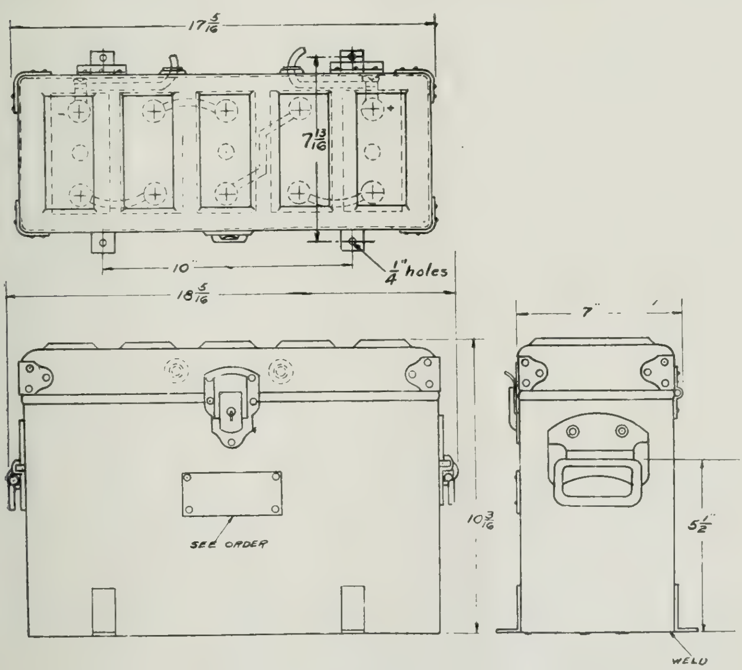


FIGURE 8—5 Type "B-4" (6 Volts) Edison Cells in Steel Container

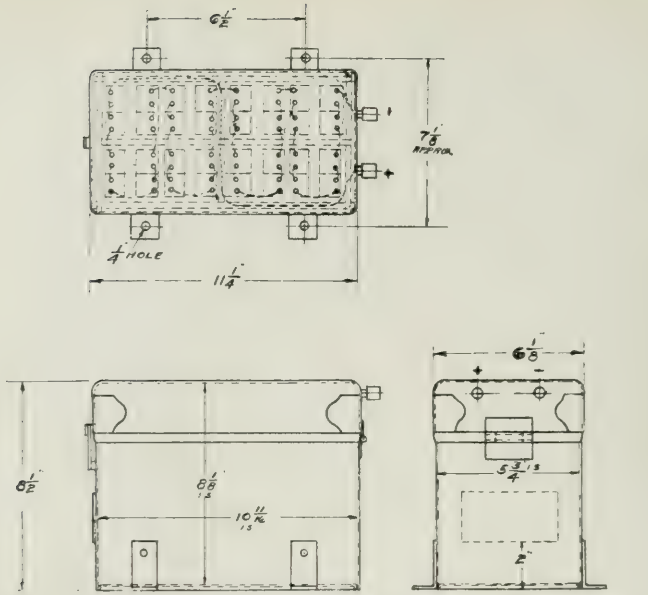


FIGURE 9—16 Type "WI-T" (40 Volts) Edison Cells, Mounted in Steel Container

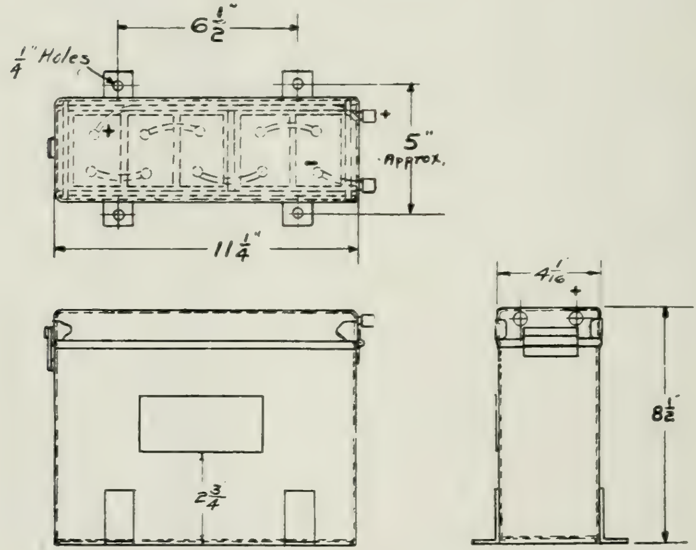


FIGURE 10—5 Type "M-20" (6 Volts) Edison Cells in Steel Container

SUMMARY: The development of a new type of Edison nickel-steel storage battery for use in the plate circuit of electron relays is described. This is a small 1.25 ampere-hour, 2.56-volt twin-cell. Normal discharge lasts several thousand hours.

Various assemblies of these for 40 and 110 volts, in conjunction with various filament-lighting Edison storage battery sets, are described in detail, with electrical operating data, weights, and dimensions

FURTHER DISCUSSION ON "ON THE USE OF CONSTANT POTENTIAL GENERATORS FOR CHARGING RADIOTELEGRAPHIC CONDENSERS AND THE NEW RADIOTELEGRAPHIC INSTALLATIONS OF THE POSTAL AND TELEGRAPH DEPARTMENT OF FRANCE" BY LEON BOUTHILLON

By

J. F. J. Bethenod

(Paris, France)

In the issue of June, 1917, of the "Proceedings of The Institute of Radio Engineers," page 199 and following, Mr. L. Bouthillon had published an interesting paper on the use of constant electromotive force for charging the radiotelegraphic condensers. I desire to mention that I have already indicated in British Patent 27,247 (1910), page 2, lines 14-30, the enormous advantages of the "periodic charging" versus the "aperiodic charging."

Moreover, the system described by Mr. Bouthillon is practically the same as the one experimented with by Major Fracque in 1910 at the Eiffel Tower station.

A very complete theory of this charging method has also been published by Major Fracque (see for instance "La Lumière Electrique," January 16, 1915, pages 45-46), so that I cannot agree with Mr. Bouthillon when he says that the above system is new.

In concluding, let me say that the use of constant electromotive force for charging does not give the *maximum maximorum efficiency* (greatest of the maximal efficiencies).

Let us suppose that a condenser is connected across the terminals of a direct-current supply by means of a self-inductance L and an ohmic resistance R (this resistance taking account of the dielectric losses in the condenser). We propose to find the wave of the charging current which will give the maximum maximorum efficiency, the time, and the increase of the charge having given values τ and ΔQ. It seems admissable to reduce to such a problem that of the most economical charging

of a radiotelegraphic condenser. If the current is i at the time t, this problem consists in fact to determine the minimum value of the integral

$$W = R \int_0^\tau i^2 \, d\, t.$$

the integral

$$\Delta Q = \int_0^\tau i \, d\, t$$

having a fixed value.

In other words, we ought to try to find the shape of wave which gives the maximum square root of the mean square, or effective value, the average value being fixed.

It is possible to find the answer by means of the general method which is applicable to such a problem; but we prefer a very elementary solution, which has been already applied by M. Marius Latour for a problem concerning the electric transmission of energy.[1]

Put:

$$i = \frac{\Delta Q}{\tau} + x.$$

We can immediately write from (2):

$$\int_0^\tau x \, d\, t = 0.$$

Therefore, when substituted in (1):

$$W = R \frac{\Delta Q^2}{\tau} + 2\, R \frac{\Delta Q}{\tau} \int_0^\tau x \, d\, t + R \int_0^\tau x^2 \, d\, t,$$

the first integral of the second member is zero, and the minimum value of W manifestly occurs when $x = \text{constant} = 0$, since the second integral contains only essentially positive elements.

Consequently it may be said that the most economical charging for a radiotelegraphic condenser is obtained when the current has a constant value during the whole charging. This value is naturally:

$$I = \frac{\Delta Q}{\tau}.$$

If we call Q the initial charge of the condenser, the efficiency can be defined by the ratio: $\dfrac{\text{stored energy}}{\text{stored energy} + \text{wasted energy}}$ or:

[1] Cf. Marius Latour, "L'Eclairage Electrique," February 23, 1901, page 279, number 8, volume XXVI.

$$\eta = \frac{\frac{1}{2} \times \frac{(Q+\Delta Q)^2}{C} - \frac{1}{2} \times \frac{Q^2}{C}}{\frac{1}{2} \times \frac{(Q+\Delta Q)^2}{C} - \frac{1}{2} \times \frac{Q^2}{C} + R\frac{\overline{\Delta Q}^2}{\tau}}$$

$$= \frac{\tau\left(1+2\frac{O}{\Delta Q}\right)}{\tau\left(1+2\frac{Q}{\Delta Q}\right)+2CR}$$

or finally

$$\eta = \frac{\tau\left(1+2\frac{Q}{\Delta Q}\right)}{\tau\left(1+2\frac{Q}{\Delta Q}\right)+\frac{\delta T}{\pi^2}},$$

δ being the logarithmic decrement, and T the natural period of the circuit.

It will be noted that with the Poulsen arc working with oscillations of the 2nd type of M. Blondel, one realizes in practice the charging with a current of constant value by means of the extremely high inductance inserted in the charging circuit.[2]

It is theoretically possible to determine the shape of wave for applied electromotive force which allows us to realize exactly such a method of charging. But it will not be of practical interest, because, for one thing, the initial conditions are always uncertain.

[2] H. Barkhausen, "Das Problem der Schwingungserzeugung," Leipzig, 1907.

NOTES RELATIVE TO THE IMMEDIATELY PRECEDING DISCUSSION OF MR. BETHENOD*

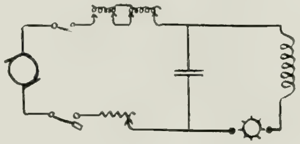
By

Leon Bouthillon

It was a pleasure to learn of the confirmation by Mr. Bethenod of what I consider to be one of the most important conclusions of my paper, dealing with the high efficiency of the system developed by me.

But I cannot agree with the statement that my system is practically the same as that which was experimented with by Commandant Fracque in 1910. The following passage, in which Commandant Fracque describes his experiments, is pertinent ("La Lumière Electrique," January 16, 1915, page 46):

"I had an opportunity to study the charging phenomena (using high voltage direct current), in 1910, at the radio station of the Eiffel Tower, using a Gramme dynamo (2,000 volts, 1.5 amperes), a paper condenser built according to the Boucherot system, two high voltage choke coils with taps on the windings from Mr. Blondel's laboratory, and a resistance made up of incandescent lamps (by means of which resistances up to 10,000 ohms could be obtained). (See the accompanying Figure.)



"The spark gap used was made up of a toothed disc which turned between two fixed discharging studs, and was driven by a small electric motor the speed of which could be very closely regulated.

"By choosing a speed of the wheel and distance between the electrodes in such a way that the time of passage of two consecu-

* Received by the Editor, May 8, 1918.

tive teeth past the fixed electrodes was equal to the time required for the generator to charge the condenser to the highest voltage

$$t = \frac{\pi}{m} = \frac{\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}},$$

I succeeded in obtaining very regularly spaced discharges, and, in consequence, a pleasant musical note.

"I attempted to choose such values of C, L, R, that, without ceasing to be a oscillatory circuit, the charging circuit had a large resistance, which much diminished the likelihood of an arc being produced in the discharger.

"A comparison of the energy taken from the dynamo with that appearing in the form of oscillations in the condenser discharge circuit, showed clearly that the efficiency, while not equal to unity, was nevertheless very high."

It can be seen from this quotation that one of the characteristics of the system employed by Commandant Fracque was the insertion into the charging circuit of a large resistance (namely, a rheostat capable of going up to 10,000 ohms), the presence of which rheostat would instantly reduce the efficiency considerably. My system is distinguished, on the other hand, by the care with which all resistances in the charging circuit are reduced to a minimum. Thus it happens, as explained by Commandant Fracque, that his charging circuit being of high resistance without ceasing to be oscillatory, the efficiency never seems to have been greater than 0.6; that is, of the same order of magnitude as that obtained when the charging is done by alternating current. In my system, efficiencies greater than 0.9 have frequently been obtained. An increase of efficiency of 50 per cent. seems to me a sufficiently interesting improvement, being an improvement of which Mr. Bethenod recognizes the importance since he claims the honor of having first described it.

As regards the theoretical part of my paper, I desire to state that in his paper in "La Lumière Electrique," Commandant Fracque considered only the particular case when the charging time of the condenser was equal to half the natural period of oscillation of the charging circuit. My theory was much more general and extends to all possible cases of musical tone systems, no matter what the duration of charge; and my theory further indicates that musical tone systems are the only systems which are stable as well as possible. The importance of this much more

extended theory is indicated by the fact that the less complete discussion of Commandant Fracque led him to exaggerate the precision with which the discharger must be regulated.

"At the exact time when V reaches its greatest maximum; that is to say at the time $\frac{\pi}{m}$, it is necessary by appropriate arrangement to release the condenser charge."

On the other hand, from the complete theory which I have given, it can be seen (and this has been repeatedly verified by frequent experimentation) that the speed of the discharger can be varied between wide limits on each side of the best value without markedly diminishing the efficiency. For example, with the charging circuit having a decrement equal to 0.4, which corresponds to the maximum efficiency of 0.91, the efficiency is not diminished by more than 10 per cent. below the maximum value if the speed of the discharger is 0.4 less or 1.5 times greater than the best value.

(Translated from the French by the Editor.)

THEORY OF FREE AND SUSTAINED OSCILLATIONS*

By

H. G. CORDES

(RADIO RESEARCH ENGINEER, BREMERTON, WASHINGTON)

The amplitude of a free oscillation decreases with time because of the dissipation of energy in the circuit. The energy is generally considered as being dissipated by a series resistance or by the equivalent of a series resistance. It is often useful to differentiate between damping due to resistance in series and damping due to conductance in parallel with the capacitance of an oscillatory circuit. The current taken by a detector circuit will produce conductance damping in a receiving oscillatory circuit. In a transmitter the losses thru insulators, corona and dielectric losses cause conductance damping. During each oscillation the conductance loss is a maximum when the potential is a maximum while the resistance loss is greatest when the current is a maximum. The damping due to radiation is generally stated as an equivalent resistance damping altho its exact nature has not been determined experimentally.

FREE OSCILLATIONS

Let Figure 1 represent a circuit having concentrated inductance, capacitance, resistance and conductance. The resistance r is in series with the inductance L and the conductance g is in parallel with the capacitance C.

Assume the direction of the arrows positive.

The equation of potential thru inductance L, resistance r and conductance g is

$$L\frac{di}{dt}+ir-\frac{i_g}{g}=0 \tag{1}$$

Equating potentials in the circuit formed by capacitance C and conductance g gives

$$\frac{i_g}{g}+\int\frac{i_c\,dt}{C}=0 \tag{2}$$

* Received by the Editor July 1, 1917.

where g is the reciprocal of the insulation resistance.

The current relation is

$$i_c = i + i_g \tag{3}$$

Eliminate i_c from (2) and (3), and differentiate

$$\frac{1}{g} \cdot \frac{d\, i_g}{d\, t} + \frac{i}{C} + \frac{i_g}{C} = 0 \tag{4}$$

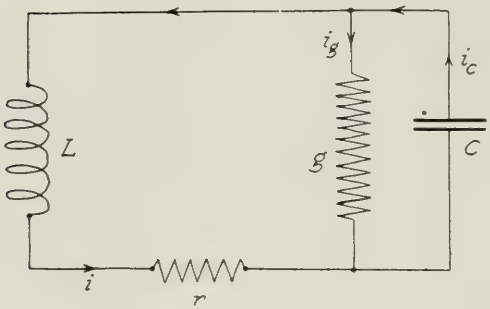


FIGURE 1

Solve (1) for i_g and substitute in (4)

$$\frac{d^2\, i}{d\, t^2} + \left(\frac{r}{L} + \frac{g}{C}\right)\frac{d\, i}{d\, t} + \frac{1 + g\, r}{L\, C}\, i = 0 \tag{5}$$

The auxiliary equation of (5) is

$$m^2 + \left(\frac{r}{L} + \frac{g}{C}\right) m + \frac{1 + g\, r}{L\, C} = 0 \tag{6}$$

Solving (6) for m

$$m = -\left(\frac{r}{2L} + \frac{g}{2C}\right) \pm j\sqrt{\frac{1}{L\,C} - \left(\frac{r}{2L} - \frac{g}{2C}\right)^2}.$$

Let $\qquad \alpha = \dfrac{r}{2L} + \dfrac{g}{2C}$ and let $\omega = \sqrt{\dfrac{1}{L\,C} - \left(\dfrac{r}{2L} - \dfrac{g}{2C}\right)^2}.$

When $t = 0$, $i = 0$, therefore the solution of (5) is

$$i = I\,\varepsilon^{-a\,t} \sin \omega\, t \tag{7}$$

The damping factor α shows the obvious fact that increasing $\frac{L}{C}$ decreases the resistance damping and that decreasing $\frac{L}{C}$ decreases the conductance damping. Conductance and resistance

damping have the same effect upon the frequency when considered individually but when both are considered present they neutralize each other; so that when the resistance and conductance damping are equal, the frequency is independent of the damping.

A free discharge becomes non-oscillatory when $\left(\frac{r}{2L}-\frac{g}{2C}\right)$ is equal or greater than $\frac{1}{LC}$. It is seen that a non-oscillatory circuit may become oscillatory by the introduction of resistance or conductance damping.

The potential of the capacitance is

$$v=L\frac{di}{dt}+ri. \tag{8}$$

Substitute (7) in (8)

$$v=I\varepsilon^{-at}\left[\omega L\cos\omega t+(r-a)\sin\omega t\right] \tag{9}$$

When $t=0$, $v=E_o$, therefore

$$E_o=\omega LI \tag{10}$$

Substitute (10) in (9)

$$v=E_o\varepsilon^{-at}\left(\cos\omega t+\frac{r-a}{\omega L}\sin\omega t\right) \tag{11}$$

which may be written in the form

$$v=E\varepsilon^{-at}\cos(\omega t-\phi) \tag{12}$$

where $\qquad E_o=E\cos\phi$ and $\phi=\tan^{-1}\frac{r-a}{\omega L}$.

Sustained Oscillations

An alternating current in a circuit having comparatively large inductance and capacitance and small impedance constitutes a sustained oscillation. The circuit is in resonance when the reactance of the circuit is zero at a given frequency of the impressed e. m. f. The impedance of the circuit is generally stated in terms of resistance, inductance, capacitance, and frequency. The effect of conductance is not always negligible nor can an equivalent resistance always be substituted for it.

In Figure 1, let v = the e. m. f. of the capacitance.

Then

$$i_g=gv. \tag{13}$$

Eliminate i_g and i_c from (2), (3), and (13) by differentiating and rearranging

$$i=-C\frac{dv}{dt}-gv \tag{14}$$

Assume a sinusoidal e. m. f., $e = -E \sin \omega_1 t$, impressed upon the inductance L. Equation (1) will then become

$$L \frac{d i}{d t} + r i - v = e \tag{15}$$

Let

$$\omega_2 = \sqrt{\frac{1}{L C} - \left(\frac{r}{2L} - \frac{g}{2C}\right)^2} \tag{16}$$

which may also be written

$$\frac{1 + g r}{L C} = \omega_2^2 + a^2 \tag{17}$$

Substitute (14) in (15) and divide by $L C$

$$\frac{d^2 v}{d t^2} + 2 a \frac{d v}{d t} + (\omega_2^2 + a^2)\, v = \frac{E \sin \omega_1 t}{L C} \tag{18}$$

Differentiate (18) twice, solve for $E \sin \omega_1 t$, and substitute in (18)

$$\frac{d^4 v}{d t^4} + 2 a \frac{d^3 v}{d t^3} + (\omega_1^2 + \omega_2^2 + a^2) \frac{d^2 v}{d t^2} + 2 a \omega_2^2 \frac{d v}{d t} + \omega_1^2 (\omega_2^2 + a^2)\, v = 0 \tag{19}$$

The auxiliary equation of (19) is

$$m^4 + 2 a m^3 + (\omega_1^2 + \omega_2^2 + a^2)\, m^2 + 2 a \omega_2^2 m + \omega_1^2 (\omega_2^2 + a^2) = 0 \tag{20}$$

which consists of the factors

$$m^2 + \omega_1^2 = 0, \quad \text{and} \quad m^2 + 2 a m + \omega_2^2 + a^2 = 0 \tag{21}$$

Therefore the roots of the above are

$$m = \pm j \omega_1, \quad \text{and} \quad m = -a \pm j \omega_2 \tag{22}$$

Hence the solution of (19) is

$$v = A\, \varepsilon^{j \omega_1 t} + B\, \varepsilon^{-j \omega_1 t} + C\, \varepsilon^{-a + j \omega_2 t} + D\, \varepsilon^{-a - j \omega_2 t} \tag{23}$$

which may be written in the form

$$v = V \sin(\omega_1 t + \phi) + V_1\, \varepsilon^{-a t} \sin(\omega_2 t + \theta) \tag{24}$$

The transient component, $V_1\, \varepsilon^{-a t} \sin(\omega_2 t + \theta)$, disappears in a very short time; therefore the sustained oscillation is represented by

$$v = V \sin(\omega_1 t + \phi) \tag{25}$$

in which V and ϕ are constants of integration to be determined for the sustained component of the oscillation. V is the maximum potential and ϕ represents the phase difference between the impressed e. m. f. amd the oscillatory e. m. f.

Substitute (25) in (18) and let $t = 0$.

$$V \omega_1^2 \sin\phi + 2\alpha V \omega_1 \cos\phi + (\omega_2^2+\alpha^2) V \sin\phi = 0 \qquad (26)$$

Substitute (25) in (18) and let $\omega_1 t + \phi = 0$

$$2\alpha V \omega_1 + \frac{E \sin\phi}{LC} = 0, \text{ or } \phi = \sin^{-1}\left(-\frac{2\alpha V \omega_1 LC}{E}\right) \qquad (27)$$

Eliminate ϕ from (26) and (27)

$$VLC\,[\omega_1^2 - (\omega_2^2+\alpha^2)] = -\sqrt{E^2 - (2\alpha V \omega_1 LC)^2}$$

or

$$V = \frac{E}{LC\sqrt{(2\alpha\omega_1)^2 + [\omega_1^2 - (\omega_2^2+\alpha^2)]^2}} \qquad (28)$$

which may also be written

$$V = \frac{E}{\omega_1 C\sqrt{\left(r+\frac{L}{C}\cdot g\right)^2 + \left(\omega_1 L - \frac{1+gr}{\omega_1 C}\right)^2}} \qquad (29)$$

Substitute (25) in (14)

$$i = -V\sqrt{(\omega_1 C)^2 + g^2}\,\cos\left[\omega_1 t + \phi + \tan^{-1}\left(\frac{-g}{\omega_1 C}\right)\right] \qquad (30)$$

Let the maximum value of $i = I$, then

$$I = V\sqrt{(\omega_1 C)^2 + g^2} \qquad (31)$$

There are two ways of securing maximum potential or current in an oscillatory circuit. Either the inductance may be varied while the capacitance remains constant or the capacitance may be varied while the inductance remains constant. If ω_1 and C remain constant, and L is varied, an inspection of (31) shows that both I and V are simultaneously a maximum for all values of r and g.

For maximum V by varying L, let $\frac{dV}{dL} = 0$; this gives

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{g}{C}\right)^2} \qquad (32)$$

For maximum V by varying C, let $\frac{dV}{dC} = 0$; this gives

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{r}{L}\right)^2} \qquad (33)$$

It is seen from (32) and (33) that with large conductance and small resistance in the oscillatory circuit, it is necessary to vary the capacitance to obtain maximum V at resonance; and, with large resistance and small conductance the inductance must be varied to get maximum V at resonance.

The potential energy, $\frac{1}{2} C V^2$, is always a maximum at resonance unless the conductance is large, then the maximum is obtained by varying the capacitance when

$$\omega_1 = \frac{1+g\,r}{\sqrt{L\,C - \dfrac{g\,L\,(r\,C+g\,L)}{1+g\,r}}} \tag{34}$$

and is obtained by varying the inductance when

$$\omega_1 = \sqrt{\frac{1}{L\,C} - \left(\frac{g}{C}\right)^2}. \tag{35}$$

Substituting (29) in (31), the maximum current amplitude

$$I = \frac{E\sqrt{\omega_1^2 C^2 + g^2}}{\omega_1 C \sqrt{\left(r + \dfrac{L}{C}\,g\right)^2 + \left(\omega_1 L - \dfrac{1+g\,r}{\omega_1 C}\right)^2}}. \tag{36}$$

The condition for resonance is that $\omega_1 = \sqrt{\dfrac{1+g\,r}{L\,C}}$. It will be noted that the reactance depends to a small extent upon the resistance and conductance when both are considered present. In (36) g is generally negligible compared to $\omega_1 C$ and the product $g\,r$ is negligible compared to unity, therefore (36) may be written

$$I = \frac{E}{\sqrt{\left(r + \dfrac{L}{C}\,g\right)^2 + \left(\omega_1 L - \dfrac{1}{\omega_1 C}\right)^2}} \tag{37}$$

When $g=0$ this reduces to the familiar expression

$$I = \frac{E}{\sqrt{r^2 + \left(\omega_1 L - \dfrac{1}{\omega_1 C}\right)^2}} \tag{38}$$

And when $r=0$ equation (37) becomes

$$I = \frac{E}{\dfrac{L}{C}\sqrt{g^2 + \left(\omega_1 C - \dfrac{1}{\omega_1 L}\right)^2}} \tag{39}$$

The product of $\dfrac{L}{C}$ times a conductance or susceptance has the dimensions of a resistance or reactance.

To determine the maximum I of equation (36) by varying C, let $\dfrac{d\,I}{d\,C} = 0$; this gives

$$\omega_1 = \sqrt{\frac{1+2\,g\,r}{L\,C} + \left(\frac{g}{C}\right)^2} \tag{40}$$

Equation (32) applies to both potential and current; but equations (33) and (40) show that by varying the capacitance, the potential and current do not attain a maximum value simultaneously; the potential reaches a maximum at a frequency less than resonance and the current at a frequency greater than resonance frequency. When the conductance is negligible, all maximum values are coincident with resonance except maximum potential when the capacitance is varied for a maximum.

The radiation resistance is not large enough to change appreciably the frequency for maximum values of current and potential, otherwise equations (32), (33), and (40) could be used to determine experimentally whether radiation causes resistance or conductance damping; furthermore, maximum radiation probably takes place when the product of instantaneous potential and current, vi, is a maximum; that is, when the transfer of energy in the oscillatory circuit is a maximum. Maximum vi is displaced 45° from both maximum potential and maximum current and occurs at double the frequency.

The expressions derived for maximum potential and current are based upon a constant frequency of the impressed e. m. f. A slight fluctuation in the frequency introduces a reactance into the oscillatory circuit. The detrimental effect of this reactance increases as the ratio $\frac{L}{C}$ increases and as the effective resistance, $r+\frac{L}{C}g$, decreases.

In equation (37), let $\omega_1 L = \frac{1}{\omega_1 C}$ and let ρ represent $r+\frac{gL}{C}$, then the resonance current

$$I_r = \frac{E}{\rho}. \tag{41}$$

Let the fluctuation in frequency be equivalent to changing the frequency by a factor a. Substituting $a\,\omega_1$ for ω_1 in (37) then (41) will become

$$I_a = \frac{E}{\sqrt{\rho^2 + \left[\frac{1}{\omega_1 C}\left(\frac{a^2-1}{a}\right)\right]^2}} \tag{42}$$

where I_a is the measured resonance current.

The term "frequency factor" may be applied to the ratio

$$\frac{I_a}{I_r} = \frac{\rho}{\sqrt{\rho^2 + \left(\frac{a^2-1}{a\,\omega_1 C}\right)^2}} \tag{43}$$

which is the power factor of a circuit in which all of the reactance is due to frequency fluctuations of the impressed e. m. f.

In a receiving antenna circuit, damping is due to the losses in the circuit and to the energy withdrawn for useful work. The former consists of re-radiation, resistance, and conductance losses; and the latter consists of the energy withdrawn by the detector circuit or its equivalent. To obtain maximum energy in the detector circuit, the familiar principle applies, viz., that the damping due to useful energy withdrawn must equal the damping due to energy loss.

Applications and Numerical Examples

Some of the equations will be stated in a form required for the substitution of practical units. Numerical examples will refer, unless otherwise noted, to a standard antenna of 0.002 microfarad capacitance, 10^5 cycles frequency (that is, of wave length 3,000 meters), and 5 ohms effective resistance. Let $C_m=$ capacitance in microfarads, $L=$ inductance in cm., $r_o=$ the resistance in ohms, $\rho_o=$ the effective resistance in ohms, and $g_m=$ conductance in mhos.

To estimate the effect of conductance upon damping, the conductance may be expressed as an equivalent resistance by the relation $\frac{r}{2L}=\frac{g}{2C}$, or, $r=\frac{L}{C}g=\frac{g}{\omega^2 C^2}$, which shows that an equivalent resistance is directly proportional to the conductance. In practical units

$$r_o=\frac{10^{12}}{4\pi^2}\cdot\frac{g_m}{f^2 C_m^{\,2}} \tag{46}$$

When $g_m=10^{-6}$ mhos, $r_o=0.62$ ohms; i. e., one megohm antenna insulation is equivalent to $5/8$ ohm antenna resistance in its damping effect.

The ratio of resonance frequency to the frequency of a free oscillation is expressed by

$$\frac{\sqrt{\dfrac{1+gr}{LC}}}{\sqrt{\dfrac{1+gr}{LC}-\dfrac{\rho^2}{4L^2}}}=\frac{1}{\sqrt{1-\left(\dfrac{\delta}{2\pi}\right)^2}} \tag{47}$$

where the logarithmic decrement, $\delta=\frac{2\pi^2}{10^6}\rho_o C_m f$. When $\delta=0.2$, the ratio $(47)=1.0005$, or a variation of 50 cycles at a frequency of 10^5 cycles. The decrement of the standard antenna is 0.02 for which the ratio (47) is practically unity.

The maximum potential in practical units neglecting g and in (32) and (33) is

$$V = \frac{10^6}{2\pi} \cdot \frac{E}{\rho_o f C_m}. \tag{48}$$

Let $E = 100$ volts, then $V = 16{,}000$ volts.

The frequency factor may be expressed, with sufficient approximation, in practical units by

$$\frac{\rho_o}{\sqrt{\rho_o^2 + \frac{1}{10}\left(\frac{10^4 h}{f C_m}\right)^2}} \tag{49}$$

where $h = 100\ (a-1)$ = the per cent. frequency variation.

The per cent. frequency variation (h) and the corresponding frequency factor are tabulated below for a standard antenna, and also for the same antenna with 10 ohms effective resistance.

TABLE I

Frequency Variation (%)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Impedance (ohms)	5	5.25	5.9	6.9	8.1	9.4	10.7	12.1	13.6	15.0	16.6
Frequency Factor (%)	100	95	85	72.5	62	53	46	41	37	33	30
Impedance (ohms)	10	10.1	10.5	11.1	11.8	12.7	13.8	14.9	16.1	17.4	18.7
Frequency Factor (%)	100	98.8	95	90	85	79	73	67	62	57	53

The ratio $\frac{L}{C}$ is naturally high in a receiving antenna circuit while the efficiency is greatly increased by reducing the effective resistance. The re-radiation resistance, which may be considered equal to the radiation resistance, cannot be reduced, but all joulean loss should be made as small as practicable.

The effect of resistance and conductance upon the frequency is generally only of theoretical interest, but becomes appreciable in a sustained oscillation receiving or measuring circuit which is highly damped or sharply tuned.

The expressions derived are based upon Ohm's law; conductances, such as detector current and corona current, do not follow this law but approach more nearly to it than does an equivalent resistance.

SUMMARY: The free oscillations produced on a circuit having capacity, inductance, resistance, and conductance (leakance) are studied. The transient and permanent conditions with sustained oscillations are similarly treated. The resonance and energy relations of such circuits are carefully considered, together with the influence of conductance and resistance on decrement of the circuit and period thereof.

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THE INSTITUTE OF RADIO ENGINEERS
announces with regret the deaths of

Edgar H. Fessenden

and

Eugene M. Murray

Mr. Fessenden was born in Brooklyn, N. Y., and trained in the public schools of that City and in the Brooklyn Commercial High School. After considerable experience as a radio amateur, he passed thru the Marconi School for Operators, thereafter becoming a professional steamship operator.

Prior to the war, he joined the First Battalion of the New York Naval Reserve. At the entrance of our country into the war, he entered the United States Navy as third-class Radio Electrician. While in charge of the radio service of one of the United States Naval vessels, he acted as instructor and passed the examinations for Chief Radio Electrician. An illness contracted in service caused his death on March 15, 1918.

Mr. Murray was an Associate member of The Institute of Radio Engineers and a skilled radio worker. He was formerly an Installation Engineer of the Marconi Wireless Telegraph Company of America. He was commissioned an Ensign in the United States Naval Reserve Force, and called to active duty on April 6, 1917. While in service, he received injuries resulting in his death on May 30, 1918. His residence was at Haverford, Pennsylvania.

RADIO COMMUNICATION WITH MOVING TRAINS*

By

Dr. Frederick H. Millener

(Experimental Engineer, Union Pacific Railroad, Omaha, Nebraska)

The following paper is a brief resumé of the more or less laborious and comprehensive research work undertaken by the writer for the Union Pacific Railroad Company during the period from 1906 to 1916, inclusive. The bare details of this work form a fairly consecutive study of the arts of radio telegraphy and telephony as they are generally known today.

The Coherer and Other Detectors for Radio Signals

In October, 1906, the management of one of the great railroads of the country requested me to devote my time to finding a method of signalling the cab of a locomotive or communicating with a train without interfering in any manner with the right of way or placing any obstruction along it. By this is meant that obstructions which could or would interfere with the work of a snow plow or with the present equipment of the railroad were excluded. For the purpose of experiment, an electric storage battery truck made by the Westinghouse Manufacturing Company, weighing 5,500 pounds (2,500 kg.) with the batteries in place, made of steel, and running on the Shop Yards narrow gauge track at Omaha was utilized. This car was equipped with an antenna, and on it all my purely experimental work was carried out, in such a way as not to interfere with the operating officials or to become in any way a menace to the right of way. Nothing has ever been suggested as an addition to or change in the right of way that has not actually worked, in a very practical manner on this machine.

The first great difficulty which was experienced was to secure a good ground. The axles were packed and ran in vaseline. The only way we could secure any effective radio control on the car was by having a steel brush forcibly pressed into contact with the rail. This was a nuisance, and would not work over

* Received by the Editor, November 17, 1917. Presented before The Institute of Radio Engineers, New York, December 5, 1917.

ice and snow. It was found that if the ground wire came into contact with the steel frame of the car (this frame acting as a counterpoise), no further trouble was experienced in receiving the impulse or wave train which caused the relay to work.

An antenna was placed on the Boiler Shop, near the laboratory, 140 feet (43 m.) above the ground, having a natural wave length of about 500 meters. This electric truck has been exceedingly valuable for experimental purposes because it had available a source of 186-volt direct current, and sufficient

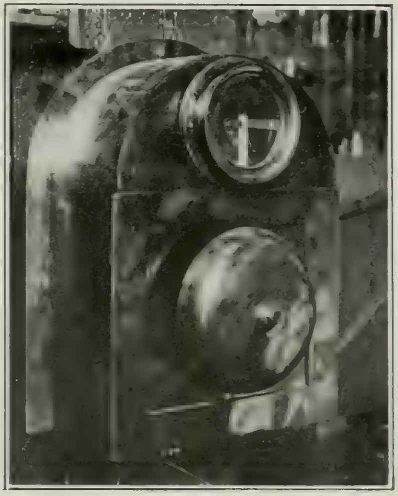


FIGURE 1—Signal Box, with Arm at "Block" ("Danger"); bell ringing

power. Our object at this time was to operate a block signal in the cab of a locomotive from a block signal tower, the idea being that when the arm on the tower went to "Danger" it would cut in storage batteries connected to an induction coil, a tuned circuit, and other equipment necessary to radiate electromagnetic waves over a short distance, such waves to be received by the antenna on the approaching locomotive and to cause the device in the cab to set a like arm at "Danger." Figure 1, illustrating this machine is shown herewith.

However, on a railroad, a danger signal must operate at least 100,000 times in succession without a failure, and must not be operated by any other means than the means intended. As the lighting of an arc light, an atmospheric disturbance, or

casual spark will sometimes cause this signal to operate, and as the best coherer or valve detector which we were able to secure actually, or to learn of, would not last for anything like 100,000 operations without a failure, work was discontinued on the mechanical radio cab signal. Altho no *mechanical* difficulty which could not be overcome had presented itself, a physical one had; namely, that the device was not sufficiently reliable tho it was possible to see its value easily during sleet storms, rain storms, and similar disturbances. This, of course, was one of the earlier reasons why the coherer was displaced in radio telegraphy, and the telephone and detector substituted. The first radio equipment utilized, together with the interior of the place that was then our laboratory is shown in Figure 2.

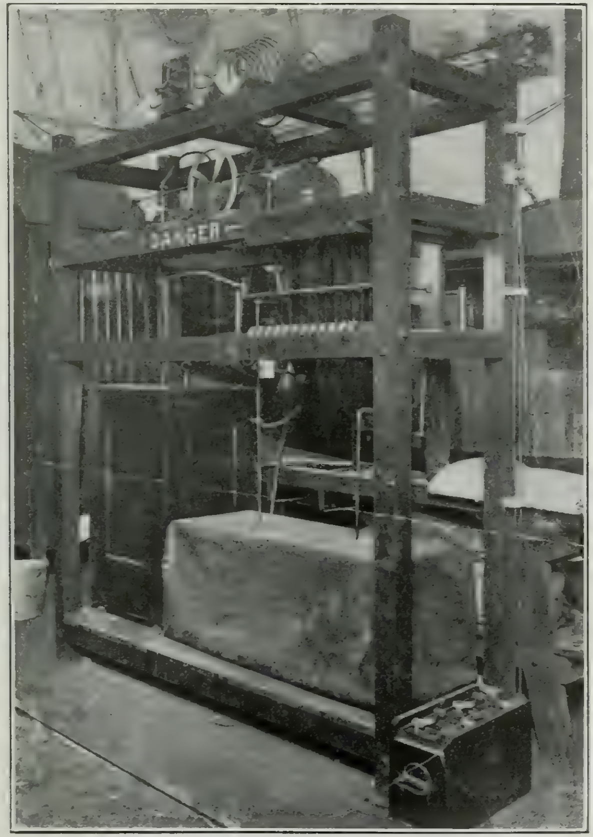


FIGURE 2—First Radio Laboratory at Union Pacific Railroad Shops

There were numerous difficulties in the way of successful experimentation. Standard instruments were exceedingly costly and rather unreliable. The patent situation was such that we could not tell whom one would ultimately have to pay for the use of any device nor the amount to be paid. There were many "systems," and many of the patents covering them were possible infringement of patents in this country or abroad. Quite regularly some corporation would threaten our Law Department with suits for using any form of radio equipment but theirs.

In developing this cab signal, it was necessary to be absolutely certain that it would work the mechanism on the locomotive, and to determine how often and how well it would work. The electric relay was caused to operate the driving mechanism of the electric truck at four speeds ahead, four speeds back, and to ring a bell. This was done of course, by means of the coherer, a four-ohm track relay, such as used by the Signal Department and a portion of the Independent Telephone Company's jenny and a large relay. The complete apparatus is shown in Figure 3.

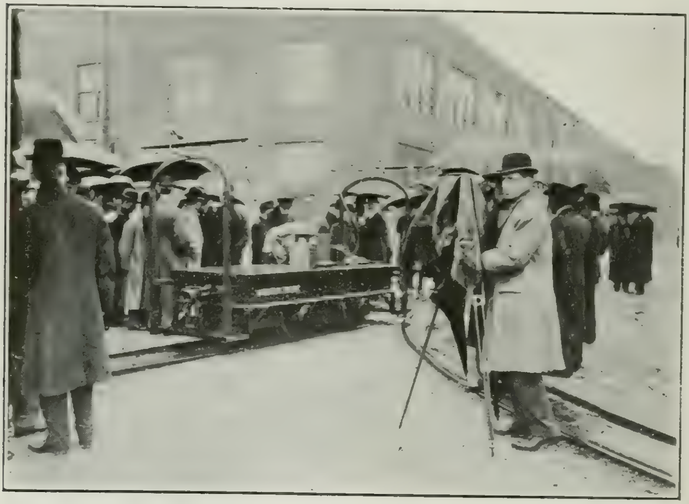


FIGURE 3—Radio Controlled Truck Under Test

All of these instruments were, of course, hand made, by myself and my friends and assistants in the Shops.

We found, that if we could control this car in this manner, we might be able to make the block signal work in the cab of

the locomotive, and a picture of the locomotive equipped with such a cab signal is given in Figure 4.

As the result of the application of radio to a car, which was believed to be the first piece of heavy machinery ever so con-

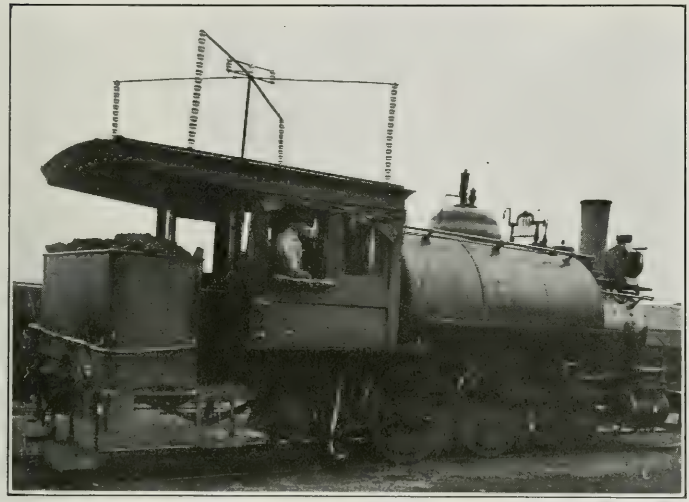


FIGURE 4—Engine Equipped with Radio Controlled Signal

trolled, our work was broadened. We were given a better laboratory and more facilities and were requested to devote our whole time to the work.

RADIO TELEGRAPHY—SYSTEM AND STATIONS

The Union Pacific Railroad has always been considered more or less a military road, it having the lowest grade of any road over the mountains and its work has been essentially that of a transcontinental railroad. Those people of the East, who have not been in the West, can scarcely realize that it is a country of magnificent distances. Such a country develops its population on the same broad lines. For miles and miles there are no trees, homes, or habitations. In winter it is not an uncommon sight to see along the track a line of telegraph poles and wires which have been put out of commission by ice, heavy wind or sleet storms and floods. So it seems that in such a country as this, there might be an opportunity to develop radio telegraphy. It was decided that some stations of a complete sys-

tem in this region were to be merely for receiving and some to be for both sending and receiving.

We were therefore directed to prepare plans and specifications for such a system, and if possible to make it flexible enough to telegraph to a moving train. It was thought that stations could be erected at certain cities and then outfit cars fitted up with one or two kilowatt sets, which were to be kept at division points. These cars were to contain a mast, which could be raised thru the roof of a car when on a siding, and a temporary station installed at either side of a wire break; communication being thus established until such time as the line men could complete their work. The one thought which was strictly impressed upon our minds at all times was that this apparatus must be cheap and simple, so that local men could be taught its use and operation to avoid the necessity of having experts, and that its whole value would be entirely dependent on this.

For 600 miles (1,000 km.) west of Omaha the country is comparatively level and straight. The elevation at Omaha is 1,200 feet (350 m.), gradually rising until an elevation of 8,000 feet (2,500 m.) is reached at the summit of Sherman Hill, Wyoming. In planning for radio telegraph stations, this elevation must be taken into consideration. Another thing which needs very close attention, and which causes very serious conditions is *static electricity*. In summer, the static electricity between Sidney, Nebraska; Cheyenne, Wyoming and Sherman Hill, Wyoming, causes all manner of disturbances with block signals burning them out, and occasionally putting the wire telegraph out of commission. Very little was *generally* known about the arc method of Poulsen and about the experimental work that was then being done on the 500-cycle spark work of Marconi and others, and the rotary synchronous spark gap of Fessenden. But the least expensive method and the one which was decided upon was the 60-cycle closed core transformer, the rotary spark gap, and the tinfoil condenser transmitter, which was then used by the Telefunken Company. These stations were to be of sufficient power to be heard night or day, summer or winter, at least as far as the next sending station. We expected the station at Cheyenne to work as far east as Omaha and an equal distance the other side of the mountains. The other sending stations were to be at Sidney, Grand Island, North Platte, Green River, and Omaha. Experiment taught that we would be able to hear the high pitched note of these stations above the strays, and later developments conclusively

proved this to be true. We were then requested to work out a system of radio telegraph and radio telephony, which could be applied to railroad uses where it would have value in protecting human life.

Figure 5 is a detailed drawing of the house for the radio station as it appeared when ready for use. These houses were 14 feet (4 m.) front and 16 feet (5 m.) deep, and contained two rooms. The rear room was the power room and contained the heavy machinery, cupboards, and other equipment. The front room contained the sending and receiving instruments and the operator's table. The wires were brought in carefully insulated thru a concrete duct under the floor. This arrangement is shown in Figure 5.

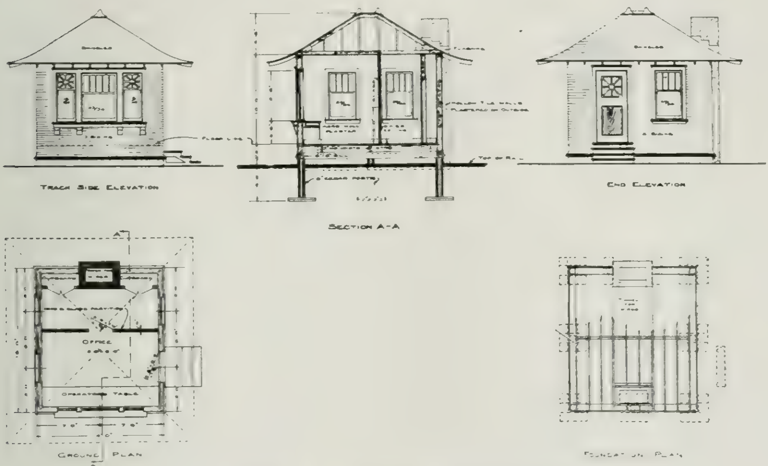


FIGURE 5—Railroad Radio Station

ANTENNAS

In railroad work, it is absolutely necessary to have the antennas close and compact. It seems that there is no doubt that the flat top antenna is the best and most practical antenna for every day use, either for railroads or in the field. (But the initial expense in erecting the same under the best working conditions is sometimes prohibitive.) In the first installation of the railroad radio telegraph, it was absolutely necessary not to complicate matters. The antennas had to be simple and easy to repair. We were obliged to endeavor to utilize men in the employ of the railroad and not to import experts.

It is, then, desirable to have a flat top antenna at stations, parallel to the track, and supported by two self-sustaining towers, the height of which should be at least 180 to 210 feet (55 to 65 m.) and which should be constructed to stand a wind stress of 90 miles per hour (150 km. per hour). There were, however, places where we could only have one mast. Examples of both kinds with drawings are given in Figures 6, 7, and 8.

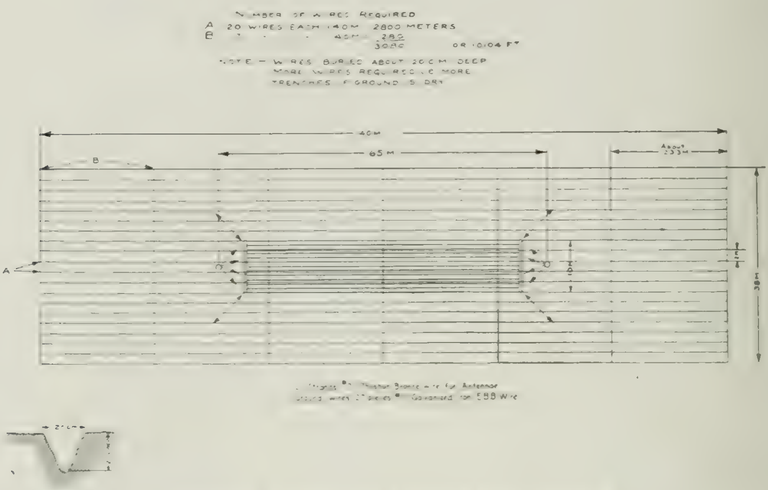


FIGURE 6—Flat Top Antenna and Ground

It was necessary that the antenna be so arranged that it could easily be freed from snow and ice. The umbrella antenna requires too many guy wires and at the critical moment frequently fails on account of snow and ice. It is also difficult to repair in the yards and stations, and it takes up too much space. The single mast antenna which is shown below was designed for this purpose, and has porcelain insulators at the top. The units are made in such a manner as to loosen the snow and ice.* A specially constructed ground or counterpoise is used. Figure 9 represents the mast and tower with the house at the base; and the arrangement of the antennas. Figure 10, is a detailed drawing of the insulators at the top of the mast, and of a detail of the inner ring of the insulator and the ring around the mast. Figure 11 represents a detail of the weaving

* U. S. Patent Number 1,206,353; filed January 19, 1912.

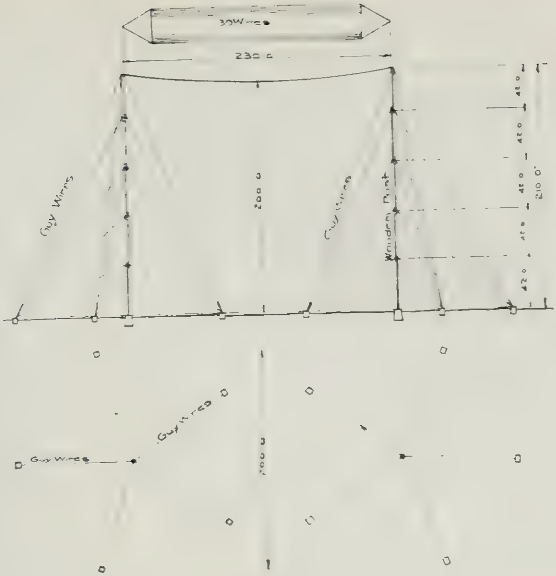


FIGURE 7—Flat Top Antenna, Wooden Masts

antenna for breaking off ice or clinging snow, and also a counterpoise utilizing the unbonded sections of track in the yard and

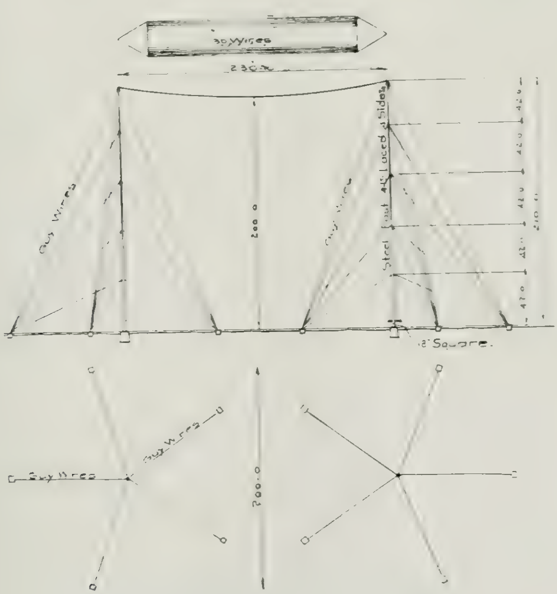


FIGURE 8—Flat Top Antenna, Steel Masts

such other material as might run on the track to increase the capacity.

In the west, a good ground is hard to secure on account of the light sandy soil which is at times very dry. The ground

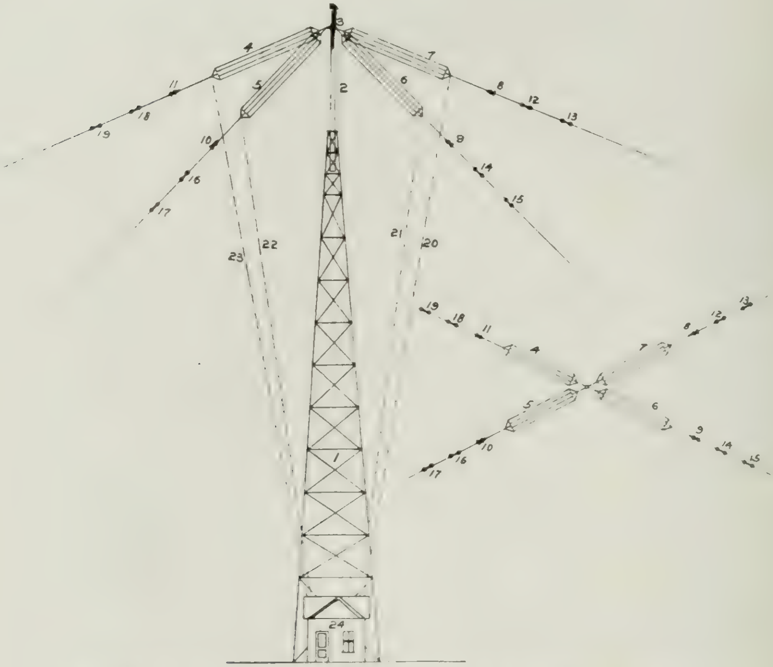


FIGURE 9—Structure of Special Umbrella Antenna for Railroad Radio Stations

or counterpoise of Figure 6 was intended to be located at Cheyenne, in the park adjacent to the railroad and belonging to the depot. The various styles of mast construction are shown in Figures 7 and 8. These were intended to be used in the installations.

TRANSMITTING APPARATUS

It was planned to have radio stations at Ogden, Utah; Green River, Wyoming; Cheyenne, Wyoming; North Platte, Nebraska; Grand Island, Nebraska; and the plant at Omaha was already in existence. There was thus covered the entire system west of Omaha. Where one support was already in place, the flat top antenna was to be used. Where both supports

would have had to be built, an umbrella antenna was planned. Where the antenna was of the flat top type, we used yards or spreaders made up of 30 feet (9 m.) of 4-inch (10 cm.) wrought iron pipe. These will space and carry twelve copper wires,

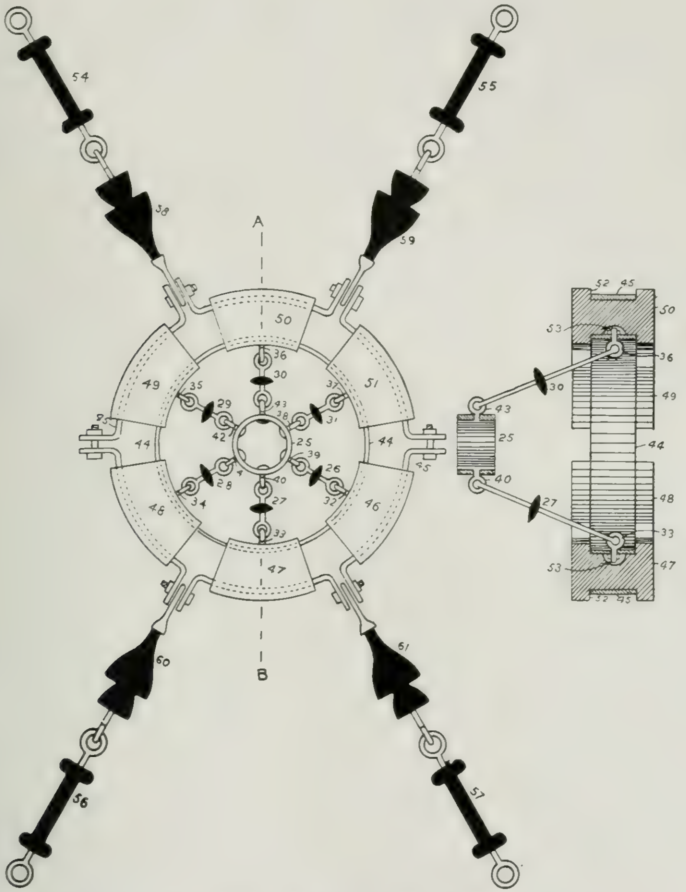


FIGURE 10—Top-of-Mast Fittings of Special Umbrella Antenna

25 feet (75 cm.) apart, 700 feet (200 m.) long. The spreader in question is illustrated in Figure 12. The umbrella masts have been previously explained.

The placing of a network of wires in the ground at the parks insured a perfect ground or counterposie connection at all times, wet or dry, and if more connections were needed, the unbonded

yard tracks would be used without interfering with the traffic or signals. The railroad tracks are all connected by wires at the end of each rail for the block signals. This is called bonding.

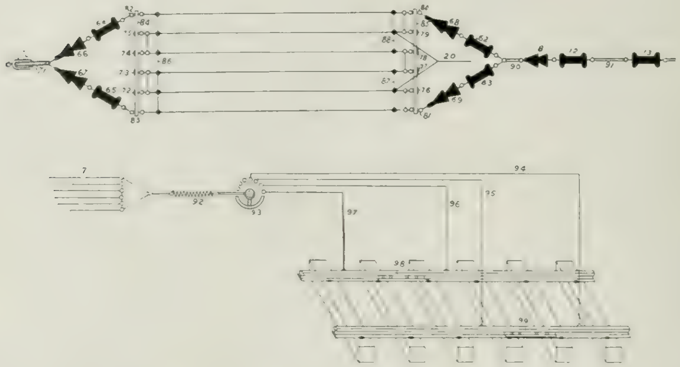


FIGURE 11 —Spreader Structure of Antenna; Counterpoise Rail Ground

It was found that placing the masts parallel with the tracks and close to them was of great assistance in working east or west and in transmitting over the mountains, the rails seeming to assist transmission.

The station at Green River was to have been a ten-kilowatt sending station, the station at Cheyenne a five-kilowatt, and the stations at Ogden, North Platte, and Grand Island were each to have been two kilowatts. The wave lengths were to be 600 to 800 meters.

The altitude at Green River, Wyoming, was 6,082 feet (1850 m.) and at Cheyenne, 6,068 feet, and the highest point on the railroad was at 8,000 feet (2,500 m.). Taking into consideration the atmospheric conditions surrounding the mountains (especially on Sherman Hill), it was desired that these two stations should radiate directly over the mountains to Omaha. In order to be sure to secure this result, the ten-kilowatt station at Green River and the five-kilowatt station at Cheyenne were chosen.

From experiments conducted and observations taken, the possibilities are that both of these stations would be read as far as Omaha without any difficulty since a high pitched spark note was to be used. On account of the differences in the tones of the emitted sparks, the two stations would be readily recognized

by the note in the telephone receiver and interference prevented as much as possible by the long wave length used. The total distance which had to be covered by this installation and for regular work was only 1,000 miles (1,600 km.); and we have no doubt now that we would not have required a ten-kilowatt station at Green River. The coming of the more sensitive audion detector has made this certain.

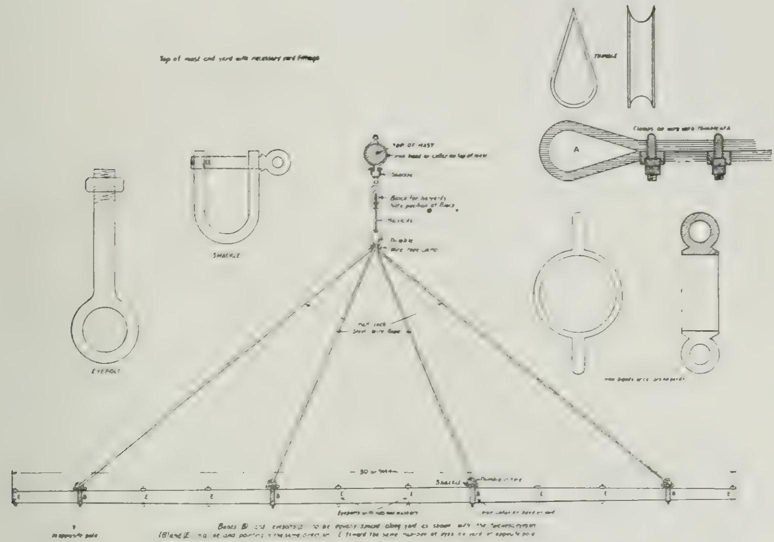


FIGURE 12—Standard 12-wire 30-foot (9 m.) Spreader

RADIO TELEPHONE TESTS

During our experiments, we have been piling up considerable data and making thoro studies of the conditions in our section of the country. The question had come up some time before as to which system would go farthest with the least amount of energy: the arc or spark method; and the results seemed to us very clear: that for short wave lengths the spark method was best, and the arc method for long wave lengths. From the very beginning of the study of radio telegraphy, it has always seemed that the ideal instrument would be a radio telephone; and from the very beginning we hoped to be able to communicate by telegraph and telephone to moving trains. For some time, we worked on a radio telephone which seemed to be perfect so far as oscillation generation was con-

cerned. It was not till 1909, under rather critical circumstances, that Mr. Harvey Gamer and I devised a successful modulation method.

Our first radiophone was made up of a series of four arcs—water-cooled, in a hydrocarbon atmosphere and under the influence of a magnetic field. The first of these is shown in

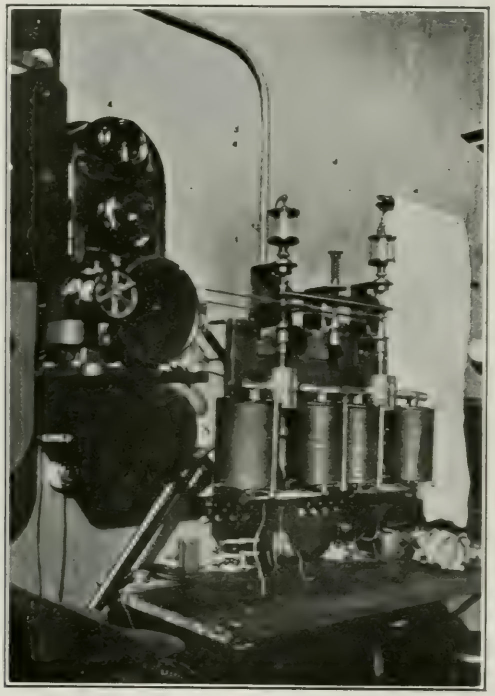


FIGURE 13—First Radiophone Arc Transmitter

Figure 13. Figure 14 illustrates the second attempt made to generate continuous oscillations by the arc method. A still later type is shown in Figure 15. In this installation the positive electrodes were made of copper, and the negative electrodes were made of some other electrical conducting material, usually carbon. The combination of a hollow metallic electrode having a chamber or cavity in its arcing end, the axial length of which was at least equal to its transverse diameter, and a negative electrode with its hemispherical end introduced within the cavity and substantially at the focal point thereof was found

useful. A diagrammatic representation is shown in Figure 16, together with our method of modulating as much energy as possible by means of the microphone.

It is well known that an arc burning in the presence of oxygen, or in an atmosphere containing oxygen, will at times

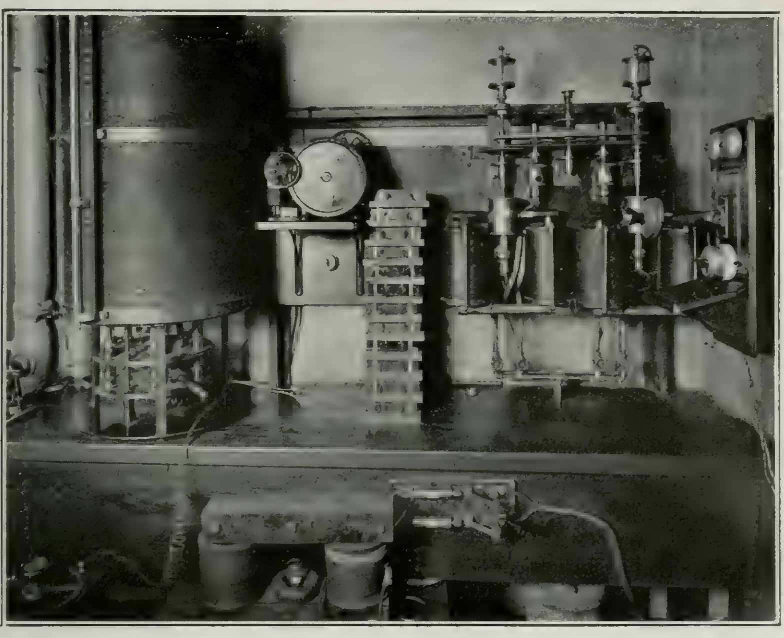


FIGURE 14 Second Radiophone Arc

hiss; and it is also known that when the hissing occurs there is a marked decrease in the difference of potential between the electrodes. This drop in potential, or any considerable variation thereof, interferes in greater or less degree with the operation of a radio transmitter in which the arc is employed to convert, or aid in converting, a direct current into an alternating current of radio frequency. Where several arcs are so employed, some or all commonly go out or cease to burn when the hissing begins. Such hissing is peculiarly detrimental when it occurs in a system of the character here described, that is, one used for speech transmission, because, in addition to its other bad effects, the hissing of the arc in great measure drowns the sound waves produced by the voice of the speaker, and precludes a clear

transmission and the certain and distinct hearing and interpretation thereof at the receiving point of the system. To prevent this hissing and to secure increased energy of the oscillations produced by an arc, the arc or arcs are enveloped in an atmosphere of hydrogen or other gas, or other means are adopted to exclude oxygen from the arc, the crater, or the vapor generated in and by the arc. While such means will give satisfactory

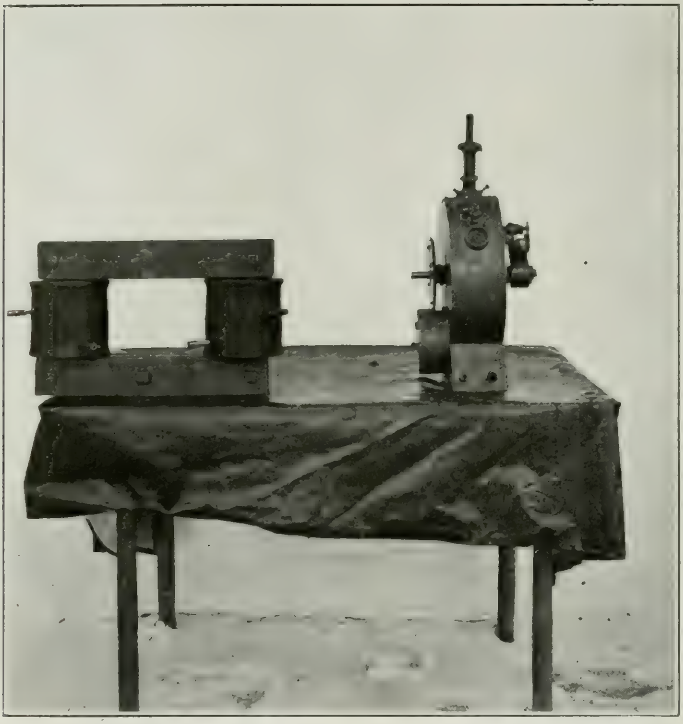


FIGURE 15—Third Radiophone Arc

results (and while we had, in a patent application, described a simple and efficient way of producing and applying such gas), we had concluded that if the positive electrode be kept cool, and the negative electrode be surrounded thereby, from its upper or arcing extremity downward to a reasonable distance, hissing of the arc may be equally well prevented, and without the necessity for any other provision or apparatus. The suppression of the hissing we believe to be due to the fact that with the positive electrode kept cool there is almost entire

absence of fusion of the electrode or of the formation of metallic globules, roughened portions or projections, depressions or pittings, which might, and in fact do, produce a variation in the length of the arc, accompanied by variation in the difference of potential between the electrodes; and to the further fact that whatever oxygen may be contained in the cup or inner cylinder of the positive electrode is speedily consumed, and the resultant or remaining gas or vapor, of whatever character it may be, rising or tending to rise by reason of its heated state,

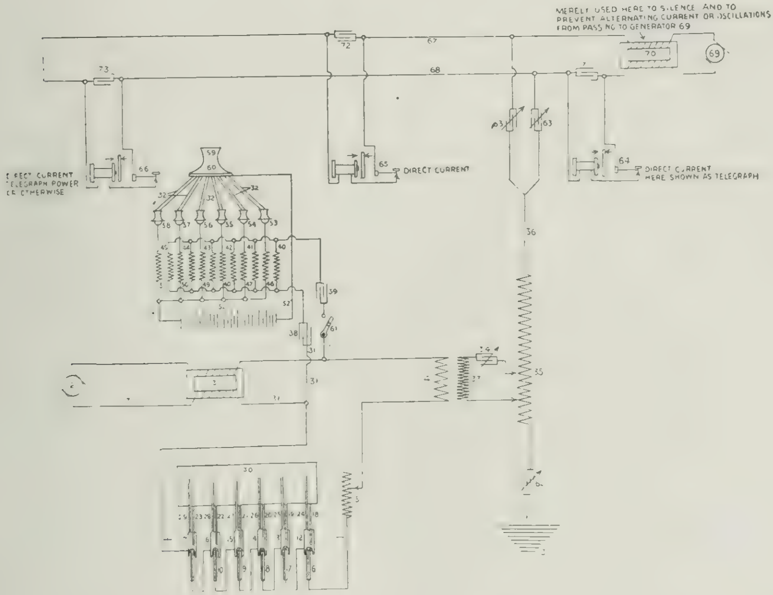


FIGURE 16—Radio Telephone Transmitter Circuit

fills and occupies the entire interior of such inner cylinder or cup, and completely envelops the enclosed extremity of the negative electrode, thereby precluding the ingress of oxygen or the contact of such oxygen with the arc, the vapor thereof, or the crater. Not only is the hissing suppressed or eliminated, but with this arrangement of the electrodes, after striking the arcs the electrodes may be gradually separated, making the arcs longer and drawing more current without causing hissing thereof, provided the voltage be adequate. We also found that with the current from sources of 500 volts or more and with arcs of from one to one and one-half inches (2.5 to 3.75 cm.), and with

seven to ten amperes supply current, the results with distant reception are improved, and the electrodes can be adjusted and the consequent arcs varied without producing the hissing sound. When several arcs are formed or burning in series, and the voltage is sufficient only to maintain them, even tho the separation of the electrodes be as slight as a quarter of an inch (6 mm.) or less, it is found that if air comes in contact with the positive electrode, and the crater runs up the side thereof, hissing begins. Such an arc absorbs more than its proper proportion of the electromotive force, there is a decrease of potential in the other arcs, and they cease or go out. If there is an available surplus voltage, this action does not take place. Steady sustained oscillations are produced, and articulate speech may be clearly and distinctly transmitted and received by apparatus constructed as above set forth. The same construction and arrangement is adopted for each pair of electrodes used, and by reason of suppression of the hissing and its attendant effects, we were enabled to maintain in proper arcing condition for prolonged periods of time, a considerable number of pairs of electrodes, arranged in series as above described, and to vary the length of such arcs within wide limits, without causing hissing.

While we are on the subject of arc generators for oscillations, it may be noted in the latter arcs that the magnetic blast and gas has been done away with. The arcs work more steadily without the magnetic blast; and there is a definite value of inductance for any given capacity, which gives a maximum current in the shunt circuit. If gas is used, as the gas pressure rises the steadiness of the arcs diminish.

Altogether we made four radiophone generators of the arc type, and they worked very satisfactorily and over considerable distances. They worked very well for radio telegraphy or for any use where they could be maintained in a stationary position, but they were heavy and ungainly and required considerable attention. In fact, they required a skilled operator.

Continuing our consideration of the arc radiophone, it was found satisfactory for the continuation of our experimental work in talking to a moving train. The oscillation generator was to be placed in the baggage car of the train, and also in the station with which we intended to talk. Experiments with these arrangements were made in the shop yards and were rather satisfactory. We even combined some of our previous experiments with the electric truck, and not only controlled it by

radio, but also talked to it while in motion by the radio telephone. The intention was to use two telegraph wires made of copper or two parallel conductors thoroly insulated along the right of way, these to be used for wire telegraphy if desired; and by proper placing of condensers at stations they were also to act as long antennas.

FIGURE 17—Umbrella Antenna for Radiophone Reception

Figure 17 illustrates what is literally an "umbrella antenna" for portable radiophone receiving stations. The tuning apparatus necessary for this work and the detector are placed in the upper ribs of the umbrella. (The umbrella was primarily used to convince doubting individuals of the actual radiophone transmission.) With the crude methods of tuning we had with this apparatus, we were able to talk or play a phonograph into the transmitter of the radio telephone, the speech or music being distinctly heard by the person with the umbrella three-quarters of a mile (1.2 km.) away. Transmission was, of course, possible

over much greater distances than this, but not with such crude receiving apparatus. When the radiophone transmitter was connected directly with telegraph wires properly provided with condensers to avoid the telegraph sounders and other telegraphic equipment, the voice appeared to be very distant and it was

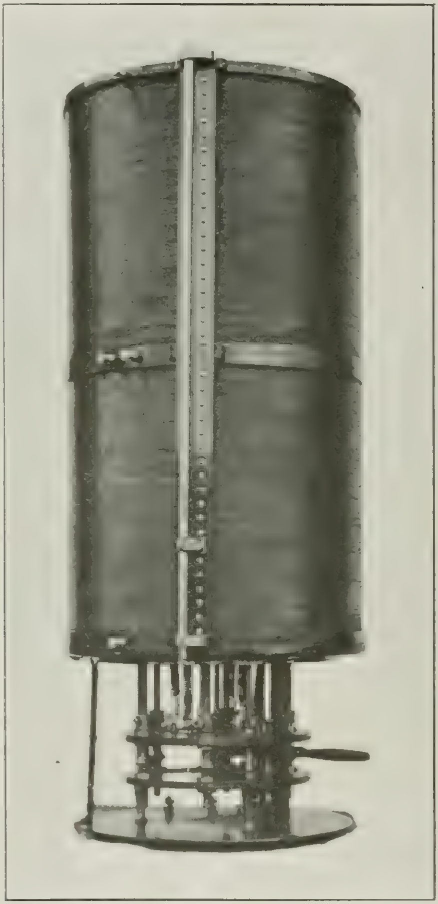


FIGURE 18—Arc and Direct Coupler for Radiophone

very difficult to recognize the inflections of the voice or to identify the speaker, altho one could recognize what was said. The voice sounded very much as it does on the transcontinental telephone. As soon as the voice was received from an antenna

not connected with the wire in any way, inflections of the voice could easily be caught and the person recognized. Our latest multiple arc direct coupler is shown in Figure 18, and the remainder of the radiophone transmitter in Figure 19.

In connection with this work, we present in Figure 20 a diagram of a train equipped with a radiophone station. The antenna is on top of the cars and is connected by couplers for the full length of the train. The coupler is shown in Figure 21.

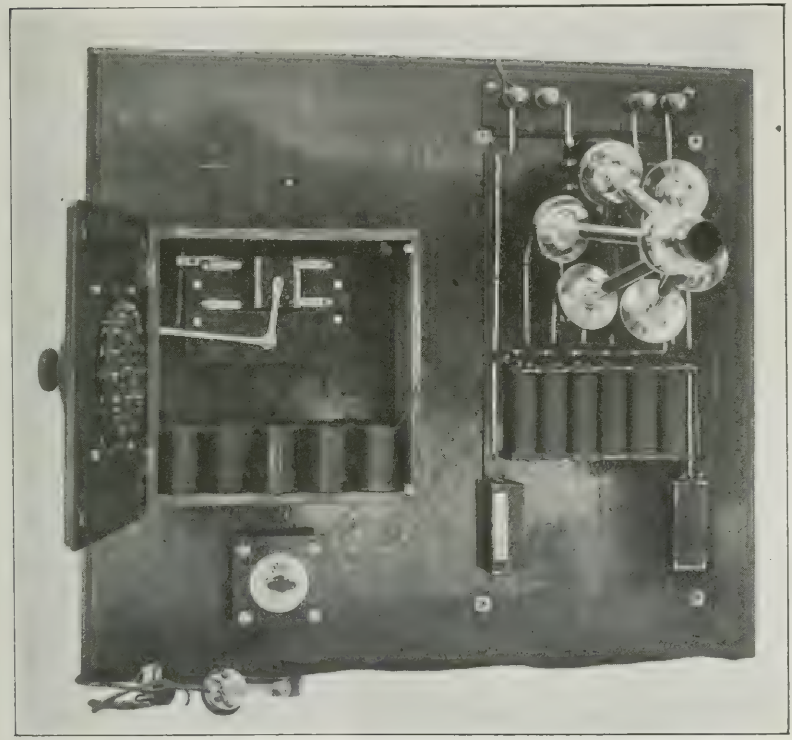


Figure 19—Arc Radiophone Transmitter

The train is wired for an intercommunicating set of telephones between cars. A typical station of the intercommunicating system is illustrated in Figure 22. An arrangement is here made so that persons in an observation car may talk with the baggage or dining car, or, if a train happens to be at a station, communication may be had with the city system. Figure 23

shows the experimental equipment for this system set up in the laboratory, and Figure 24 a line coupler. If, after the train had left the station, it is desired to converse with the station, this may be accomplished by connecting any one of these telephones with what would correspond to the ground wire of the antenna circuit in the baggage car. By first calling the baggage

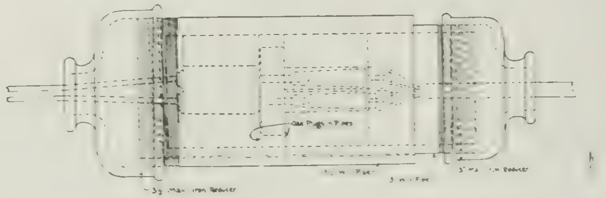


Figure 21—Coupler Between Cars Equipped for Radio Service

car and requesting them to provide a radiophone connection (that is, to start the oscillation generator and then cutting in the microphone on the antenna circuit), such communication could be obtained. This system worked perfectly in the shop grounds using the electric truck and several small cars which ran on the narrow gauge railway.

The next laboratory was a movable one, and a photo-

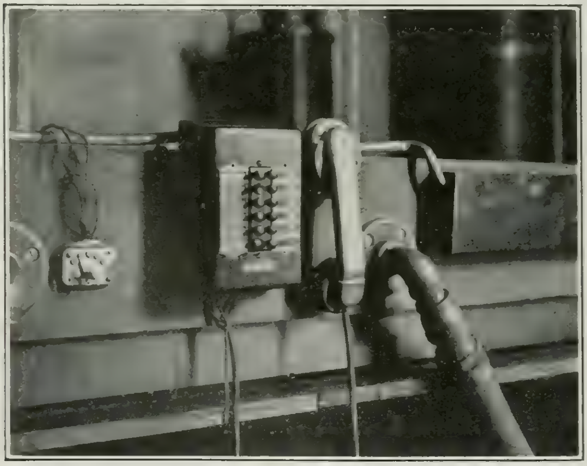


Figure 22—Telephone Intercommunicating Station for Wire or Radio Communication

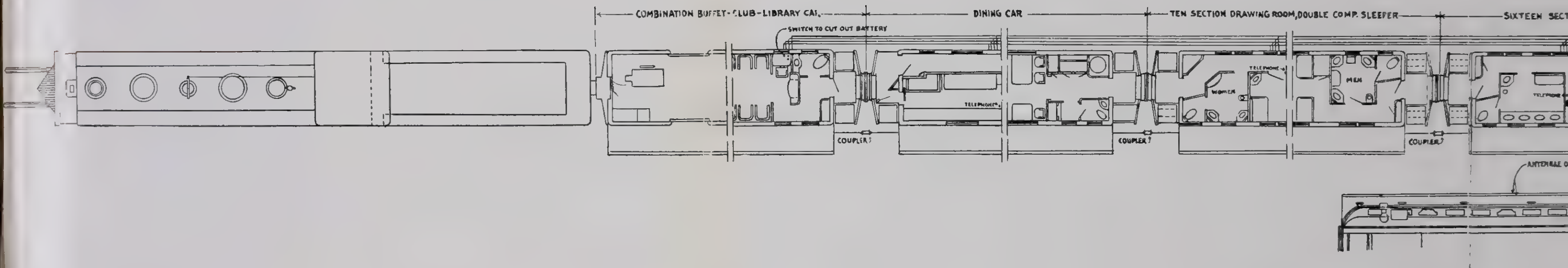
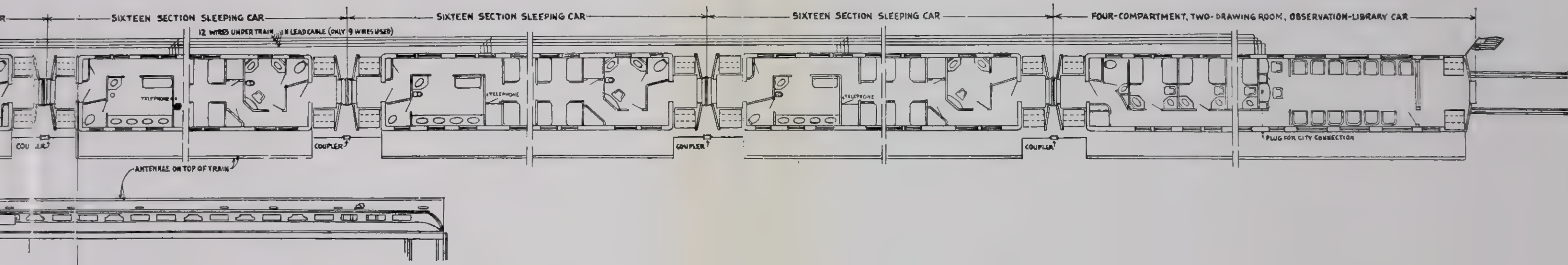


FIGURE 20—System of Train Wire and I



of Train Wire and Radio Telephony

graph thereof is given in Figure 25. Car 399 was designed as a laboratory to carry on these experiments in connection with the first class trains of the railroad. The war, however, had begun; and in 1916 new men took charge of the railroad and this experimental work was temporarily discontinued.

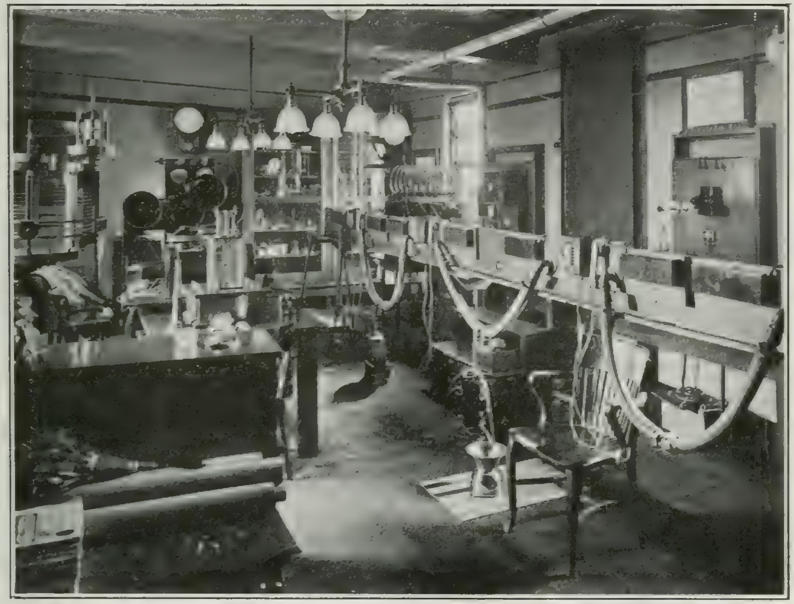


FIGURE 23—Model of Train Wire and Radio Telephone System

THE RADIO LABORATORY CAR

The radio laboratory car was made over from a diner. The kitchen and two of the refrigerators, together with the water tank and hot and cold water, were allowed to remain. The pantry was converted into an engine room (shown in Figure 26) in which we had a 40-horse-power (30 kilowatt) gasoline engine; the shelves of the pantry were used for the glass condensers, and in the roof of the car in the pantry were gasoline tanks, holding a barrel of gasoline, and refillable from the roof. The car was renumbered 399 and classified as an educational car (similarly to the air brake cars and medical examiner's cars, etc.). In the kitchen nothing was changed. A three-phase generator and switchboard, together with the gasoline engine

mentioned. were placed in the pantry, and a door cut thru from the kitchen to the pantry so that access from the body of the car could be had thru the engine room. The shelves for kitchen

FIGURE 24—Inter-Car Coupler of Wire and Radio Telephone Railroad System

utensils, etc., were left in place but soldering irons, a Wheatstone bridge, and vises and mechanical tools were also added so that

FIGURE 25—Railroad Radio Laboratory, Showing Antenna and Car Couplers

the kitchen could be turned into a work shop when desired. Curtains were placed so that the main portion of the car could be utilized for sleeping purposes, the cots being placed over the toilet and other convenient places at the end of the car under the roof when not in use. Where the sideboard used to be, the

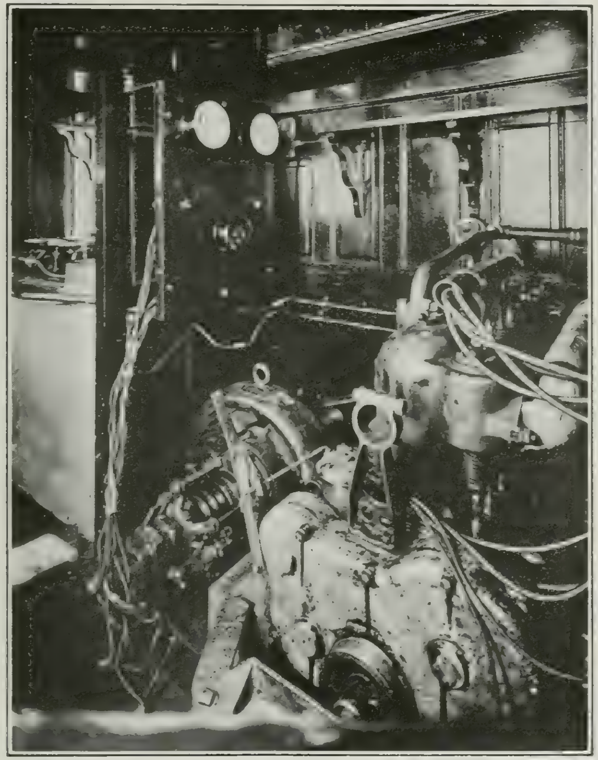


FIGURE 26—Engine Room of Radio Laboratory Car During Installation

coil of the direct coupler was placed and an opening made thru roof for the antenna. The car was 67 feet 9 inches (20.6 m.) over all and the antenna contained twenty-one wires. The highest wires on the frame were connected, forming one antenna; and the other wires were so connected that we had three antennas available which could be switched into one or connected in the car in any manner we desired. On the platform under the dome at the kitchen end of the car, it was intended to place a

"Komet" mast. This is a sectional mast made in Germany, which is elevated by means of a crank and shaft. When telescoped, it occupied a space 6 inches (15 cm.) in diameter and 8 feet (2.4 m.) high. When extended its height was 82 feet (25 m.). This was provided in case we wished to use the car as a telegraph office and desired to transmit over greater distances than we could with the antenna on the car. This type of mast is shown in Figure 27.

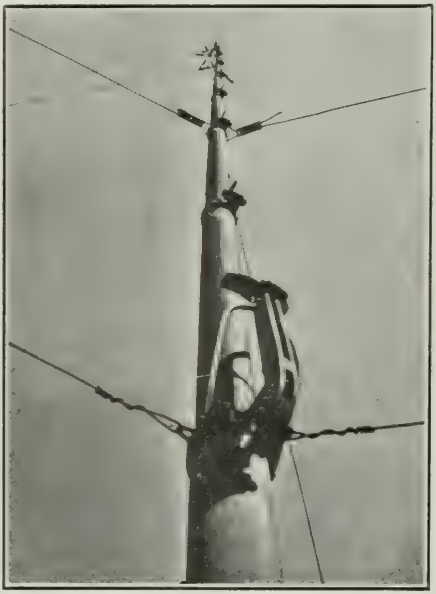


FIGURE 27—"Komet" Mast

It struck us at the time how singularly well adapted this laboratory was for portable long range communication purposes, since it contained both electric light and gas; and, if necessary, two other cars could be lighted from the power plant on the car (which included a three-phase generator). It would have made an ideal headquarters car for the receiving and dispatching of telegraphic work—both wire and radio. The interior view of this car is given in Figure 28. At the forward end of the car was the operator, and to the right of him was the transformer and rotary spark gap. As was stated above, the antenna wire left the car at about its middle. The ground wires resembled

a letter "H" and were fastened to the trucks underneath the car and to the truck boxes, on the inner lid of each of which was a spring ending in a cone which fitted into the conical depression

FIGURE 28—Operator on Duty in Radio Laboratory Car

in the end of the axle, so that when the box was closed tightly we had a friction ground on the axle of the car.

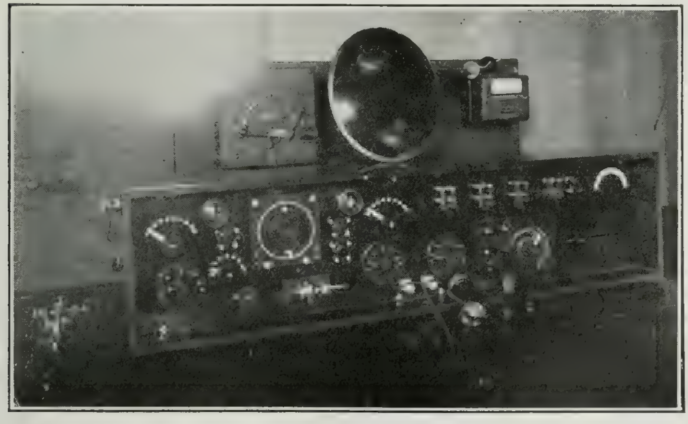


FIGURE 29—Receiving Station at Union Pacific Headquarters

On taking measurements with the Wheatstone bridge it was found that the resistance between the truck and the rail and between the axle and the rail was on an average as follows:—between truck and rail —0.365 ohm, between axle and rail —0.26 ohm.

Of course the greater the number of connections which the ground wire has to the different axles, the less will be the resistance between the car and the ground, providing we do not increase the amount of wire resistance.

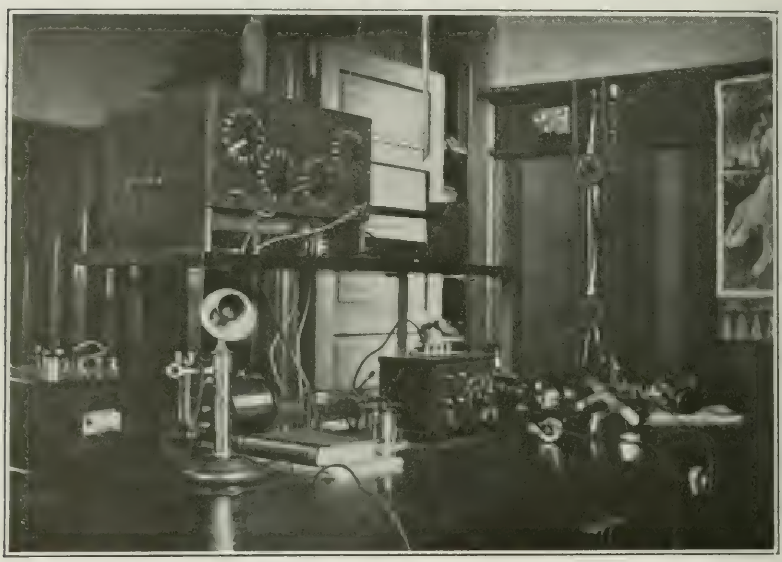


Figure 30—Receiving Set, Including Coaxial Cylindrical Coupler, and Loading Coil

Large Cylindrical Coupler of Coaxial Construction

The first operating room at Union Pacific headquarters is illustrated in Figure 29. A more recent receiver is shown in Figure 30. The antenna used therewith is a five-wire one, wires 2.5 feet (75 cm.) apart, 80 feet (25 m.) long and seventy-five feet (22 m.) above ground and of the "L" type. The couplers, detectors, and inductance are all mounted on one table. At all times, day or night (whenever they are working), we are able to copy the following stations. While there is nothing unusually extraordinary about this reception, it is regarded as

satisfactory, because, until recently, we used an umbrella type antenna supported on a field mast.

"K I E"	Honolulu	"N A A"	Arlington
"N P L"	San Francisco	"N A D"	Boston
"W S L"	Sayville	"W I I"	Belmar
"W G G"	Tuckerton	"W S E"	Seagate

"N A T" New Orleans

The wiring diagram of the receiver is given in Figure 31.

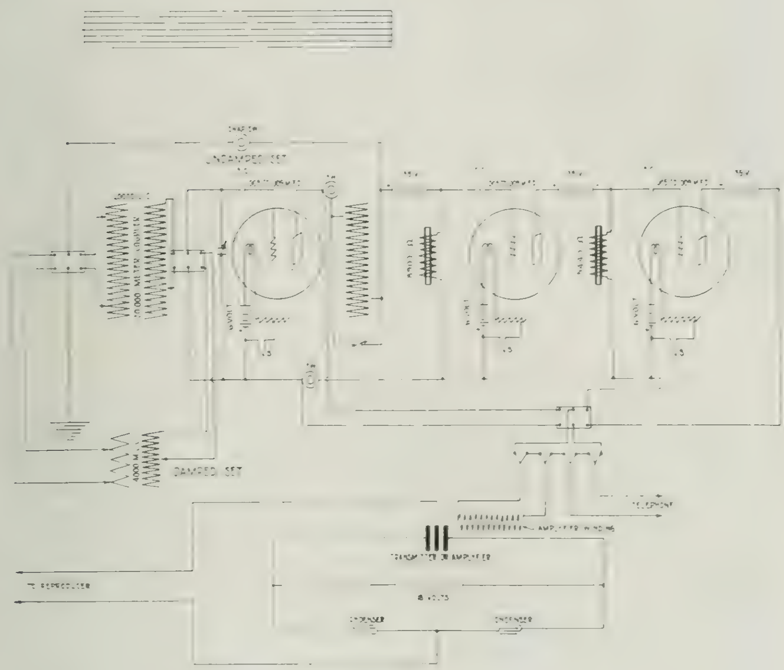


FIGURE 31.—Receiving Set

RECORDING RECEIVED SIGNALS

Early in these experiments, we tried to copy and preserve radio messages on the phonograph. Figure 32 shows a telegraphone recorder while Figure 33 shows equipment adapted to the Columbia dictaphone. With these, however, we did not entirely succeed because we did not have a receiver of constant adjustment. At the present time, we are having fair success using a powerful amplifying relay shown in relay Figures 34 and 35. This is really a powerful telephone transmitter with associated circuits of special design.

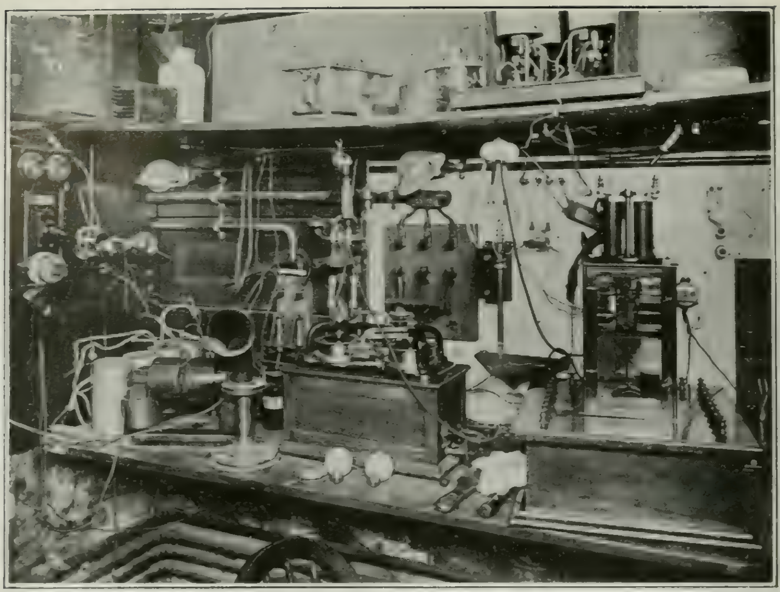


FIGURE 32—Telegraphone Recorder of Radio Signals

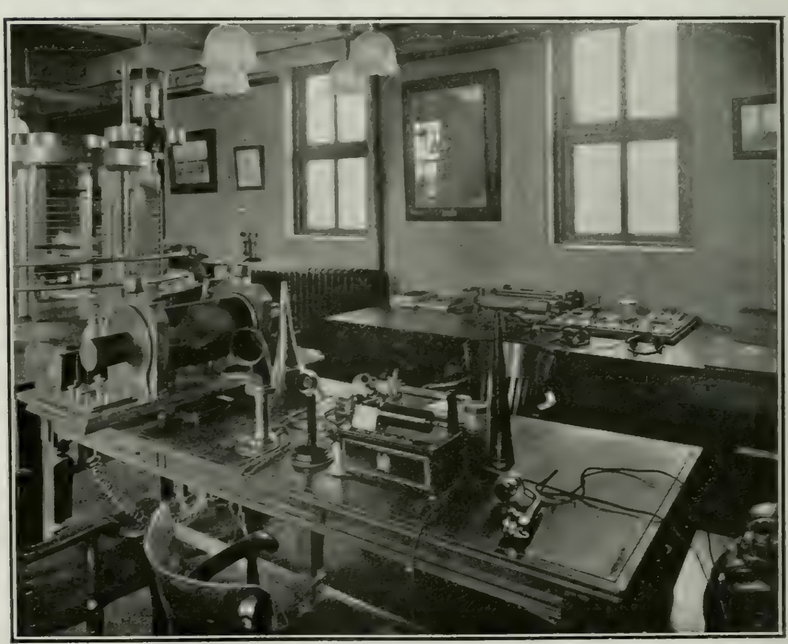


FIGURE 33—Dictaphone Recorder for Radio Signals

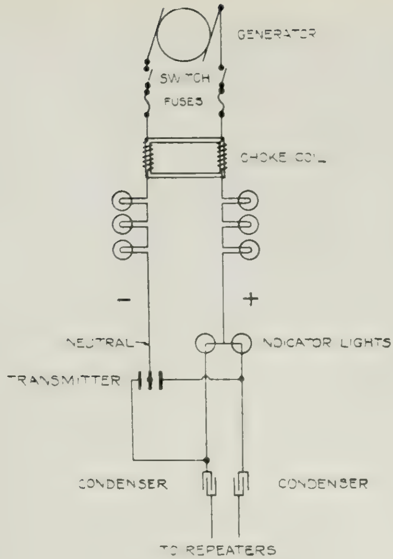


FIGURE 34—Equipment Associated with Amplifier

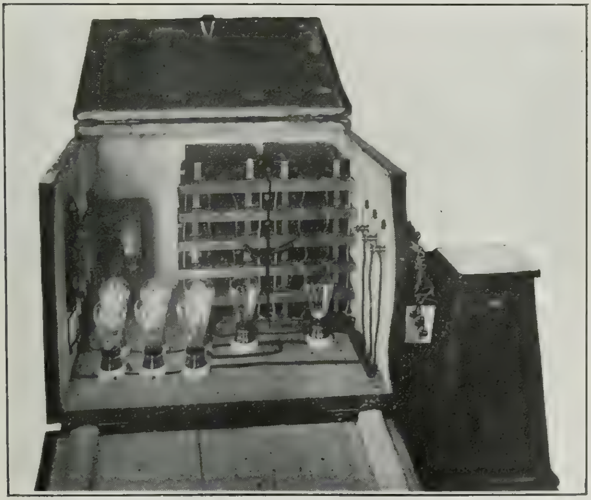


FIGURE 35—Amplifying Relay Equipment

SUMMARY: The radiotelegraphic researches carried on under the auspices of the Union Pacific Railroad are described. A radio "danger" signaling system was first worked; and while operative, did not give the extreme reliability required by such systems. A radio controlled car is then described.

A series of experiments with arc radiophone equipment are considered. Their applications to inter-car wire telephone communication and train-to-station radiophone communication are given in detail. The necessary fixed stations are also described.

The experimental radiotelegraphic car laboratory is then described, together with the antennas, ground, engine, and transmitter thereof.

Finally, receiving sets and a telephone relay amplifier (intended to facilitate telegraphone or dictaphone recording of signals) are considered.

DISCUSSION

George B. Joslin: I recall an experience which I had similar to that of several of the gentlemen present, which illustrates the general opposition with which old railroad employees who had always been accustomed to Morse methods of communication, met an early attempt to experiment along the line of telephonic train dispatching. It was at the time I was in charge of the installation of a railway composite telephone circuit between ———— and —— ——. A wire was selected for the experiment, supposed to be the best between the two points, and at each intermediate station into which this wire looped for telegraph purposes, a condenser was supposed to be bridged across the pin board terminals for the purpose of shunting the telegraph relay against higher frequency telephonic currents. In most cases this wire entered railroad switching and signaling towers, over which presided an old railway employee thoroly familiar with Morse, and bitterly opposed to any innovation which would tend, if it succeeded, to relieve him of his job. These men were of the unanimous opinion that telephonic train dispatching was doomed to failure as it would be impossible to transmit orders by telephone and have them thoroly understood and executed, as, according to them, there was a tendency to take telephone messages in one ear and out of the other. They said instructions communicated in this manner could never stick in a man's mind, but orders communicated by telegraph and as transmitted by a telegraph sounder were literally hammered into the conscience of the man at the receiving end in a manner that he could not forget. We had a howler device for receiving high frequency signaling currents which would not affect or disturb the operation of the telegraph relays on the line; it was what is known to-day as a loud-speaking telephone or receiver in the form of a horn, operated by a diafram mounted on the wall above the telephone instrument. It was loud speaking in that it spoke Morse as well as high frequency notes, so that altho there was telephone secrecy on the telegraph line there was no telegraph secrecy at stations where the composite equipment looped in. From a telephone standpoint, however, our experimental wire was very unsatisfactory as it took up inductively all Morse impulses or rather a composite conglomeration from the neighboring wires, which ran parallel to it, along the pole line a distance of over 200 miles (320 kw.).

The telephonic apparatus to which I refer, was a device tried out in New England about twelve years ago and called a "Railway Composite." It was designed to utilize existing telegraph wires for the electrical transmission of speech without interfering with the telegraphic functions of the wire. My experience with a single untransposed wire—a wire that took the same identical pin on the same particular cross-arm of the telegraph pole line over a distance of 200 miles—was similar to some mentioned. Under this condition, we could hardly avoid getting everything on our wire, by induction, that took place on its neighboring wires thruout this distance, and it occurred to me that the trouble mentioned in early experiments may have been of the same nature, and due to the fact that the line was not transposed or otherwise prepared for telephonic transmission of speech.

FURTHER DISCUSSION ON "OSCILLATING AUDION CIRCUITS" BY L. A. HAZELTINE*

By

August Hund

Professor Hazeltine's paper in the April, 1918, issue of the Proceedings of The Institute of Radio Engineers is certainly of interest to many radio engineers, since it gives the explanation for almost any oscillator arrangement of the three-element type of hot cathode apparatus, using primary emission of electrons. His treatment by means of the loss method has great inherent advantages, since for certain types of coupled systems it may avoid equations of higher degree. Altho the determinant solution at the end of his paper is very broad, it seems worth while to give here a solution which the writer developed several years ago in connection with the oscillating pliotron. The method employed makes use of the generalized symbolic method and is illustrated by the most common arrangement such as indicated in Figure 1, the nomenclature being the same as in Professor Hazeltine's paper. The procedure gives

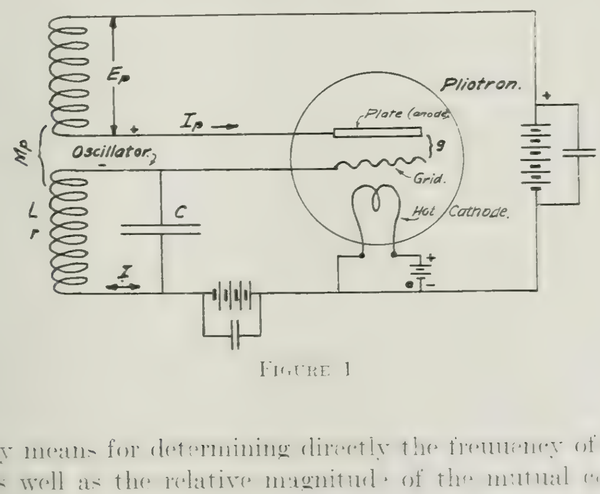


Figure 1

a ready means for determining directly the frequency of oscillation as well as the relative magnitude of the mutual conduct-

* Proceedings of The Institute of Radio Engineers, April, 1918.

ance, g, mutual inductance, M_P, capacity, C, and ohmic resistance, r, simultaneously indicating how plate and oscillator (grid—in this special case) are to be connected. The solution is:

$$\dot{I}\left[r+nL+\frac{1}{nC}\right]+\dot{I}_P\, n\, M_P=0$$

combined with the relation

$$\begin{aligned} I_P &= \dot{E}_G \cdot g \\ &= \frac{\dot{I}}{nC} \cdot g \end{aligned}$$

results in

$$n^2 L+n\left[r+\frac{M_P}{C}g\right]+\frac{1}{C}=0$$

which for the oscillatory case leads to the two complex and conjugate angular velocities

$$\begin{aligned} n &= a \pm j\,\omega \\ &= -\frac{1}{2L}\left[r+\frac{M_P}{C}g\right] \pm j\sqrt{\frac{1}{CL}-\left[\frac{1}{2L}\left(r+\frac{M_P}{C}g\right)\right]^2} \end{aligned}$$

For sinusoidal oscillations, $a=0$, that is,

$$g^{mhos}=\frac{C^F \cdot r^\Omega}{[-M_P]^H}$$

and

$$f^{\sim sec}=\frac{1}{2\pi\sqrt{C^F L^H}}$$

The negative sign belongs necessarily to M_P and indicates that the plate coil must at any instant be of opposite polarity to the grid (oscillator) coil, both being referred to the negative end of the hot cathode.

ON THE INTERPRETATION OF EARLY TRANSMISSION EXPERIMENTS BY COMMANDANT TISSOT AND THEIR APPLICATION TO THE VERIFICATION OF A FUNDAMENTAL FORMULA IN RADIO TRANSMISSION*

By

Leon Bouthillon

(Engineer in Charge of the Radio Telegraphic Service of the Postal and Telegraph Department of France)

For radio transmission with damped oscillations over flat and perfectly conducting ground, with the air regarded as a loss-free dielectric, and for distances greater than several times the wave length, the following expression for the current at the base of the receiving antenna may be deduced theroetically:

$$I_r = 377 \frac{I_s}{R} \cdot \frac{h_1 h_2}{\lambda d} \cdot \frac{1}{\sqrt{1+\frac{\delta_1}{\delta_2}}}$$

where I_r is the current at the base of the receiving antenna;

- I_s the current at the base of the transmitting antenna;
- R the total resistance of the receiving antenna, including the radiation resistance;
- h_1 and h_2 the effective heights of the two antennas;
- d the distance between the antennas;
- λ the wave length; and
- δ_1 and δ_2 the decrements of the sending and receiving antennas respectively.

In this equation, the lengths are expressed in centimeters (0.4 inch), the currents in amperes, and the resistances in ohms. Even if the upper layers of the atmosphere are regarded as an imperfect dielectric, the formula given is applicable to short-range transmission over the ocean.

* Received by the Editor, April 17, 1917. Translated from the French by the Editor.

The proportionality between the current and the effective heights has been verified by the experiments of W. Duddell, J. E. Taylor, L. W. Austin, and others; while the proportionality between received and transmitting current and the influence of the resistance has also been shown to be correct.

The experiments carried on by J. E. Taylor and by Commandant Tissot have similarly shown that, for small distances, the current at the foot of the receiving antenna is in inverse ratio to the distance between the stations. The value of the numerical factor entering into the expression has not, up to the present, been determined experimentally with sufficient exactitude. Barkhausen showed that, on the basis of certain plausible hypotheses, the experiments of L. W. Austin led to the value 377 for this factor. On the other hand, the experiments of W. Duddell and J. E. Taylor are too incomplete to permit deducting this factor from them.

It seems interesting to show that the experiments of Commandant Tissot were of such nature as to constitute a valuable contribution to the study of this question. These experiments, tho carried on at an early date, were so carefully performed and the deductions therefrom were published so precisely[1] that it is possible to calculate the desired numerical factor from them with a close degree of approximation.

The transmitting and receiving antennas consisted each of a single vertical wire 50 meters (162 feet) high and 0.4 centimeter (0.16 inch) in diameter. The transmitting antenna was on board ship, and the transmission was direct.

The current measured at the base of the transmitting antenna was $I_s = 0.95$ ampere. The receiving antenna was on land. A bolometer of resistance $\rho = 17.5$ ohms was inserted at the foot of the antenna. The received current at this point was $I_r = 1.5\ (10)^{-3}$ ampere. The wave length was 210 meters. The distance d between the stations was 1,700 m. (somewhat over a mile). The decrement at the transmitter was $\delta_1 = 0.24$. The measured capacity of the antenna was $C = 300$ cm. (E. S. U.) or 0.00033 microfarad. We shall first calculate the quantities entering into the formula for I_r.

The resistance R of the receiving antenna consists of an ohmic resistance which was almost entirely that of the bolometer, namely $\rho = 17.5$ ohms, and of the radiation resistance R_{Σ}.

[1] In the publication "Sur la resonance des systemes d'antennes," Paris 1905.

The receiving antenna was exactly similar to the transmitting antenna, and, since the transmission was direct, the current distribution was sinusoidal in both antennas and the radiation resistance was given by the expression

$$R_{\Sigma} = 160\,\pi^2 \left(\frac{2}{\pi} \cdot \frac{h}{\lambda}\right)^2$$

h being the actual height of the antennas. In the present case:

$$R = 160\,\pi^2 \left(\frac{2}{\pi} \cdot \frac{50}{210}\right)^2 = 36.5 \text{ ohms.}$$

From this we get $R = R_{\Sigma} + \rho = 36.5 + 17.5 = 54$ ohms. Since the current is sinusoidal, the effective heights of the antennas, h_1 and h_2, are

$$h_1 = h_2 = \frac{2}{\pi} h = \frac{2}{\pi} 50 \text{ meters.}$$

We must still calculate the damping of the receiving antenna. This is given by the equation

$$\delta_2 = \frac{R}{2\,nL} \cdot \frac{\pi}{2}$$

where n is the frequency and L the audio frequency inductance of the system. (The equivalent inductance with sinusoidal distribution is $\frac{2}{\pi}L$.) The inductance L can be deduced from the experimentally determined value of C, namely 300 cm., by the equation $CL = h^2$, (C, L, and h being expressed in centimeters), assuming a sinusoidal current distribution along the antenna. Thus we obtain:

$$L = \frac{(5\,(10)^3)^2}{300} = 0.833(10)^5 \text{ cm. or}$$

$$L = 0.833(10)^{-4} \text{ henry.}$$

Also
$$n = \frac{3(10)^{10}}{\lambda} = \frac{3(10)^{10}}{210(10)^2} = \frac{(10)^6}{7}.$$

If, in the expression for δ_2, we replace, R, L, and n by their values, we obtain:

$$\delta_2 = \frac{54 \cdot 0.7 \cdot \pi}{2(10)^6 \cdot 0.833(10)^{-4} \cdot 2} = 0.356.$$

Finally we obtain the value of the numerical coefficient in the formula:

$$k=\frac{I_r}{I_s}\cdot\frac{R\lambda d\sqrt{1+\frac{\delta_1}{\delta_2}}}{h_1 h_2},$$

$$k=\frac{1.5(10)^{-3}}{0.95}\cdot\frac{54\cdot 210\cdot 1700\sqrt{1+\frac{0.24}{0.356}}}{\left(\frac{2}{\pi}50\right)^2},\quad\text{or}$$

$$k=390.$$

This value is in perfect agreement, as far as can be expected from the degree of precision of the experiments, with the theoretical value of the constant, namely 377.

Thus the experiments of Commandant Tissot may be considered as verifying the important formula which is at the basis of all electromagnetic wave transmission much more satisfactorily than those made before.

SUMMARY: Some early experiments by Commandant Tissot (1905) are used to calculate the value of the constant entering into the usual transmission formula (neglecting the transmission absorption term). The value of the numerical constant in the formula as determined from the experiments agrees well with the theoretical value.

DISCUSSION

Oscar C. Roos (communicated)*: In connection with M. Bouthillon's paper, it may be interesting to call attention to the fact that the current distribution might not be sinusoidal in the antenna vibrating at 210 meters, which is a longer wave-length than its fundamental of 200 meters under the classical theory. However, according to Conrad and Pierce 4.2 times the height gives the fundamental wave-length which in this case is exactly 210 meters. These considerations affect the form-factor and the value of k in the Austin-Cohen formula.

The experiments of Tissot on current distribution are very interesting, especially on the receiver side of the problem. It is entirely possible that the proper form-factor has not been deduced by M. Bouthillon for reasons which follow, and hence the decrement of the receiving antenna $\frac{\pi}{4} \cdot \frac{R_o}{L_o n}$ will change in direct proportion to the error; as R varies as the square of the form-factor and L inversely as the form-factor. Hence, in the final expression for k which is

$$\frac{I_r}{I_s} \cdot R_o \lambda d \sqrt{\frac{1+\delta_1}{\delta_2}},$$

the radiation term 36.5 ohms (see Zenneck's "Wireless Telegraphy," page 40) in R might be corrected in the ratio $\frac{73.2}{80}$; i.e., it may be taken as 33.4 ohms, and the total R would then be $33.4+17.5$ or 52 ohms closely instead of 54. Now the ratio $\frac{52}{54}$ applied to the factor $k=390$ deduced in the paper brings it to 369 which is in error by 8 parts instead of 13 out of 377.

Taking into account the same ratio of change in δ_2, we must apply a correction-factor to the radical when evaluated, of $\frac{1.318}{1.293}$, or to the value $k=369$ above. This, when applied, gives $k=376+$, which is remarkably close to the theoretical value, 377, in fact, too close—there must be other disturbing factors.

Now, if it seems advisable to modify the corrective factors introduced above, there is still the residual source of doubtful receiver form-factor to fall back upon. The current distribution in a receiving antenna vibrating harmonically is not, in general, sinusoidal. This is due to the fact that the differential

* Received June 14, 1917, by the Editor.

equations when correctly based on the e m f. induced per unit length of the receiver-antenna, contain a quadratic plus a circular (or hyperbolic) function of the frequency, of opposite signs. The sender solution contains only the latter types of function, by way of contrast.

Consider a receiver-wire vibrating at its fundamental with a resistance R_o in series to earth. The current in R_o is C_o and

$$C_o = \frac{e\,v}{R_o\,n}$$

where v = velocity of light in cm. per second,

n = vector velocity,

e = e m f. per cm.,

R_o = resistance in $\frac{\text{cm.}}{\text{sec.}}$.

The current C_x at *any* point, x cm. distant from the base of the receiver antenna or the current distribution, under the same circumstances, is given by

$$C_x = \frac{e\,i}{L\,n}\left[1 - \left\{\frac{R_o - L\,v\,i}{R_o} \cdot \cos\frac{n\,x}{v} - \sin\frac{n\,x}{v}\right\}\right] \tag{1}$$

In this equation, when $x = \frac{\lambda}{4} = a$, at the top of the antenna, $C_x = 0$; and when $x = 0$, $C_o = \frac{e\,v\,i}{R_o\,n}$ as above, at base of antenna.

This distribution-curve is a sinusoidal curve the zero point or antinode of which is displaced downward from the top of the antenna—(its normal position in the *sender* vibrating at its fundamental)—by a distance $(a - x_o)$ where

$$x_o = \frac{v}{n}\tan^{-1}\sqrt{\frac{R_o^2 + L^2 v^2}{R_o}} \tag{2}$$

It is important to note, in passing, that L is the specific inductance per unit length of the antenna and is *not* constant along its length; hence it should be replaced by an empirical value or the calculated average value, as was done in 1905 by the writer using Heaviside's rational current element.

This receiver current distribution-curve has its geometrical node displaced away from the *axis* of abcissas by a value equal to $\frac{e}{L_m}$; because the receiver current curve has a point of inflection here.

This unusual current curve has been checked by Tissot in his bolometer measurements and was evaluated by the writer in 1907 from the differential equations appearing in the "Electrical

Review" of October 15, 1904, in an article by Mr. John Stone Stone, and by the latter independently in 1908 before the Society of Wireless Telegraph Engineers,* in Boston.

Figure 1 shows the general outline of the current distribution-curve at the fundamental with only resistance in the base of antenna. The potential distribution is the first derivative of the current distribution. The greater the surge resistance $L v$ is, relatively to the receiving resistance R_o—which includes the resistance of re-radiation—the higher does the point of inflection climb on the curve toward the top of the antenna and the closer becomes the resemblance between the sending and receiving distribution curves.

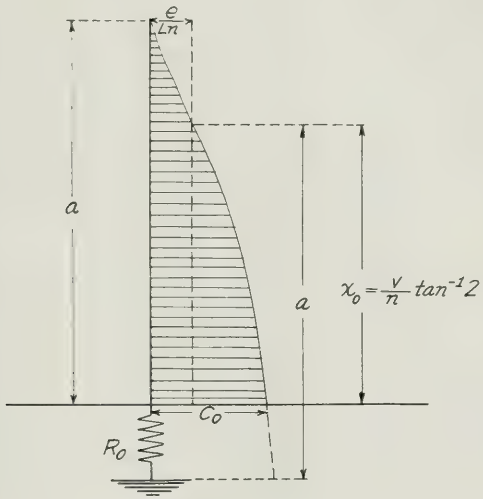


FIGURE 1

Let $L^2 v^2 = 3 R_o^2$, then $x_o = \tan^{-1} 2$ and from equation (2), the point of inflection x_o is about $\frac{2}{3}$ of the way to the top of the antenna. The form-factor is lower than in a transmitter, other things being equal. The interesting question arises as to the relative currents with the same voltage at the base of two antennas, receiver and sender, respectively.

$$C_o = \frac{e\,\lambda}{R_o\,2\,\pi} \text{ at base of receiver,} = \frac{2\,a\,e}{\pi\,R_o} \qquad (3)$$

* One of the societies which combined to form THE INSTITUTE OF RADIO ENGINEERS.

Here $a\,e$ is the potential above ground of the base of the antenna or the top of the receiving equivalent resistance. In a transmitter, at the fundamental, however, $C_o = \frac{E}{R_o} = \frac{a\,e}{R_o}$, hence the current at the base of the receiver under these conditions is $\frac{2}{\pi}$ of the value it would have, other things being equal, as a transmitter.

The final general expression for receiver antenna current-distribution is given below:

Let K be a lumped reactance at the base of the antenna. Then in vector notation $\overline{Z_o = R_o + K_o}$.

Let $p = n\,j = n\sqrt{-1}$, a differential operator for harmonic functions.

a = antenna height in cm.,

n = vector velocity,

$v = 3 \times 10^{10}$ cm. per sec. = velocity of light,

L = average specific inductance of antenna,

$$C_x = \frac{e}{L\,p}\left[1 - \frac{p}{v}\left\{\frac{\frac{v}{p} + \frac{Z_o}{L\,p}\sinh\frac{p\,a}{v}}{\cosh\frac{p\,a}{v} + \frac{Z_o}{L\,v}\sinh\frac{p\,a}{v}} \cdot \cosh\frac{p\,x}{v} + \frac{1 - \cosh\frac{p\,a}{v}}{\frac{L\,p}{Z_o}\cosh\frac{p\,a}{v} + \frac{p}{v}\sinh\frac{p\,a}{v}} \cdot \sinh\frac{p\,x}{v}\right\}\right] \tag{4}$$

This is the solution of Stone's differential equation, given in 1904, for all frequencies and distances on antenna, and it shows that a receiver can vibrate at the octave, which a transmitter can not do. Neither can vibrate when $\cosh\frac{p\,a}{v} = +1$.

When the antenna resistance at the fundamental is large compared to the average surge resistance, $L\,v$, then the point of inflection is half-way down the antenna practically, and theoretically never gets more than half-way down, since $\frac{v}{n}\tan^{-1} 1 = \frac{\lambda}{8} = \frac{a}{2}$.

In conclusion, it is of interest to note that while a transmitting Marconi vertical antenna can not vibrate at any multiples whatever of the octave of the fundamental, a receiver *can* vibrate at the octave and all odd multiples of same, i. e., whenever $\cos\frac{n\,a}{v} = \pi,\ 3\,\pi,\ 5\,\pi$, etc.

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of

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(INCORPORATED)

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THE INSTITUTE OF RADIO ENGINEERS
announces with regret the death of

Morris N. Liebmann

Mr. Liebmann was a Westerner, a graduate of the University of Nebraska, and served with a Western regiment in the Spanish-American War. He came to New York several years later.

In 1901, he joined Company I of the old Twenty-third Regiment, of Brooklyn, rising in rank until he became its Commander. He served as Captain on the Mexican border in June, 1916.

He was Vice-President and Secretary of a large New York company manufacturing scientific and engineering equipment. He was a member of The Institute of Radio Engineers.

In May, 1917, he was commissioned Lieutenant-Colonel, and was repeatedly commended, at Spartanburg and elsewhere, for the excellent showing made by the men under his command.

On August 8, 1918, Lieutenant-Colonel Liebmann was killed in action while leading his men in a charge at the front in Flanders.

He is remembered among his many friends in the radio field as an able and indefatigable worker and a man of loyal and attractive personality.

FEASIBILITY OF THE LOW ANTENNA IN RADIO TELEGRAPHY *

By

Edward Bennett

(Professor of Electrical Engineering, University of Wisconsin)

Contents

I. Considerations Applying to Both Transmitting and Receiving Antennas

* Received by the Editor, September 7, 1917.

1. The Practice in the Construction of High Power Radio Stations

The present practice in the construction of high power radio telegraph stations is to mount the conductors constituting the upper capacity area of the antenna at the greatest elevation above the ground which is deemed to be mechanically feasible. For example, the Government Naval Stations recently erected at Darien in the Canal Zone and at San Diego, Honolulu, and the Philippine Islands, have the capacity areas in the form of horizontal triangular shaped platforms of wire suspended at an elevation of approximately 150 meters (500 feet) above the ground. The wires are suspended from three steel towers each 183 meters (600 feet) high, and the towers themselves are placed at the vertices of a triangle which is approximately equilateral, with the sides 320 meters (1,100 feet) in length. The antennas of the stations at Nauen (near Berlin) and at Tuckerton (near Atlantic City) are carried to even greater heights. These stations have antennas of the umbrella type supported from insulated towers 305 meters (990 feet) and 252 meters (850 feet) high, respectively. It will be noted that the mean radius of the capacity area in the Naval stations above referred to is approximately equal to the height above the ground at which the capacity area is mounted.

In a bulletin entitled, "High versus Low Antennas in Radio Telegraphy and Telephony"[1] it is shown that if the capacity area of the antenna of a radio telegraph station consists of an extended horizontal network of wires mounted above a conducting plane, and if the mean radius of the capacity area is two or more times as great as any height above the ground at which it is feasible to mount the capacity area, then the electric and magnetic forces at a great distance from such a radiator are practically independent of the height of the network above the ground, provided the frequency of oscillation and the operating voltage from the network to ground are kept the same for the different mounting heights. That is to say, an extended network of wires charged

[1] "High Versus Low Antennas in Radio Telegraphy and Telephony," by Edward Bennett, "Bulletin of the University of Wisconsin," number 810, Engineering Series, Volume 8, number 4, pages 179–248, September, 1916.

The conclusions arrived at in this bulletin with reference to the properties of the low antenna are based purely upon a mathematical analysis of the case, and not upon actual experiments between stations with low antennas. Since the publication of the bulletin, the writer has been advised that an experimental trial of a network of wires mounted at a low elevation was made by R. A. Fessenden in 1908. The manner in which the antenna was constructed, and the tests conducted, or the results obtained (save that they were not promising) have not been disclosed.

to a given voltage and allowed to discharge to earth thru an inductance tuned to give a frequency of say 100,000 cycles per second, sets up the same electric and magnetic forces at distant points whether mounted 10 feet (3 m.) or 200 feet (60 m.) above the ground. In these two cases the rate of radiation (in kilowatts) from the two antennas is the same but the initial store of energy in the case of the 10-foot (3 m.) mounting height is about 15 times as great as in the case of the 200-foot (60 m.) mounting height. Therefore, the oscillation in the former case is much more persistent than in the latter; in fact, the oscillation becomes so persistent for low mounting heights that the power condensers and coupled circuits at present required in spark systems of radio telegraphy may be dispensed with and a simple series circuit comprising capacity area, tuning inductance and spark gap, may be used. Such a circuit has the merit of oscillating at a single frequency, whereas the coupled circuits have two frequencies of oscillation.

In a comparison of the receiving properties of two such stations with networks at different elevations it is shown that the above stations are ultimately able to abstract energy at the same rate from passing electromagnetic waves, provided these waves are persistent and not rapidly damped,—the plane of the wave front of the advancing waves being assumed to be normal to the surface of the earth. The high antenna is shown to abstract energy at a greater rate than the low antenna during the initial stages (first few swings) of the oscillation. The high antenna will, therefore, respond much more readily to highly damped waves than will the low antenna. This means, of course, that when receiving undamped or slightly damped waves, the high antenna will be subject to greater interference from atmospheric disturbances than will the low antenna. To reduce this interference in the case of stations with high antennas, additional capacity is used in the "interference preventer" circuits. In other words, the low antenna is to be regarded as the equivalent of a high antenna and "interference preventer" combined.

The summary of the bulletin above referred to contains the statement—"It is feasible to construct a radiator with a capacity area at a very moderate elevation which will have a radiation figure of merit equal to or greater than the values which are at present attained by mounting the capacity area at a great elevation in long-distance radio stations."

The disclosure that the low antenna mounted above a *conducting plane* will initially radiate energy at the same rate as the

elevated antenna, and that it will *ultimately* be able to abstract energy from impinging waves at the same rate as the elevated antenna is by no means a demonstration of the feasibility of the low antenna. Its feasibility hinges upon a score of other considerations such as the following:

(a) the relative effects produced by resistance losses in the earth in the vicinity of antennas of the two forms—high and low.

(b) the relative response of antennas of the two forms to oscillations for which they are tuned and to disturbing sources, such as strays and detuned stations.

(c) the feasibility of obtaining from generators of undamped waves the large currents required by the low antenna.

(d) at low frequencies, the time constant of the receiving antenna may be so great that a dot or a dash is completed before the oscillation in the receiving antenna has built up to its full value.

This paper presents calculations and data relating to the feasibility of the low antenna. It deals mainly with those wasteful antenna and earth resistances which are common to the use of the low antenna both in sending and receiving. It is the purpose to discuss the other aspects of the feasibility of the low antenna in subsequent publications under the following headings:

II. The Low Antenna for Receiving Purposes
III. The Low Antenna as a Radiator
IV. Mechanical Considerations

2. Distinction Between the "Low" Antenna and the "Ground" Antenna

Before proceeding, it may be well to distinguish between two types of antenna, namely, the *Marconi* antennas of Figure 1, 2, and 3, and the *ground*, or *Hertzian* antenna, of Figure 4,—and between three forms of the Marconi antennas, namely, the low power Marconi antenna of Figure 1, the high power *elevated* Marconi antenna of Figure 2, and the high power *low* Marconi antenna of Figure 3. The features of these types and forms are shown in the figures. By the high power *elevated* antenna is meant an antenna in which the radius of the capacity area and the elevation of the capacity area above the ground are about equal in value. By the high power *low* antenna is meant an antenna in which the elevation of the capacity area is equal to 1-10th or less of the radius of the capacity area.

The Marconi antennas (for sending and receiving purposes) are most effective if mounted over a plane of infinite conductivity, while the *ground* antenna (which apparently was first investigated by Kiebitz[2]) would not function at all as a receiver if mounted adjacent to such a plane. The *low* antenna of Figure 3 should not be confused with the *ground* antenna of Figure 4.

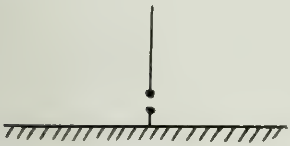


FIGURE 1—Marconi Antenna of Low Power

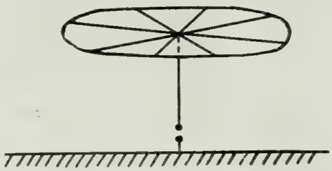


FIGURE 2—"Elevated" Marconi Antenna of High Power

With the low antenna it is advisable to use as the lower capacity area a network of wires mounted a few feet above the earth's surface, because low resistance in the lower capacity area is absolutely essential to the success of the low antenna On the other hand, high conductivity in the surface layers of the earth would render the ground antenna inoperative. The ground

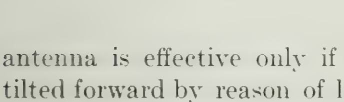


FIGURE 3—"Low" Marconi Antenna of High Power

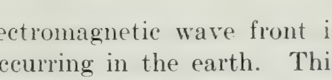


FIGURE 4—Hertzian "Ground" Antenna

antenna is effective only if the electromagnetic wave front is tilted forward by reason of losses occurring in the earth. This paper deals only with the relative merits of the Figure 2 and Figure 3 forms of the Marconi antennas, and not at all with the ground antenna.

3. ILLUSTRATIVE LOW AND ELEVATED ANTENNAS

In order that the comparison between the low and the elevated antenna may be as specific as possible, the proportions and

[2] F. Kiebitz, "Jahrbuch der drahtlosen Telegraphie," 1912, Volume 5, page 360; Volume 6, pages 1, 554.

constants of an extended horizontal antenna having a height of 10 meters (33 feet) and a radiation figure of merit equal to that of the antenna of the United States Government Naval Station at Darien in the Canal Zone will now be computed. The treatment which follows will then be illustrated by comparing the properties of the "10-meter Darien-equivalent" antenna with the corresponding properties of the Darien antenna.

By the "radiation figure of merit" of an extended antenna is meant the product obtained by multiplying the capacity of the elevated network of the antenna by its height above the lower capacity area—generally the surface of the earth. Two extended antennas having the same figure of merit and located upon a plane of infinite conductivity radiate energy at the same rate if they are operated at the same frequency[3] and at the same voltage between the networks and earth. When in *full oscillation* as receiving antennas, they abstract energy at the same rate from sustained waves, provided that in each case the ohmic resistance of the antenna is made equal to its radiation resistance.

The constants[4] of the Darien antenna are as follows:

Capacity of network to earth	0.01 microfarads
Effective height of network	146. meters
Figure of merit of antenna	1.46 microfarad-meters

Therefore the constants of the 10-meter Darien-equivalent antenna will be:

Height of network	10. meters
Capacity of network to earth	0.146 microfarads
Area of network (approximately)	165000. sq meters
Radius of circular network	229 meters

The network area and radius are first approximations only, calculated upon the assumption that the wires of the network

[3]The statement that the radiating and absorbing powers of a given extended horizontal platform antenna (having a radius equal to two or more times its mounting height) are independent of the mounting height, is applicable only to the usual condition of operation in radio telegraphy. The usual practice is to have much of the inductance in the tail of the antenna, and to operate at a frequency much lower than the natural frequency of the unloaded antenna. If the inductance in the tail of an antenna is made so low and the operating frequency so high that there is at any instant a large difference in potential between points of the extended capacity area, the case becomes so complex as to transcend present powers of analysis. The above conclusions are only approximations in such a case.

[4]For a description of the Darien Station, see Lieutenant R. S. Crenshaw, "The Darien Radio Station of the U. S. Navy," "Proceedings of The Institute of Radio Engineers," 1916, Volume 4, page 35. Also L. W. Austin, "Experiments at U. S. Naval Radio Station, Darien, Canal Zone," "Proceedings of The Institute of Radio Engineers," 1916, Volume 4, page 251.

are so close that the network may be treated as a conducting sheet or plate. The spacing of the wires of the network will be discussed later.

4. Electrical Constants of the High and Low Antenna Circuits

Before attempting to discuss such features of the antenna as the spacing of the wires, conductor resistance, earth resistance, etc., it is helpful to consider the electrical constants of hypothetical antenna circuits free of all resistance save the radiation resistance. Table I is a compilation of the electrical constants of the hypothetical Darien and the hypothetical 10-meter Darien-equivalent antennas for the frequencies corresponding to wave lengths of 2,500 and 10,000 meters. In calculating these constants, the antenna circuits have been regarded as series circuits containing segregated inductance and capacity and a resistance equal to the radiation resistance.

TABLE I

ELECTRICAL CONSTANTS OF HYPOTHETICAL ANTENNA CIRCUITS CONTAINING NO RESISTANCE SAVE THE RADIATION RESISTANCE

	Varies as	Darien Antenna $C=0.01$ μf. $h=146$ meters		10-meter Darien-equivalent Antenna $C=0.146$ μf. $h=10$ meters	
Wave length (meters)		2,500	10,000	2,500	10,000
Frequency (cycles per sec.)		120,000	30,000	120,000	30,000
Inductance (μ henrys)	$h\,a^{-1}f^{-2}$	176	2,820	12.1	193
Capacity reactance (ohms)	$h\,a^{-1}f^{-1}$	132	530	9.08	36.3
Critical resistance (ohms)	$h\,a^{-1}f^{-1}$	265	1,060	18.16	72.6
Radiation resistance (ohms)	$h^2 a^0 f^2$	5.37	0.336	0.0252	0.00158
Current at 100 r. m. s. kv. (amp)	$h^{-1}a\,f$	754	188	11,020	2,755
Power radiated at 100 kv. (kw.)	$h^0 a^2 f^4$	3,080	12	3,080	12
Time constant (seconds)	$h^{-1}a^{-1}f^{-4}$	0.000066	0.0168	0.00095	0.244
Time constant (periods)	$h^{-1}a^{-1}f^{-3}$	7.87	504	114	7,320
Resistance ratio	$h^{-1}a^{-1}f^{-3}$	49.4	3,160	718	46,000

Referring to Table I,

Line 3 shows the inductance necessary to make the antenna circuit reasonant to the wave lengths and frequencies tabulated in lines 1 and 2.

Line 4 shows the capacity reactance to these frequencies. The inductive reactance is, of course, substantially equal to the capacity reactance.

Line 5 shows the critical resistance R_c of the circuit,—the resistance which would render the circuit just non-oscillatory. The critical resistance is substantially equal to twice the reactance.

Line 6 gives the radiation resistance R_r of the antennas as computed from the formula,

$$R_r = \frac{160\,\pi^2\,h^2 f^2}{s^2}$$

The efficiency of the antenna as a radiator is equal to the ratio of its radiation resistance to its total resistance. The low value to which the radiation resistance of the 10-meter antenna falls is striking,—0.00158 ohm for the wave length of 10,000 meters.

Line 7 gives the r. m. s. value of the current which would be observed in the tail of the antenna with a sustained voltage of 100 r. m. s. kilovolts from the capacity network to earth.

Line 8 gives the rate of radiation in kilowatts from the antenna at the above current and voltage. It will be noted that a sustained voltage of 100 r. m. s. kv. between antenna and earth at a frequency of 120,000 cycles per second would mean a rate of radiation of 3,080 k.w. This means that the voltages attained at this frequency under sustained wave operation are far lower than 100 kv.

Lines 9 and 10 show the *time constants* of the circuits, expressed in seconds and in periods respectively. In the case of a receiving antenna, the time constant is the interval of time required, after the alternating voltage is first impressed upon the circuit, for the current and the condenser voltage to build up to 73.2 per cent. of their final or full oscillating values. It is to be noted that the time constant of the hypothetical 10-meter antenna at a frequency of 30,000 cycles is 0.24 seconds. In such a circuit the oscillation would not have sufficient time during a Morse dot interval to build up to its full value, since the approximate length of the dot interval is only 0.05 seconds.

Line 11 gives the "*resistance ratio*" of the hypothetical antennas. By the *resistance ratio* of a series oscillatory circuit is meant the ratio of the *critical resistance* of the circuit to its *actual*

resistance. This ratio is equal to twice the ratio of the reactance (at the natural frequency of the circuit) of the inductance, or of the condenser, to the actual resistance of the circuit. Therefore under sustained wave operation in either a receiving or radiating antenna, the ratio of the condenser voltage (voltage from network to ground) to the induced voltage is equal to one-half of the resistance ratio. The high values this ratio attains for the 10-meter antenna should be noted. For example, at a frequency of 30,000 cycles, the condenser voltage would build up to a value equal to 23,000 times the induced voltage.

The column headed "*Varies as*" shows the manner in which the values of the quantities listed above vary with the height and area of the antenna, and with the frequency of oscillation. For example, the time constant (expressed in seconds) of an extended antenna varies inversely as the first power of the height, the first power of the area, and the fourth power of the frequency.

Reference will be made to this table in succeeding sections.

5. Power Losses in the Antenna Circuit

The efficiency of the antenna circuit as a radiator of electromagnetic energy is the ratio of the power radiated to the total power expenditure in the antenna in radiation, wire and earth resistance, etc. The power radiated from a given extended antenna *at a given frequency* is directly proportional to the square of the current measured in the tail of the antenna. Since the power radiated is proportional to the square of the current, it is convenient to introduce the quantity known as the *radiation resistance*, R_r, of the antenna—a fictitious resistance (not a constant, but a variable having a value proportional to the square of the frequency) of such a value that the product $I^2 R_r$ is equal to the power radiated. It is then convenient to compute the *equivalent resistances* of all other sources of loss for comparison with the radiation resistance. By the *equivalent resistance* of such a source of loss as the ionization of the air around the antenna wires at high voltages is meant a fictitious resistance of such a value that the product obtained by multiplying this resistance by the square of the antenna current will give the power expended as the result of ionization. For a given antenna, these fictitious resistances are not constant, but they are variables the values of which depend on the frequency, and in some cases on the voltage impressed upon the insulating medium in which the loss occurs. The sum of these resistances may be termed the *wasteful*, or *dissipative* resistance, R_w, of the antenna, while the

sum of the *radiation* resistance plus the wasteful resistance may be termed the antenna resistance, R_a. If the equivalent resistances of all sources of loss are so expressed, the efficiency of an antenna circuit as a radiator of electromagnetic energy is the ratio of its radiation resistance to its antenna resistance.

When used as a receiver, the energy abstracted from the passing waves by the antenna circuit is expended partly in the detector, partly in the other resistances, and is partly re-radiated. When receiving sustained oscillations, the power expenditure in the detector is a maximum if the equivalent resistance of the detector is made equal to the sum of the radiation resistance plus the wasteful resistance of the antenna. With such an adjustment, the power abstracted by an antenna of given dimensions from sustained waves of a given frequency and intensity and delivered to the detector is inversely proportional to the antenna resistance. Since the radiation resistance of an antenna of given form and dimensions is inherent in the form and dimensions, it follows that the efficiency of a given antenna as a receiver is 100 per cent. if the equivalent resistance of the detector is made equal to the radiation resistance and if the wasteful resistance is made negligibly small in comparison with the radiation resistance. If the wasteful resistance is not negligibly small, the efficiency of the antenna as a receiver is equal to the ratio of the radiation resistance to the total antenna resistance (exclusive of the detector).

Thus in either case, whether used as a radiator or as a receiver, the efficiency of an antenna circuit is equal to the ratio of the radiation resistance to the sum of the radiation plus the other antenna circuit resistances. There are, then, the following resistances to be considered.

R_r representing the radiation resistance of the antenna
R_w representing the wasteful resistance of the antenna
R_d representing the equivalent resistance of the detector (reduced to the antenna circuit)
$R_a = (R_r + R_w)$ representing the antenna resistance
$R_t = (R_r + R_w + R_d)$ representing the total resistance

The regions of power expenditure in the antenna have been listed in Table II. The computed or estimated equivalent antenna resistances of some of these regions of loss have been given in this table for the 10-meter Darien-equivalent antenna at frequencies of 120,000 and 30,000 cycles per second. The methods used in arriving at these equivalent resistances are discussed in succeeding paragraphs.

TABLE II

EQUIVALENT ANTENNA RESISTANCES OF THE TEN-METER DARIEN-EQUIVALENT ANTENNA

(Resistances are in ohms)

Frequency (cycles per second)	120,000	30,000
Wave length (meters)	2,500	10,000
Region of power expenditure		
Common to sending and receiving		
1. Radiation resistance	0.0252	0.00158
2. Conductor resistance—upper network	0.0034	0.0017
3. Conductor resistance—lower network	0.0034	0.0017
4. Earth resistance from surface to buried wires	0.0007	0.0007
5. Earth resistance from choking effect	0.0016	0.0001
6. Extra-peripheral earth resistance	0.0012	0.0006
7. Grass resistance	0.003	0.004
8. Insulator dielectric hysteresis	———	———
9. Insulator wet weather leakage	———	———
10. Supporting poles or structures	avoidable	avoidable
Additional regions when receiving		
11r Tuning inductance and secondary	———	———
12r Detector	———	———
Additional regions when sending		
11. Ionization losses around wires	avoidable	avoidable
12. Ionization losses at insulators	———	———
13. Tuning inductance	———	———
14. Spark or	———	———
14a Poulsen arc generator, and radio frequency transformer or	———	———
14b Radio frequency alternator, and radio frequency transformer	———	———

6. Spacing of the Antenna Wires

The radiation resistance of the low antenna is so low that ordinary methods of grounding the antenna may easily lead to a power expenditure in the earth resistance far in excess of the power radiated. An extended low antenna will therefore necessarily contain a lower as well as an upper network of wires. In some cases the lower network may be buried in the earth at a depth of 0.2 meter or more. In other cases it may be advisable to mount the lower network at an elevation of two or three meters above the surface of the earth, in order to avoid the earth losses discussed later. In either case, the lower network should not only be co-extensive with the upper network, but it should extend beyond the periphery of the upper network to a distance of 30 meters (100 feet) or more, and it should be thoroly grounded at many points of its periphery.

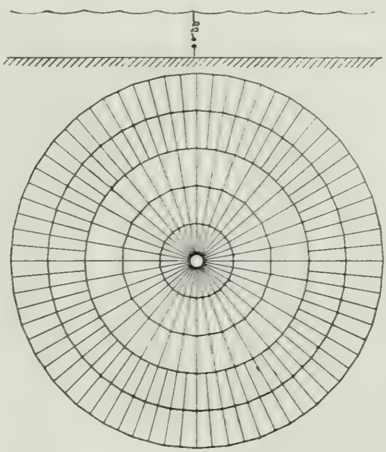


Figure 5

The conductors constituting either network will necessarily issue from common points in a radial manner, either as the spokes of a wheel as in Figure 5 or as the ribs of a fan. The diameter of the conductor used in the networks will be determined by the requirement of mechanical strength. To meet this requirement, it is assumed in the following calculations that a hard drawn copper wire (or a copper clad steel-core wire) having a diameter of 4.1 mm. (0.16 inch) (number 6 B. and S. gauge) will be used in networks above the ground and a copper wire

having a diameter of 3.3 mm. (0.13 inch) (number 8 B. and S. gauge) in buried networks.

The greater the mean distance between the wires of a network, the less will be the weight of the copper required to obtain a given capacity, and the less will be the mechanical difficulties of suspension. On the other hand, the less the distance between the conductors, the less will be the ohmic resistance of the network, the smaller will be the ground area required to obtain a given capacity, and the higher will be the sending voltage between network and earth which may be used without incurring ionization losses in the air around the conductors.

The effect of the spacing of the wires upon the steady-state capacity to earth of a horizontal harp of given area made up of 4.1 mm. (0.16 inch) wires stretched parallel to each other at an elevation of 10 meters above the ground is shown in Figure 6.

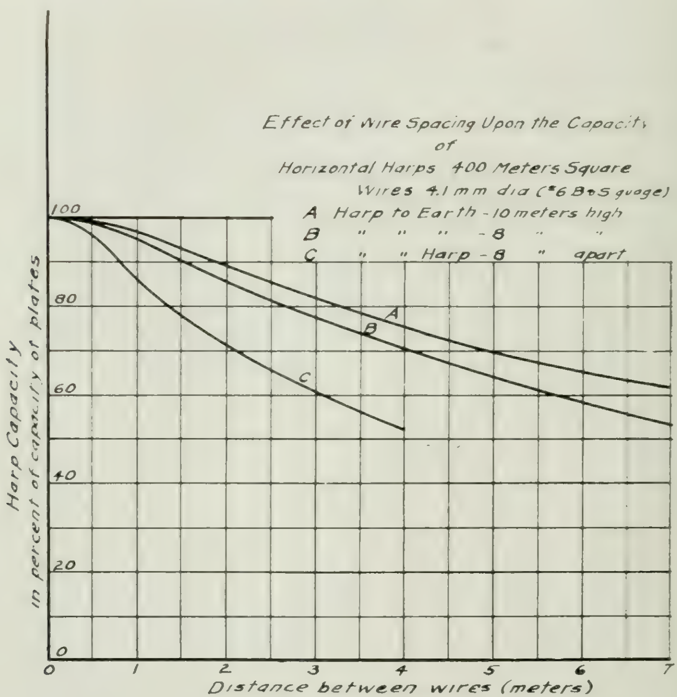


FIGURE 6

The points for this curve have been computed by Kelvin's method of images for a square harp 400 meters (1,350 feet) on a side. The computations are based upon the assumption that

the charge per unit length of wire is uniform over the entire harp. As a matter of fact, the charge per unit length of wire is somewhat greater near the edges of the harp than near its center, but the error in the curves resulting from the assumption of uniform distribution is very slight indeed. The capacities of the harps with different wire spacings have been plotted in per cent. of the capacity which the same area would have if it were made up of a continuous sheet, or plate, of conducting material. Figure 6 also contains a similar curve for the capacity to earth of a harp 8 meters (25 feet) above the earth, and also for the capacity between two parallel wire harps mounted 8 meters apart in space.

No ionization losses in the air surrounding the conductors are to be expected if the electric intensity or potential gradient at the surface of the conductors during the peak of the voltage wave does not exceed 30 peak kilovolts per cm. Figure 7 shows for different spacings of 4.1 mm. (0.16 inch) wire the root-mean-square value of the voltage between the harp and earth which will give rise to a gradient of 30 kilovolts per cm. at the surface of the wire. Since corona does not form around polished 4.1 mm. wires except at gradients 67 per cent. greater than 30 kv. per cm., these voltages may, somewhat arbitrarily, be designated as the "safe non-ionizing sending voltages."

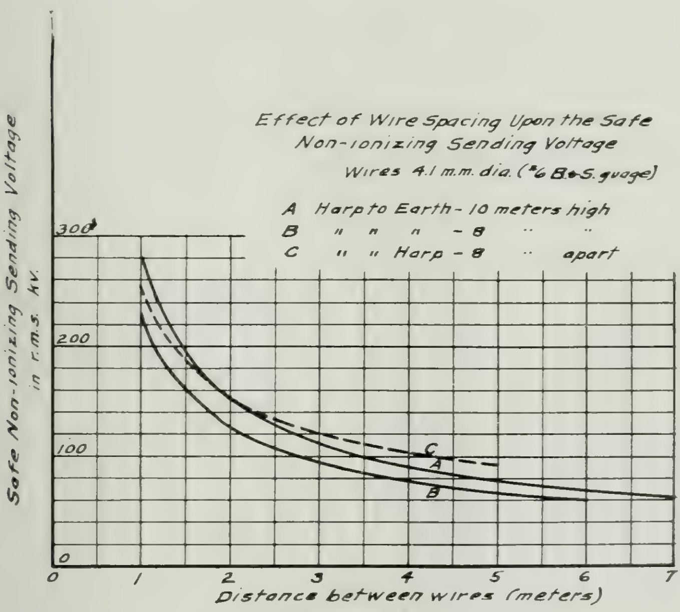


FIGURE 7

The voltages plotted in Figure 7 may be obtained as follows: The electric displacement or the electrostatic flux density, D, at the surface of the wire under a gradient, F, of 30,000 volts per cm. is

$D = pF = 30{,}000\,p = 30{,}000\ (8.84 \times 10^{-14})$ coulombs per sq. cm.

(p represents the permittivity of air)

The quantity of electricity Q per centimeter length of 4.1 mm. wire at this flux density is

$$Q = 0.41\,\pi\,D = 3.42 \times 10^{-9} \text{ coulombs}$$

The voltage to earth necessary to cause this charge per centimeter length of wire may readily be obtained from the capacity curves plotted in Figure 6.

An inspection of Figure 6 shows that the capacity of the 10-meter Darien-equivalent antenna is less than the capacity between plates of the same dimensions by only 4 per cent. if the wires are spaced 1 meter apart, by 11 per cent. with a spacing of 2 meters (6 feet 7 inches), 18 per cent. at 3 meters (9 feet 10 inches), and 25 per cent. at 4 meters (13 feet 1 inch). Considering the effect of the wire spacing upon the capacity of the antenna, a spacing of at least 3 or 4 meters is preferable if land is cheap.

An antenna designed to be used mainly at the lower frequencies (20,000 to 50,000 cycles) will, for two reasons, require a closer spacing of the antenna wires than if it were designed to be used at the high frequencies.

The first reason for a closer spacing of wires in low frequency stations is that high operating voltages must be used in order to radiate any great amount of power at the low frequencies. For example, the rate of radiation from the Darien antenna, and from the 10-meter Darien-equivalent antenna, at a frequency of 30,000 cycles is only 12 kw. at 100 r.m.s. kv. (See Table I). The higher the operating voltage, the closer must be the spacing of the antenna wires if ionization losses at the conductors are to be avoided. An inspection of Figure 7 shows that for a working voltage of 100 r. m. s. kv. the distance between wires need not be less than 3.5 meters (12 feet). In stations operated from Poulsen arcs at the higher frequencies (120,000 cycles), the sending voltage will not exceed 40 r. m. s. kv., since the rate of radiation of the Darien antenna at this frequency and at 100 r. m. s. kv. is 3,080 kw.—a rate 8 times as great as the capacity of any radio frequency generating plant at present in use. For a voltage of 40 kv. a wire spacing of 10 meters would be sufficient to avoid ionization losses around the wires.

The second reason for the closer spacing of wires in low frequency stations is the low radiation resistance of the antenna at the low frequencies. At $1/n$th the frequency, the radiation resistance is $1/n^2$ as great. From this it follows that to obtain the same antenna efficiency at two different frequencies (either in sending or receiving), the conductor resistance for the low frequency antenna should be made $1/n^2$ as great as that of the high frequency antenna. This lower antenna resistance can in practice be obtained only by using more wires spaced closer together.

7. Resistance of Antenna Wires

With the wires of the capacity area arranged as in Figure 5, namely, with 270 copper wires of 4.1 mm. (0.16 inch) diameter (number 6 B. and S. gauge) extending in a radial manner from a common point to a circle having a diameter of 280 meters (918 feet), and with 540 wires extending radially from this circle to a circle having a diameter of 460 meters (1,508 feet), the direct current resistance of the network from the common point to the center of load is approximately 0.0006 ohms. The alternating current resistance of 4.1 mm. (0.16 inch) copper wire at frequencies of 30,000 and 120,000 cycles is 2.9 and 5.6 times its d. c. resistance. Therefore the a. c. resistance of the network will be 0.0017 and 0.0034 ohms to the two frequencies. At 30,000 cycles the resistance of the conductors in the upper network is substantially equal to the radiation resistance, while at 120,000 cycles the conductor resistance is only 14 per cent. of the radiation resistance. The conductors of the lower network will be regarded as arranged to have substantially the same resistance as the upper. For example, if buried, the lower network might consist of 360 copper wires of 3.3 mm. (0.13 inch) diameter (number 8 B. and S. gauge), extending in a radial manner to a circle having a diameter of 280 meters (918 feet) with 720 wires extending from this circle to a circle having a diameter of 520 meters (1,705 feet.)

8. Losses in the Earth

If the lower network is buried beneath the surface of the earth, or is laid upon the surface of the earth, the damping losses due to the flow of current in the earth may be considered under three headings.

The specific resistance of the earth is so high in comparison with that of copper that the current which flows radially inward and outward from the ground end of the antenna tail is carried

mainly by the buried network. Conduction currents flow from the earth's surface (at which the lines of displacement from the upper network terminate) and converge upon the conductors of the buried network in the manner shown in Figure 8. That resistance which when multipled by the square of the antenna current will give an I^2R product equal to the power expenditure due to the currents pictured in Figure 8, will be termed the "*earth resistance from surface to buried conductors.*" This is the first resistance to be considered.

Altho the current which flows out radially from the foot of the antenna tail is carried mainly by the buried network, still a part of the current streams out thru the earth itself, being confined to a surface layer the depth of which varies from about one meter (3.28 feet) for sea water at a frequency of 120,000 cycles to 100 meters (328 feet) for earth of fairly high resistivity at a frequency of 30,000 cycles. While the conductance of the earth is in parallel with the conductance of the buried network and would at first sight seem to give rise to a smaller loss than if the network alone carried the current, yet paradoxically, the result is just the opposite. The effective resistance of the two conductors in parallel—the buried wires and the conducting earth—is greater than the resistance of the wires alone. The explanation of this paradox is that the concentration of the current in the buried wires leads to the distribution of magnetic flux about the wires pictured in Figure 9. As a result, the voltage consumed by inductance is greater in the copper conductor than in like filaments of earth at some distance from the conductor. This means that the high resistance earth filaments at some distance from the wire are forced to carry a greater current than they would carry with a continuous electromotive force impressed, and the resultant loss is greater than if the copper conductors carried all the current. That resistance which when multiplied by the square of the antenna current will give an I^2R product equal to the power expenditure due to the earth currents just pictured, will be termed the "*earth resistance from the choking effect.*" This is the second resistance to be considered. Both this resistance and the *earth resistance from the surface to buried conductors* may be eliminated by mounting the lower network at a height of 1 to 3 meters (3 to 10 feet) above the surface of the earth.

It is not feasible to extend the lower network much beyond the periphery of the upper. Lines of displacement originating on the upper network terminate, as illustrated in Figure 10, upon

the surface of the earth at points beyond the limits of the lower network. From the termination of the lines of displacement upon the earth's surface, conduction currents flow in the surface skin of the earth, converging in radial lines upon the grounded periphery of the lower network. That resistance which when multiplied by the square of the antenna current will give an I^2R product equal to the power expenditure caused by these currents, will be termed the "*extra-peripheral earth resistance.*" This is the third resistance to be considered.

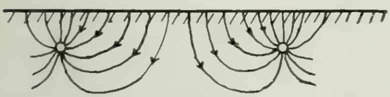


FIGURE 8—Current from Earth's Surface to Buried Wires

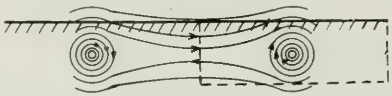


FIGURE 9—Distribution of Magnetic Lines About the Buried Wires
Wires and current flow are perpendicular to the plane at the paper

Earth resistance from surface to buried wires. The resistivity of the earth in which the network is buried will lie between the limits 1,000 ohm-cms. and 100,000 ohm-cms. In the subsequent calculations a resistivity of 10,000 ohm-cms. will be assumed.

With the 137,000 meters (449,400 feet) of 3.3 mm. (0.13 inch) (number 8 B. and S. gauge) wire buried at a depth of 0.25 meter (0.82 feet), and with an average spacing of approximately 1.5 meters (4.9 feet) between wires, the computed resistance from the surface of the earth to the buried conductors is of the order of 0.0007 ohms. This resistance is equal to 44 per cent. and 3 per cent. of the radiation resistances at 30,000 and 120,000 cycles respectively. When the ground is frozen a resistance much higher than the value above computed may be expected.

Earth resistance from the choking effect. As pointed out above, the concentration of the current in the buried wires leads to the distribution of magnetic lines of force pictured in Figure 9. As a result, the voltage consumed by inductance is greater in the case of a filament of the copper wire than in the case of a parallel filament of earth at some distance from the wire. In other words the "choking action" of the flux which encircles the copper wire but does not encircle the earth filament forces the earth filament of high resistivity to carry more current than would be determined by the ratio of the conductances of the two filaments.

The ratio of the current density in the film of earth on the surface of the copper wire to the current density in the surface film of copper will equal the ratio of the conductivity of earth to the conductivity of copper. For earth of the conductivity previously assumed, this ratio is 10^{-4} to (5.5×10^5) or 1 to (5.5×10^9).

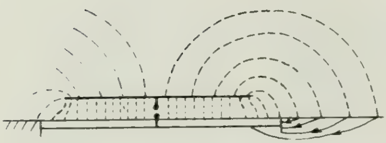


FIGURE 10—Current Causing Extra-peripheral Loss

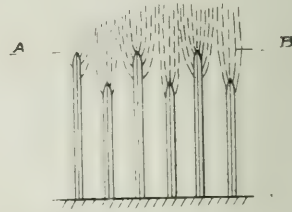


FIGURE 11—Lines of Displacement from Upper Portions of Grass

As the film of earth under consideration is taken farther and farther from the surface of the copper wire, the current density increases with the distance in the manner shown in Table III. This table has been computed for a 3.3 mm. (0.13 inch) wire and an earth resistivity of 10,000 ohm-cms. at a frequency of 30,000 cycles per second. There is some uncertainty as to the correctness of the values given for the current densities at distances greater than 5 centimeters (2 inches) from the center of the wire.

An extreme upper limit for the magnitude of the I^2R loss occasioned by these earth currents may be rapidly arrived at in the following way: Assume that the current density in the entire block of earth enclosed by the dashed lines in Figure 9 is 30 times as great as the actual current density in the earth adjacent to the wire. This block of earth, containing a number 8 wire at its center, is 1.5 meters (4.9 feet) wide by 0.5 meters (1.6 feet) deep. For the above current density the current carried by the earth works out to be 0.15 of one per cent. of the current carried by the wire, and the loss in the earth works out to be 6 per cent. as great as the loss in the wire. Since the conductor resistance of the lower network is 0.0017 ohms, the *earth resistance from the choking effect* at a frequency of 30,000 cycles is 0.0001 ohms, or it is 6 per cent. as great as the radiation resistance. At 120,000 cycles the current flowing in the earth will be four times as great as at 30,000 cycles, or it will be 0.6 of one per cent. of the current

flowing in the wire. The equivalent resistance will be 16 times as large, or 0.0016 ohms. This resistance is 6 per cent. as great as the radiation resistance at 120,000 cycles. It is interesting to note that the higher the conductivity of the earth (within the limits encountered in practice), the greater will be the loss now under discussion. For example, if the resistivity of the earth is 1,000 ohm-cms., the loss and the resistances will be ten times as great as the values estimated above for a resistivity of 10,000 ohm-cms.

TABLE III

CURRENT DENSITY IN THE EARTH IN TERMS OF THE DENSITY IN THE EARTH AT THE SURFACE OF THE WIRE

Distance from center of wire		Current density
r	0.163 cm.	D
$2\ r$	0.326 cm.	4.5 D
$3\ r$	0.489 cm.	6.6 D
$4\ r$	0.652 cm.	8.2 D
$5\ r$	0.815 cm.	9.4 D
$6\ r$	0.98 cm.	10.3 D
$8\ r$	1.30 cm.	11.9 D
$10\ r$	1.63 cm.	13.1 D
$20\ r$	3.26 cm.	16.8 D
$40\ r$	6.72 cm.	20.6 D
$60\ r$	9.8 cm.	22.7 D
$100\ r$	16.3 cm.	25.5 D
$200\ r$	32.6 cm.	29. D
$400\ r$	67.2 cm.	33. D
$600\ r$	98. cm.	35. D

r represents the radius of the wire

D represents the current density in the earth at the surface of the wire

Extra-peripheral earth resistance. An estimate of the magnitude of this resistance for the 10-meter (33-foot) Darien-equivalent antenna at a frequency of 30,000 cycles may be arrived at in the following manner:

1. Let the lower network, which may either be buried or mounted above the surface of the earth, be assumed to extend 30 meters (98 feet) beyond the upper network, and to be thoroly grounded at frequent intervals around its circular periphery.

2. Imagine the surface of the earth beyond the grounded periphery to be divided into narrow circular zones for a distance of a quarter wave length (2,500 meters) (8,200 feet) beyond the periphery. Let the steady state surface density of charge over each of these zones corresponding to a given uniform surface density of charge q upon the upper network be calculated. This calculation may be made by introducing the image with reference to the earth's surface of the charge upon the upper network, and then by an arithmetical integration determining the electric intensity set up at the mid point of each zone by the two distributed charges. The error in the final result occasioned by the assumption of uniform surface density on the upper network is very slight in comparison with the errors which may be due to the assumption of steady state conditions.

3. Let the assumption be made that when the antenna is oscillating the current which flows radially inward across any zone toward the periphery of the grounded network is the current necessary to supply the computed surface density of charge beyond the zone in question.

4. Let the current crossing each zone be so computed; and let the $I^2 R$ loss due to the flow of current across each zone be then computed on the basis that these currents flow in a surface layer or skin of the earth whose depth is taken as

$$\frac{5033}{\sqrt{\gamma f}} \text{ centimeters.}$$

γ represents the conductivity of the earth.

5. Let the power expenditure in all the zones be summed up, and a resistance be then computed which multiplied by the square of the antenna current will give an $I^2 R$ product equal to this power expenditure.

In the manner thus outlined, I have obtained 0.0006 ohms as the estimated equivalent resistance of the losses beyond the periphery of the grounded network at a frequency of 30,000 cycles. It will be noted that this treatment is far from rigorous. The problem is not susceptible of rigorous treatment, and it is extremely difficult to visualize the state of affairs in the vicinity of an *actual radiating antenna*. In the judgment of the writer, the extra-peripheral earth resistance will not exceed the value estimated above by more than 100 per cent., and may possibly be somewhat less than the estimate. The extra-peripheral resistance estimated above is 38 per cent. as great as the radiation resistance. At 120,000 cycles the extra-peripheral resistance may be put at something less than twice the above value—twice

since the skin thickness of the earth at 120,000 cycles is only one-half as great as at 30,000 cycles. A resistance of 0.0012 ohm at 120,000 cycles is an extra-peripheral resistance of only 5 per cent. of the radiation resistance at that frequency.

9. Losses in the Grass

Of greater magnitude than the power losses in the earth is the power loss which may be occasioned by conduction currents flowing in grass or vegetation growing under the antenna. This loss cannot be experimentally determined, but its order of magnitude may be estimated for the case in which the antenna is mounted over a meadow or lawn. The conditions in such a case are crudely illustrated in Figure 11. Very little of the displacement terminates upon the surface of the earth, but the lines of displacement from the upper network terminate upon the upper portions of the blades of grass in the manner illustrated in the figure. Conduction currents flow from these upper portions thro the blades of grass to the earth. The loss under consideration is the I^2R loss caused by these conduction currents.

As a justification of these statements, let us compare the condensive reactance and the ohmic resistance from the earth to a plane AB, Figure 11, touching the longer blades of grass. Consider a square meter (10.7 square feet) of the lawn under an antenna. The grass may be assumed to be 5 cm. (2 inches) deep. The condensive reactance from the surface of the earth to the plane AB one meter square and 5 cm. distant at a frequency of 30,000 cycles is 30,000 ohms. For the purpose of this calculation it may be assumed that at intervals of two centimeters (0.8 inch) a blade of grass extends about one centimeter (0.4 inch) above the blades of the next shorter group of blades, as illustrated in the figure. Ordinary lawn grass blades, 0.3 cm. (0.12 inch) wide by 0.018 cm. (0.007 inch) thick, have a resistance of about 1,000,000 ohms per cm. (0.4 inch) of length. If all the conduction current were carried by the longer blades, of which there are 2,500 per sq. m., (10.7 square feet), the ohmic resistance per square meter would be 2,000 ohms. Since the condensive reactance is 15 times as great as this ohmic resistance, this justifies the statement that the lines of displacement terminate mainly upon the upper portions of the blades of grass, and not upon the earth's surface.

The resistance of 2,000 ohms per square meter has been arrived at upon the assumption that only the longer blades of grass carry the conduction current. Of course some of the lines of

displacement terminate on the shorter blades, and the grass instead of standing vertically upright, as in the illustration, is greatly matted. Lines of displacement spring from blade to blade, and near the surface of the earth many more blades take part in the conduction of current. Under these conditions, any estimate of the resistance may be considerably in error. A resistance of 2,000 ohms per sq. m. is probably an upper limit. I would estimate the resistance to be about one-third of this or 700 ohms per sq. m. As the total area under the 10-meter (33 feet) Darien-equivalent antenna is 165,000 meters (1,760,000 square feet), the equivalent resistance of the grass losses is estimated to be of the order of 0.004 ohm. This resistance is equal to 2.5 times the radiation resistance. That is, at 30,000 cycles 2.5 kw. will be expended in heating the grass per kilowatt radiated.

At 120,000 cycles the condensive reactance per square meter (10.7 square feet) is one-quarter of 30,000 ohms, or 7,500 ohms. This means that displacement will take place more readily from blade to blade and the shorter blades will take a greater part in conveying the conduction current in the lower depths of the grass. The grass resistance will, therefore, be somewhat lower than at 30,000 cycles, possibly about 0.003 ohms. This is of the order of 12 per cent. of the radiation resistance. The grass loss may be eliminated by mounting the lower network at an elevation of 1 to 3 meters (3.3 to 9.9 feet) above the ground.

10. Pole or Steel Structure Losses

If wooden poles are used to support the antenna, the conduction currents which will flow in the poles may occasion a loss as high as 10 kw. per pole under a sustained voltage of 100 r.m.s. k.v. It will be absolutely necessary to cover wooden poles with galvanized iron or copper netting. This will make the loss inappreciable.

Steel supporting structures will have a resistance of the order of 0.1 ohm. Any conduction currents the structures may carry will cause an inappreciable loss.

11. Insulator Losses—Dielectric Hysteresis and Wet Weather Leakage

The writer is not in possession of data for estimating the magnitude of these losses. They will necessarily be greater in the case of the low antenna than in the case of the high, because it is necessary to support the low antenna at a greater number of points.

12. Conclusions

From Table II, showing the computed resistances of the 10-meter (33-foot) Darien-equivalent antenna, it may be seen that at the higher frequency of 120,000 cycles the wasteful resistance of the antenna (exclusive of the equivalent resistance of the insulators, the ionization losses and the tuning inductance) is only 53 per cent. of its radiation resistance. At the frequency of 30,000 cycles, the wasteful resistance is 5.5 times the radiation resistance. This means that as an absorber of electromagnetic energy from sustained waves, the efficiency of the low antenna (neglecting for the present the losses noted above) is 65 per cent. at 120,000 cycles and 15 per cent. at 30,000 cycles.

It should be noted, however, that if the lower network is not buried but is mounted several meters above the earth, the grass losses and all the earth losses save the "extra-peripheral earth loss" are eliminated. Under these conditions the efficiency of the 10-meter Darien-equivalent antenna at 120,000 and 30.000 cycles becomes 76 per cent. and 28 per cent. respectively.

The resistances of the existing elevated high power radio antenna have not been worked out in the detailed manner exhibited in Table II, but the best stations are reported to have wasteful resistances not less than 0.6 ohms. Assuming the wasteful resistance of the Darien antenna to be 0.6 ohms. the efficiency of the Darien antenna as an absorber of energy from sustained waves is 90 per cent. at 120,000 cycles and 36 per cent. at 30,000 cycles.

These efficiencies have been compiled in Table IV.

TABLE IV

COMPUTED EFFICIENCIES OF HIGH AND LOW ANTENNA (EXCLUSIVE OF INSULATOR, IONIZATION, AND TUNING INDUCTANCE LOSSES)

Frequency	120,000	30,000
Darien-equivalent antenna		
with lower network buried	65 per cent	15 per cent.
with lower network unburied	76 per cent	28 per cent.
Darien antenna	90 per cent	36 per cent.

The comparison of efficiencies is somewhat unfavorable to the low antenna at the lower frequencies. The relative merit

of high and low antennas is determined not alone by the radiating and absorbing efficiencies of the two types, but also by considerations having to do with the power generating devices and the selective reception of signals under the conditions of commercial operation. For example: the function of a receiving antenna is not simply "to deliver to a detecting device energy abstracted from impinging waves." Its function is "to *selectively* abstract energy from impinging waves and to deliver it to the detecting device." That is to say, if antenna H is able to deliver to the detector twice as much energy from the *correspondant* station as antenna L, it does not follow that antenna H is superior to antenna L, since antenna H may at the same time deliver to its detector—not twice—but ten times as much energy from "*strays*" or *interferent stations* as does antenna L.

The ultimate comparison of the relative merits of two antennae for receiving purposes *under the conditions to be met in commercial operation* must be a comparison of the amounts of energy delivered to the detectors from the desired correspondant when the strengths of the interfering signals or noises have been reduced to the same intensity in the two cases. As previously stated, it is the purpose to discuss these other aspects of feasibility in subsequent papers.

SUMMARY: This paper supplements a previous publication entitled "High versus Low Antennas in Radio Telegraphy," in which it is shown that if the antenna of a radio telegraph station consists of an extended horizontal network of wires mounted above a highly conducting plane, and if the mean radius of the capacity area is two or more times as great as any height above the plane at which it is feasible to mount the network, then the rate of radiation from the antenna at a given voltage and frequency, and the rate at which the antenna will ultimately be able to abstract energy from impinging (sustained) waves, are both independent of the mounting height.

The conclusions in the previous paper are the result of a mathematical analysis for the hypothetical case in which the antenna is mounted over a highly conducting plane. The present paper deals mainly with those wasteful antenna and earth resistances which are common to the use of the low antenna both in sending and receiving. The distinction between the "low" antenna and the "ground" antenna is pointed out. The electrical constants of an antenna having the same radiation figure of merit as the Darien (Canal Zone) antenna, but mounted at an elevation of only 10 meters (33 feet), are contrasted with those of the Darien antenna. The wasteful resistances of this 10-meter Darien-equivalent are then computed. Of these losses, that in the vegetation growing under the antenna is found to be the most serious. The efficiency of the 10-meter Darien-equivalent antenna, with its lower capacity area not buried but mounted above the ground, is computed to be of the order of 76 per cent. for frequencies of 120,000 cycles per sec. and 28 per cent. for 30,000 cycles. These efficiencies are slightly lower than those reported for existing high-power elevated antenna.

APPENDIX A

Electromagnetic Radiation

A compilation* of the expressions applying to stations in which the antenna is a horizontal network the radius of which is large (ten times) in comparison with its mounting height, but small (one-tenth) in comparison with the wave length at the operating frequency. The stations are assumed to be on an infinitely extended plane of infinite conductivity.

A_o, A_1 represent the areas in sq. cm. of the sending and receiving networks

h_o, h_1 represent the heights in cm. of the sending and receiving networks

Q represents the maximum charge (in coulombs) on the upper network at the sending station

r represents the distance in centimeters to the receiving station

f represents the frequency

p represents the permittivity of air $= 8.84 \times 10^{-14}$ coulomb-volt-cm.

s represents the velocity of propagation $= 3.0 \times 10^{10}$ cm. per sec.

Note: All values of current, voltage, and electric and magnetic intensity are r. m. s. values.

Sending station values

Current (amperes) $I_s = \sqrt{2}\,\pi f Q$ . . . . (1)

Voltage to earth (volts) $E_s = \dfrac{Q\,h_o}{\sqrt{2}\,p\,A_o}$ . . . . (2)

Rate of radiation (watts-hemisphere) $P = 320\,\pi^4 Q^2\,h_o^2 f^4 s^{-2}$ . . . . (3)

Rate of radiation (watts-hemisphere) $= 160\,\pi^2\,h_o^2 f^2\,s^{-2} I_s^2$ . . . . (4)

Rate of radiation (watts-hemisphere) $= \dfrac{4}{90}\pi^2\,A_o^2 f^4\,s^{-4} E_s^2$ . . . . (5)

*Compiled from Bulletin 810, University of Wisconsin, "High Versus Low Antennas in Radio Telegraphy."

Radiation resistance (hemisphere-ohms)	$R_r = 160\,\pi^2 f^2 h_o^2 s^{-2}$ . . . . (6)
Critical resistance (ohms)	$R_c = 2\sqrt{\frac{L}{C}} = \frac{1}{\pi f C} = \frac{h_o}{\pi f\, p A_o}$ . . . . (7)
Logarithmic decrement (due to radiation)	$\delta = 2\pi \frac{R_r}{R_c} = \frac{8}{3}\pi^3 f^3 h_o A_o s^{-3}$ . . . . . (8)
Time constant (in periods)	$T_c = \frac{3\,s^3}{8\,\pi^3 f^3 h_o A_o}$ . . . . (9)
Resistance ratio R_c/R_r	$R_c/R_r = 2\pi T_c = \frac{3\,s^3}{4\pi^2 f^3 h_o A_o}$ . . . . (10)
At distant points	
Electric intensity (volts per cm.)	$F = 120\sqrt{2}\,\pi^2 Q\, h_o f^2 (s\,r)^{-1} \sin\theta$ (11)
Magnetic intensity (amp-turns per cm.)	$H = \sqrt{2}\,\pi Q\, h_o f^2 (s\,r)^{-1} \sin\theta$ . . (12)
Power flow—average value (watts per sq. cm.)	$P_1 = 240\,\pi^3 Q^2 h_o^2 f^4 (s\,r)^{-2} \sin^2\theta$ (13)
Power flow—average value	$= 120\,\pi\, h_o^2 f^2 I_s^2 (s\,r)^{-2} \sin^2\theta$ (14)
Power flow—average value	$= \frac{\pi A_o^2 f^4 E_s^2}{30\, s^4 r^2} \sin^2\theta$ . . . . (15)
Receiving station values	
Induced voltage (volts)	$E = \frac{120\sqrt{2}\,\pi^2 f^2 h_o h_1 Q}{s\,r} = \frac{120\pi f h_o h_1 I_s}{s\,r} = \frac{2\pi f^2 A_o h_1 E_s}{s^2 r}$. (16)
Radiation resistance	$R_r = \frac{160\,\pi^2 f^2 h_1^2}{s^2}$ . . . . (6)

Final values when receiving sustained waves with radiation resistance alone in the antenna:

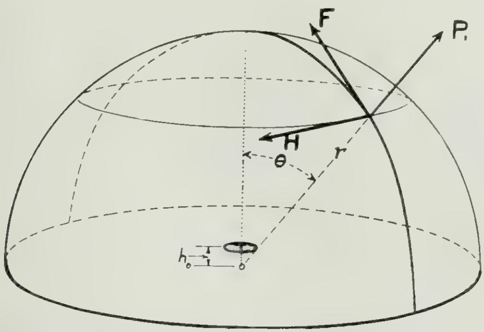
Current (amperes)	$I = \frac{3\,s\,h_o Q}{2\sqrt{2}\,r h_1} = \frac{3\,s\,h_o I_s}{4\pi f r h_1} = \frac{A_o E_s}{80\,\pi\, r h_1}$ . . . (17)
Condenser voltage	$E = \frac{90\,s^2 h_o Q}{\sqrt{2}\,r f A_1} = \frac{45\,s^2 h_o I_s}{\pi f^2 r A_1} = \frac{3\,s\,A_o E_s}{4\,\pi f r A_1}$ . . . (18)
Power re-radiated (watts-hemisphere)	$P = \frac{180\,\pi^2 f^2 h_o^2 Q^2}{r^2} = \frac{90\,h_o^2 I_s^2}{r^2} = \frac{f^2 A_o^2 E_s^2}{40\, r^2 s^2}$ . . . (19)

$$\text{Time constant (in periods)} \quad T_c = \frac{3\,s^3}{8\,\pi^3 f^3 A_1 h_1} \quad \ldots\ldots\ldots\ldots (20)$$

$$\frac{\text{Final condenser voltage}}{\text{Induced voltage}} = \pi\,T_c = \frac{3\,s^3}{8\,\pi^2 f^3 A_1 h_1} \quad \ldots\ldots\ldots (21)$$

In order to abstract and utilize the maximum power possible from the impinging electromagnetic waves, the detector resistance R_d should be made equal to the sum of the radiation resistance plus the wasteful resistances of the antenna circuit. If the wasteful resistances are negligibly small in comparison with the radiation resistance, then for maximum power utilization the detector resistance should be made equal to the radiation resistance. Under these conditions—

The final value of the current equals ½ of the above value.
The final value of the condenser voltage equals ½ of the above value.
The final value of the power utilized equals ¼ of the above value.
The final value of the power re-radiated equals ¼ of the above value.



Relation Between Electric and Magnetic Intensities

DISCUSSION

L. W. Austin (by letter): The success of Professor Bennett's plan seems to depend entirely on his ability to keep down the wasteful resistance of his antenna. I am afraid that this will prove more difficult than his calculations indicate. The minimum resistance of the best modern high power antennas, exclusive of the inductance and radiation resistances, lies between one and two-and-one-half ohms, and it seems improbable that even the most elaborate counterpoise system can ever reduce this much below one-half ohm. The problem of a further reduction to less than one-hundreth ohm will certainly be a difficult one.

L. L. Israel: If I were an engineer designing a transmitting station with the low antenna described to-night, I would be somewhat troubled by the design of the loading inductances for carrying from 1,000 to 10,000 amperes, in spite of the fact that the inductance would be somewhat less than is usual with high antennas. The difficulties in the way of placing coil conductors so that they are out of the intense magnetic fields are considerable.

The next difficulty that comes to mind is to get rid of the losses due to the magnetic fields in the antenna itself. Lowering the antenna not only increases the loss in the static field but it also increases the loss in the magnetic field. Experience with high power stations has shown that the loss in poor conductors cut by intense magnetic fields rises to enormous amounts. In one high power station the loss in concrete walls cut by the magnetic field of the loading coil was high enough to make the wall burning hot. Undoubtedly the ground in the region of the center of the low antenna is cut by strong magnetic fields with consequent high eddy current loss. It would be interesting to have a resistance figure computed for this eddy loss in the ground. I suspect that it would considerably increase the totals given by Dr. Bennett.

I would be glad to offer an explanation for an observation mentioned; namely that altering the length of the counterpoise alters antenna ammeter reading at constant output, as I once had the same trouble in similar experiments.

When the counterpoise length is changed the position of the potential node changes too, so that even tho the antenna input and resistance are the same the current indication varies. By connecting the ammeter each time at the potential node,

the antenna current changes are small or at least more in conformity with what one would expect from the other conditions of the experiment.

There is another interesting point in this paper worthy of mention, namely, the variation of radiation resistance with the height of the antenna. Dr. Bennett's research is based on the assumption that the radiation resistance varies as the square of the height of the antenna. This may be true for antennas which have a small flat top area in comparison with their height, but I doubt whether it is true for antennas which have a very large flat top area. If we apply the square law of radiation resistance to ground antennas, we are hard put to it to see why the ground antennas work at all. I feel that a paper supplementing this one and taking up this aspect of the matter would be very helpful. There are a great many engineers in the field who become confused by the new facts which are being discovered in the use of ground antennas.

At one time we thought that a ground antenna received because of sloping wave fronts, but the efficient working of ground antenna in sea water leaves us "high and dry." And how can we explain antennas working thousands of feet below the earth's surface?

These low antennas must change the energy distribution or form of the wave so that more of the wave energy is absorbed. Such variations of radiation resistance from the square law contribute greatly to the difficulty of the low antenna problem. They tend to increase the radiation resistance of the low antenna, minimizing the importance of the losses, and bringing the whole problem into the class that can only be satisfactorily solved by physical trial. Dr. Bennett's deductions will certainly form a most valuable guide thereto.

Alexander E. Reoch: About one year ago, I read extracts from a "Bulletin" by Dr. Bennett, entitled "High Versus Low Antennas for Radio Telegraphy," and having considerable interest in the design of large antennas, I obtained a copy of the bulletin. The first part of the bulletin arrives at conclusions which are generally accepted; but, with respect to the second part, I came to the conclusion that the author had not given much consideration to the wasteful resistance in antennas of present-day type. A complete understanding of the subject would be much easier for the practical engineer if the energy in the antenna was given in terms of the current rather than

the voltage. The antenna current in antennas of different capacity, maintained at the same voltage, varies considerably; and if the total resistance of the antenna remains about the same the power required to maintain the same voltage will likewise vary. The total resistance of the antenna consists of useful radiation resistance and wasteful resistance. The radiation resistance varies with the height but the wasteful resistance does not necessarily so vary.

In this bulletin is cited a case of an antenna 10 meters high with a radiation resistance of 0.011 ohm and it is suggested that the total resistance of this antenna would be 0.12 ohm. In which case the radiation efficiency would be 9 per cent.

It will be generally acknowledged, however, that in an antenna of these dimensions, with a natural wave length of the order of 4,000 meters, the total resistance cannot be reduced at the best to less than 1 ohm. In this case then, the radiation efficiency would become 1.1 per cent. and 98.8 per cent. of the energy supplied to the antenna would be utilized in overcoming wasteful resistance.

If the wasteful resistance, instead of being the controlling factor, was small compared with the radiation resistance, then the argument of this first paper would be well based and antennas of this type would call for practical consideration.

It is evident that the assumption that the total resistance of a large antenna can be reduced to an exceedingly small figure has been carried into the present paper.

That this is not the case is fully borne out by published information concerning the Marconi stations in Ireland, at Glace Bay, and New Brunswick, the arc stations at Arlington, San Francisco, and Darien, and the Sayville and Tuckerton stations. From this information, while no exact measurements are given, it can be deduced that the minimum total resistance is about 2 ohms.

Assuming the total resistance of the Darien antenna to be 2 ohms, and the radiation resistance to be 0.3 ohm, there is a wasteful resistance of 1.7 ohms to be accounted for.

Lieutenant Crenshaw in a description of the Darien station described how an antenna inductance situated near a reinforced concrete wall caused the wall to become exceedingly hot (as mentioned by Mr. Israel). Large stations have been built with umbrella antennas with a central tower of lattice work steel and the main lead to the antenna has been run parallel to this tower at a distance of thirty feet from it to a height of 500 ft. (150 m.).

Eliminating such evidently fruitful sources of antenna wasteful resistance and taking every care to reduce the wasteful resistance to the lowest possible figure, as no doubt has been done in the stations mentioned above, the wasteful resistances still remain exceedingly high in comparison to the radiation resistances of the antennas of these stations.

In the instance mentioned above in the Darien antenna there is a resistance of 1.7 ohms which remains to be eliminated or reduced. The sum of the resistances of all the elements which Dr. Bennett has considered is negligibly small in comparison with this figure, and these elements could, therefore, be passed over for the time being and other elements which have not as yet been attacked could be profitably investigated.

As the matter stands at present, however, in stations of the type considered, the higher the antenna the greater its radiation efficiency, and a compromise must be made between efficiency and cost, the price of steel being the deciding factor, the cost of copper for the construction of the antenna proper being of minor amount.

There are two particularly remarkable points raised in dealing with the details of the wasteful resistance. (1) The use of a counterpoise ground instead of buried wires does not materially reduce the ground resistance as suggested, because currents corresponding to those which flow in the counterpoise network must flow in the ground beneath the counterpoise. No great reduction in antenna resistance has been obtained in practice by the use of a counterpoise net work. (2) That the section of the ground resistance due to choking effect is increased when the ground material is of low resistance. That this has not been noticeable in practice may also be due to the fact that the maximum value of this section of the ground resistance is negligibly small.

In present-day high power stations, the efficiency of the antenna and ground systems combined is the weakest link in the chain between prime mover and radiation. Dr. Bennett's work as published in his original bulletin and the present paper, is undoubtedly of the greatest value; but in my opinion, this value lies in the unflinching attack which has been made on the inefficiency of the radiator rather than in the suggestion of the feasibility of the low antenna. An explanation of the methods which Dr. Bennett has used in his calculations would be of great value to engineers familiar with antenna design. Many engineers who are at present unable to approach intelligently these

problems would be greatly assisted thereby, and useful results would surely ensue.

Edward Bennett: I agree with Professor Hazeltine that it is largely a matter of judgment as to the distance from the periphery at which to stop in computing the extra-peripheral resistance. The question is raised as to why the zone for which the extra-peripheral loss has been computed should be arbitrarily taken as a quarter wave length in width. The practical justification for not taking the zone any wider than one-quarter wave length is that any reasonable increase in the width of the zone—for example, an increase to a width of one wave length—would not materially increase the computed resistance. The reason for this is that the surface density of charge decreases rapidly as the zones are taken farther and farther from the periphery, and at the same time the cross sectional area of the earth thru which the current flows increases, so that the loss in the outer zones drops off very rapidly indeed. The computations indicate, for example, that the current flowing beyond the grounded periphery is only six per cent. of the current to be measured in the tail of the antenna, while the current crossing a circle at a distance of one-eighth wave length from the periphery is only one-half of one per cent. of the current in the tail. The principal justification for taking the zone as wide as a quarter wave length is that by so doing an upper limit is established for a resistance of hitherto unknown magnitude. Since the computation indicates that the extra-peripheral resistance is small in comparison with the radiation resistance, the lower limit is not of much interest.

Dr. Goldsmith points out that it is not satisfactory to work large and high power stations for trans-oceanic transmission at twenty words per minute, and states that the trend is toward speeds of 100 to 1,000 words per minute. From this he concludes that the trend is toward the continued use of the high antenna, since the long time-constant of the low antenna would preclude its use at the higher speeds. High speed in the sense of a high number of words per station means high power, expensive stations, because high speed stations cannot be made very selective and the strays must be swamped by the use of high power. It does not follow, however, that high speed in the sense of a high number of words per station is synonymous with high speed in the sense of a high number of words per dollar of expenditure. In my estimation the high number of words

per dollar may possibly mean the use of several slow speed, low power, inexpensive, highly selective stations, rather than one large high power station.

Mr. Ballantine has questioned the propriety of the statement "Since the radiation resistance of an antenna of given form and dimensions is inherent in the form and dimensions, it follows that the efficiency of a given antenna as a receiver is 100 per cent. if the equivalent resistance of the detector is made equal to the radiation resistance, and if the wasteful resistance is made negligibly small in comparison with the radiation resistance." I welcome this opportunity to take issue with the practice which assigns to an antenna under the conditions described above, an efficiency of only 50 per cent. The statement that the efficiency is only 50 per cent. signifies to ninety-nine engineers out of a hundred that there may be a possibility of doubling the amount of power which the 50 per cent. antenna is abstracting from the impinging waves. The statement is, therefore, grossly misleading, because by no conceivable method can the antenna be made to abstract and deliver more power to the detector.

Mr. Israel has referred to the large eddy current losses experienced in a concrete wall in the neighborhood of the loading coil of the Darien station, and has expressed the fear that the larger current of the low antenna may lead to increased difficulty in reducing such losses. In this connection it should be recalled that the magnitude of the eddy current losses depends upon the intensity of the magnetic field. Now the intensity of the magnetic field in the vicinity of the loading coil is proportional to the product of current times the number of turns of wire. Since the low antenna has a large capacity and requires a small loading inductance or a small number of turns in the loading coil, it follows that notwithstanding the larger current, the intensity of the nagnetic field in the vicinity of the loading coil of the low antenna may not be greatly different from the intensity in the vicinity of the high loading coil.

To several of the speakers two things have seemed questionable or incredible. The first is that the wasteful resistance can be of the order of only 0.01 ohm for the low antenna as contrasted with 0.06 ohm for the high antenna of the Darien type. The second questionable point is that a low antenna in which the voltage generated by the impinging waves is only one-tenth or one-twentieth as great as in the high antenna can abstract the same power from the waves as the high antenna. As these

views have been privately expressed again and again it may be well to consider them at length.

In considering the wasteful resistances we ought first to disabuse our minds of the notion that the wasteful resistances are resistances which may be measured with a Wheatstone bridge. If with this impression, we look at two antennas, a high and a low, no reason can be seen for such a great difference in the magnitude of the two resistances. The wasteful resistances are computed resistances found by dividing the power losses by the square of the current to be measured at the base of the antenna tail. If then, for the sake of argument, we assume that the power loss is the same in the ten-meter antenna as in the Darien antenna when operated at the same voltage and at 30,000 cycles, it follows that since the low antenna current is 14.6 times as great as the high antenna current the computed wasteful resistance of the high antenna will be 14.6^2 or 215 times as great as that of the low. It may assist to refer again to the extra-peripheral resistance, the estimated value of which has been set at 0.0006 ohm. Contrast this with the resistance which would be obtained by an ammeter and a voltmeter for a surface layer of earth of depth equal to the skin thickness and extending from the grounded periphery of the lower net work to a second circle of pipes surrounding the net work at a distance of a quarter wave length. This measured resistance would be of the order of 1 ohm, but in reducing this resistance to its equivalent in the tail of the antenna, consideration must be taken of the fact that only 6 per cent. of the antenna current flows beyond the periphery of the antenna and only one-half of one per cent. flows beyond the one eighth wave length point.

To consider the effect of the height and dimensions of a flat top antenna upon the power which the antenna can abstract from sustained waves and deliver to a resistor detector, we assume that the equivalent detector resistance R_d, is equal to the sum of the radiation resistance R_r plus the wasteful resistance R_w and that the total resistance R_t is

$$R_t\,(= R_d + R_w + R_r) = k\,R_r.$$

k may be expected to lie between 3 and 8.

If an antenna has an extended capacity area at a height h and is resonant to the sustained impinging waves, the average power P, delivered to the detector after the current attains the steady-state value is

$$P = \tfrac{1}{2}\,\frac{E^2_{r.m.s.}}{k\,R_r} = \tfrac{1}{2}\,\frac{(hF_{r.m.s.})^2}{k\,R_r} = \frac{2\,(hF_m)^2}{\pi^2\,k\,R_r}$$

in which F_m represents the peak value of the electric intensity of the impinging waves.

Substituting in the above equation, the expression for the radiation resistance of an extended flat top antenna namely,

$$R_r = \frac{160\,\pi^2 h^2}{\lambda^2}$$

the expression for the power becomes

$$P = \frac{\lambda^2 F_m{}^2}{80\,k\,\pi^4}$$

In other words, the power abstracted from the impinging waves and delivered to the detector is independent of the dimensions of the antenna, except in so far as these dimensions may thru the wasteful resistance affect the value of k.

Thruout this discussion, it is assumed that the greatest dimension of the antenna is a small fractional part of the wave length.

It may be of interest to put the above expression for the power in a form fraught with greater physical significance

$$P = \frac{\lambda^2 F_m{}^2}{80\,k\,\pi^4} = \frac{\frac{1}{2}\,pF_m{}^2\,\lambda^2}{\frac{1}{2}\,p\,80\,k\,\pi^4} = \frac{3}{k\,\pi^3}\,s\,\lambda^2\left(\tfrac{1}{2}\,pF_m{}^2\right)$$

in which p represents the permittivity of air, namely $\dfrac{1}{36\,\pi\,10^{11}}$ coulombs per sq. cm. per volt per cm.

Now $\frac{1}{2}\,pF_m{}^2$ represents the energy in the dielectric per cubic cm. when the peak of the electro-magnetic wave impinges upon the antenna, s represents the velocity of propagation, and λ^2 represents the area of a square the side of which is equal to the wave length—"a wave length square." Since the electric intensity F is a sine function of time, the average value of $\frac{1}{2}\,pF^2$ taken over the wave as it streams past the receiving station is $\frac{1}{2}$ of $(\frac{1}{2}\,pF^2)$ and since the electro-kinetic energy $\frac{1}{2}\,m\,H^2$, is equal to the electro-potential energy $\frac{1}{2}\,pF^2$, it follows that $s\,\lambda^2\,(\frac{1}{2}\,pF_m{}^2)$ represents the energy which streams past the receiving station per second across an area equal to the wave-length square. That is to say, the greatest average power which can be delivered to a detector by any antenna from impinging sustained waves equals $\dfrac{3}{k\,\pi^3}$ times the power flowing across a wave length square at the receiving station.

THE AMPLIFICATION OBTAINABLE BY THE HETERODYNE METHOD OF RECEPTION*

By

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(Professor at the City and Guilds (Engineering) College, London)

A certain difference of opinion appears to exist as to the maximum amplification obtainable by the simple heterodyne method of receiving undamped signals. In a paper read before this Institute in 1915, Dr. B. Liebowitz[1] maintained that the amplification of audio power could not exceed four times, except in so far as indirect influences come into play, and cause an increase in efficiency of the receiving apparatus. This point of view was strongly opposed by Mr. Louis Cohen; both arguments were supported by mathematical proofs and Mr. Cohen maintained that his was further supported by experimentally observed facts.

In a recent paper by Mr. Armstrong[2], it is stated that the results obtained with the auto-heterodyne appear to support the view put forward by Dr. Liebowitz.

The present communication is intended to throw some light on this question.

Before considering the heterodyne, it is important that one should be quite clear with regard to the assumptions made as to the rectifying action of the detector employed. Two very simple assumptions are possible; one may assume that the detector acts as a thermo-junction, heated by the radio-frequency current passing thru it, and, owing to a combination of electrothermal phenomena, generating an electromotive force proportional to the square of the radio-frequency current; on the other hand, one may assume that the rectifier has a constant resistance in one direction, and a higher, preferably infinite, resistance in the other direction.

The former assumption is in agreement with the experimental

* Received by the Editor, March 25, 1918.

[1] "Proceedings of The Institute of Radio Engineers," Volume 3, page 185.

[2] "Proceedings of The Institute of Radio Engineers," Volume 5, page 145, 1917.

observation that, when using a crystal detector, the audibility factor, and consequently the audio current, is proportional to the square of the received radio frequency current. On the latter assumption, the audibility factor would be directly proportional to the radio frequency current. Since the former assumption implies that the power supplied by the detector to the telephone receiver increases as the fourth power of the received radio current, i. e., that the output of the detector is proportional to the square of its input, it is evidently not of unlimited application. In view, however, of its known agreement with experimental observation over a very wide range, it will be assumed that the ordinary contact detector may be considered from this point of view.

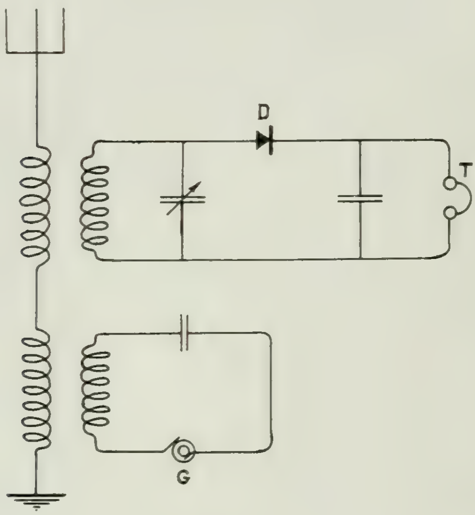

FIGURE 1

With the circuit arrangement shown in Figure 1, let the radio frequency current thru the detector D due to the received wave be $i_1 = I_1 \sin \omega_1 t$, and let that due to the local high frequency generator G be $i_2 = I_2 \sin \omega_2 t$, then the resultant current passing thru the detector is $i = i_1 + i_2 = I_1 \sin \omega_1 t + I_2 \sin \omega_2 t$. This resultant current is represented in Figure 2 (a) on the assumption that $I_2 = 2 I_1$. When the two currents come into phase, the resultant amplitude is $I_1 + I_2$, whereas, when they come into op-

position it is $I_2 - I_1$. Now the rate (averaged over a radio period) at which heat is developed at the detector contact is equal to the effective resistance of the latter multiplied by a half of the square of the amplitude of the radio current. Hence the telephone current will vary between $0.5\,k\,(I_2 + I_1)^2$ and $0.5\,k(I_2 - I_1)^2$ where k is a constant depending on the detector and on the impedance of the telephone receiver. This telephone current is shown in Figure 2 (b) in which the maximum and minimum ordinates are as 9 to 1. This is equivalent to a steady current of $0.5\,k\,(I_1^2 + I_2^2)$ upon which is superposed the real sine-wave audio-current with an amplitude of

$$0.25\,k\left[(I_2 + I_1)^2 - (I_2 - I_1)^2\right] = k\,I_1\,I_2.$$

Hence the audio current is directly proportional both to I_1 and I_2.

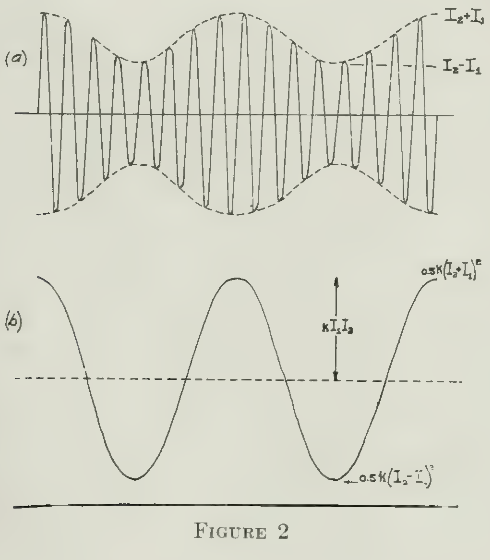

FIGURE 2

The problem may be looked at in a slightly different way, more suggestive of the method of Dr. Liebowitz. The rate at which energy is supplied to the detector is proportional to i^2, that is, to

$$(I_1 \sin \omega_1 t + I_2 \sin \omega_2 t)^2 = I_1^2 \sin^2 \omega_1 t + I_2^2 \sin^2 \omega_2 t + 2\,I_1\,I_2 \sin \omega_1 t \sin \omega_2 t.$$

Putting aside any question of the thermal capacity of the detector preventing the temperature changes following the varia-

tions of radio frequency, the electromotive force will be proportional to this expression at every moment. The effects of these electromotive forces on the telephone receiver have now to be determined. That the first two terms produce no audio current, can be shown in several ways. In Figures 3 (a) and (b), for example, the two terms are shown separately, on the left when in phase and on the right when out of phase. The resultant in

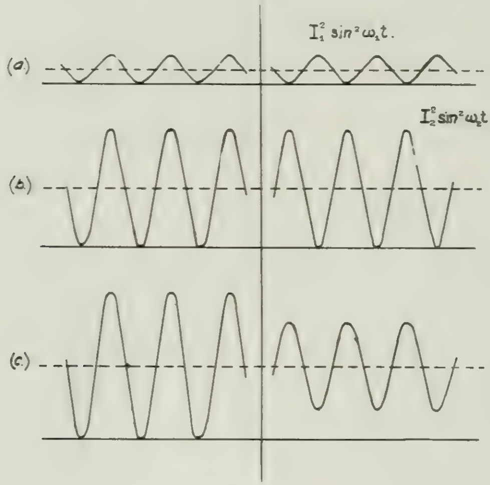

FIGURE 3

each case is shown in Figure 3 (c), and it is seen that the mean value is constant whilst the amplitude of the radio oscillation varies. These two terms, therefore, represent a steady current thru the telephone receiver, the radio oscillations passing thru the shunt condenser.

The last term is best split up into two cosine terms, thus:—

$$2\, I_1\, I_2 \sin \omega_1 t \sin \omega_2 t = I_1\, I_2 \left[\cos\ (\omega_1 - \omega_2)\ t - \cos\ (\omega_1 + \omega_2)\ t\right]$$

The latter of these two terms represents an electromotive force with a frequency equal to the sum of the two component radio frequencies, and is, therefore, without any effect on the telephone. *The remaining term* $I_1 I_2 \cos (\omega_1 - \omega_2)\, t$ *represents the electromotive force which produces the audio frequency current of amplitude* $k\, I_1\, I_2$.

If the heterodyne is not used, and the received waves are made audible by being interrupted at an audible frequency, then,

assuming equal duration for the open and closed periods, the radio frequency current thru the detector will be as shown in Figure 4 (a), where, to make the consideration more general, it is assumed that the detector carries a steady polarizing current I_o.

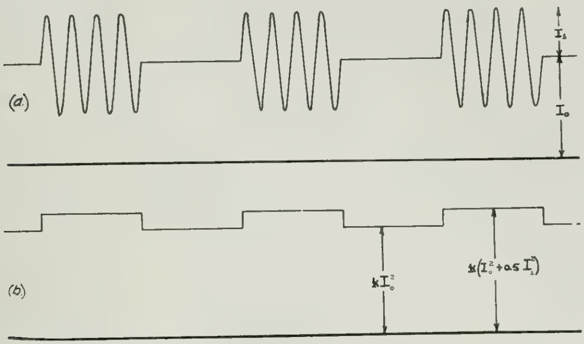

FIGURE 4

The heat production and consequent electromotive force will consist of rectangular pulses as in Figure 4 (b) and the telephone current will vary between $k\left(I_o^2+\frac{I_1^2}{2}\right)$ and $k\,I_o^2$, which is equivalent to a steady current of $k\left(I_o^2+\frac{I_1^2}{4}\right)$ upon which is superposed a rectangular alternating current with an amplitude of $\frac{k\,I_1^2}{4}$. This rectangular wave may be analysed into a number of sine waves, the fundamental of which determines the pitch of the note, whilst the higher harmonics determine its character. Altho it is difficult to determine upon what basis two sounds of different wave-forms should be compared, the simplest method and the one least open to objection is to compare the amplitudes of their fundamentals, assuming, of course, that these are of equal pitch. On this basis the amplitude of the audio current should be taken as $\frac{4}{\pi}\cdot\frac{k\,I_1^2}{4}$, and the amplification of audio current obtained by the use of the local heterodyne generator is

$$\frac{\frac{k\,I_1\,I_2}{4}}{\frac{4}{\pi}\cdot\frac{k\,I_1^2}{4}}=\pi\frac{I_2}{I_1}$$

Turning now to the other simple assumption which may be

made with regard to the operation of the detector, viz.: that it has a constant resistance to current in one direction and an infinite resistance to current in the reverse direction, it may be assumed as a close approximation that the current passing thru the detector will be simply $I_1 \sin \omega_1 t + I_2 \sin \omega_2 t$, except that all current in one direction is suppressed, as shown in Figure 5 (a).

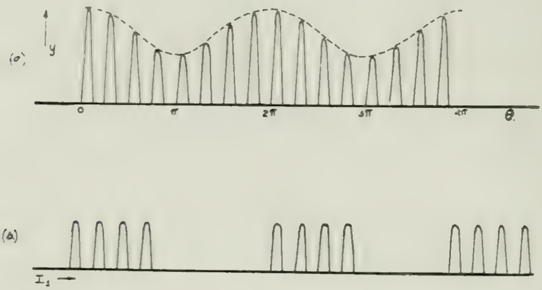

FIGURE 5

Assuming that I_2 is greater than I_1, the average value of the current over a radio period varies between $\frac{(I_1+I_2)}{\pi}$ and $\frac{(I_2-I_1)}{\pi}$; the equivalent steady current thru the telephone, and the amplitude of the fundamental audio-frequency current can be found as follows:—

Let the ordinate of the envelope shown dotted in Figure 5 (a) be y, then

$$y = \sqrt{(I_1^2 + I_2^2 + 2 I_1 I_2 \cos \theta)}$$

where θ is the instantaneous value of the phase difference between the two component currents. This may be written

$$y = \sqrt{\left[I_1^2 + I_2^2 + 2 I_1 I_2 \left(1 - 2 \sin^2 \frac{\theta}{2}\right)\right]}$$

$$= (I_1 + I_2) \sqrt{\left(1 - \frac{4 I_1 I_2}{(I_1+I_2)^2} \sin^2 \frac{\theta}{2}\right)}$$

Putting $I_2 = n I_1$

$$y = (n+1) I_1 \sqrt{\left(1 - \frac{4n}{(n+1)^2} \sin^2 \frac{\theta}{2}\right)}$$

The mean value of y over a complete period is

$$\frac{1}{\pi}\int_0^{\pi} y\,d\theta = \frac{2(n+1)}{\pi} I_1 \int_0^{\pi/2} \sqrt{\left(1-\frac{4n}{(n+1)^2}\sin^2\frac{\theta}{2}\right)}\,d\frac{\theta}{2}$$

$$= \frac{2(n+1)}{\pi} I_1 E\,(k\cdot\pi/2)$$

where E is the complete elliptic integral of the second type, and the modulus $k=\frac{2\sqrt{n}}{n+1}$. The values of E can be obtained from any table of elliptic integrals; they are given in the following table for $n=1$, 2, 3, and 4.

$n = \frac{I_2}{I_1}$	k	$\alpha = \sin^{-1}k$	E	$\frac{y_{\text{mean}}}{I_1}$	$\frac{I_{\text{mean}}}{I_1}$	$\frac{I_{\text{mean}}}{I_2}$	Amplitude of Fundamental of y	Amplitude of Fundamental Audio Current
1	1.0000	90°	1.000	1.274	0.4055	0.4055	0.849 I_1	0.27 I_1
2	0.9425	70°24′	1.118	2.137	0.68	0.34	0.968 I_1	0.308 I_1
3	0.8667	60°	1.211	3.085	0.982	0.327	0.986 I_1	0.314 I_1
4	0.8000	53°8′	1.27	4.042	1.287	0.322	0.991 I_1	0.316 I_1

Since the radio current passes for only half the total time, $I_{\text{mean}}=\frac{y_{\text{mean}}}{\pi}$. To find the amplitude of the fundamental audio current, the curve of y must be analysed into its Fourier components. This the writer has done and the results are given in the last column but one; the last column is obtained by dividing this by π.

If the heterodyne is not used, but the received waves made audible by being interrupted to give the same audio frequency, as shown in Figure 5 (b), the average current varies between $\frac{I_1}{\pi}$ and 0. The steady component and the amplitude of the rectangular alternating component are both equal to $\frac{I_1}{2\pi}$. The amplitude of the sine-wave fundamental is $\frac{I_1}{2\pi}\times\frac{4}{\pi}=\frac{2I_1}{\pi^2}$. The current amplification is obtained by dividing the figures given in the last column of the above table by this figure. Its values for $n=1$, 2, 3, and 4 are 1.33, 1.52, 1.55, and 1.56 respectively, with a maximum value of $\frac{\pi}{2}$ as n is still further increased.

It is seen therefore that the amplification of the audio current

depends on the type of detector employed. With a detector which gives an audibility factor proportional to the radio current, there is no doubt of the correctness of Dr. Liebowitz's contention that the amplification is not increased indefinitely with the current I_2. On the above assumptions the maximum amplification of audio power would be $(1.56)^2 = 2.43$. With the other type of detector, however, there are some grounds for a difference of opinion. On page 194 of Volume 3 of the "PROCEEDINGS," Dr. Liebowitz states that the maximum *true* amplification of audio power obtainable in the most efficient form of heterodyne receiver is four, and it is probable therefore that he would maintain that the amplification as here calculated is not a *true* amplification. It will be interesting then to determine to what extent the amplification as here calculated is due to increased efficiency of the detector. The efficiency will be taken to be the ratio of the audio power, which is the useful output, to the total input power. The ratio of the output with the heterodyne to that without it is $\left(\frac{\pi I_2}{I_1}\right)^2$, whilst the ratio of the corresponding inputs is $\frac{I_1^2 + I_2^2}{0.5\, I_1^2}$. Since the supply of power to the detector is interrupted during half the time when the heterodyne is not used, the power actually supplied to the detector has been put in the denominator; this is a point upon which some difference of opinion may exist.

The efficiency of the detector is increased in the ratio

$$\frac{\pi^2 I_2^2}{I_1^2} \cdot \frac{0.5\, I_1^2}{I_1^2 + I_2^2} = \frac{\pi^2}{2} \cdot \frac{1}{1 + \left(\frac{I_1}{I_2}\right)^2}$$

If $I_2 = I_1$, this equals $\frac{\pi^2}{4}$; the total ratio in which the audio power is increased is π^2, leaving a ratio of 4 to 1 for what Dr. Liebowitz calls the true amplification of the power. This is evident, moreover, from the fact that, with $I_2 = I_1$, the power supplied by the heterodyne generator is equal to that supplied from the antenna, and since they are supplied continuously, whereas without the heterodyne the current I_1 is interrupted half the total time, four times the power is supplied and the audio power given out would be increased in the same ratio if there were no change in the efficiency.

If $I_2 = 2\, I_1$, the input is ten times as great as with the interrupted continuous waves; the efficiency of the detector, as here

defined, is increased in the ratio $\frac{\pi^2}{2.5}$; the total amplification of audio power is $4\pi^2$, and the amplification apart from the increased efficiency is 10, which is, of course, the ratio in which the input has been increased.

This subdivision of the amplification, which depends on the method of defining the efficiency of the detector, is carried out here merely to show that such a procedure does not necessarily support the view put forward by Dr. Liebowitz. In the opinion of the writer, such a distinction is not warranted. Dr. Liebowitz differentiates between amplification obtained (1) by infusing new energy into the oscillations and (2) by increasing the efficiency of the receiving apparatus. In the latter example considered above, the newly infused energy was four times that already there, and if I_2 had been equal to $3\,I_1$ it would have been nine times as great. The amplification, however, depends not upon the amount of infused energy but upon its effect on the detector. By simply increasing the amplitude of I_2 from I_1 to $2\,I_1$ the audio power output is quadrupled. If Dr. Liebowitz's contention is correct, that there is no increased true amplification apart from increased efficiency, the latter must have been quadrupled, that is, the efficiency must have increased in the same ratio as the output. In the writer's opinion there is no basis for such an assumption. If such an increased efficiency could be obtained by means of a continuous polarising current, thus allowing the same amplification to be obtained with a smaller value of I_2, there would be more reason for denying the heterodyne the full credit for the total amplification, but it can be shown very simply that, on the assumptions made, the calculated amplification is independent of such a continuous polarising current.

It appears, therefore, that the large amplifications claimed by Mr. Cohen may be quite possible without tuning the audio frequency to resonance with the telephone diafram. Dr. Liebowitz on page 201 ascribes a part of the amplification to "adjusting the amplitude of the local current so as to work the crystal on the best part of its characteristic;" it should be noted, however, that the calculations of this paper are based on the assumption of an ideal detector with no "best part of its characteristic," and that, moreover, as already pointed out, the same increase of efficiency cannot be obtained by means of a continuous polarising voltage; it is essentially a part of the heterodyne amplification.

SUMMARY: The author contrasts the Cohen theory that heterodyne amplification can be increased indefinitely by increasing the local current (using an ideal detector of unlimited current-carrying capacity) with the Liebowitz theory that the maximum "true heterodyne amplification" is four.

It is then shown by several different methods of considering detector and heterodyne action as compared with chopper detection of received energy, that if the detector gives an audibility current proportional to the received current, the maximum amplification of audio power is 2.43, and does not increase indefinitely with the local current.

With detectors giving an audibility current proportional to the square of the received current (e. g., ordinary contact detectors thru a considerable range), the amplification may greatly exceed four, and its excess over four cannot be accounted for on the basis of "increased detector efficiency," since a steady polarizing current will *not* produce the same increase.

FURTHER DISCUSSION ON "ON THE INTERPRETATION OF EARLY TRANSMISSION EXPERIMENTS BY COMMANDANT TISSOT AND THEIR APPLICATION TO THE VERIFICATION OF A FUNDAMENTAL FORMULA IN RADIO TRANSMISSION" BY LEON BOUTHILLON

By

Oscar C. Roos

Equation (1) on page 226 should be, if j be substituted for $i = \sqrt{-1}$, to conform to equation (4) on page 228:

$$C_o = \frac{e\,j}{L\,n}\left[1 - \left\{\frac{R_o - jL\,v}{R_o} \cdot cos\,\frac{n\,x}{v} + sin\,\frac{n\,x}{v}\right\}\right] \quad . \quad . \quad . \quad . \qquad (1)$$

Two lines below the above equation the expression $\frac{e\,v\,i}{R_o\,n}$ becomes $\frac{e\,v}{R_{o\,n}}$. In equation (1) $\frac{n\,x}{v}$ is the electrical angle from the base of the antenna to the point x on the antenna, thru which the current stationary wave passes, as the current changes in value along the antenna.

The last line in my discussion is obviously to be read as

$$\frac{n\,a}{v}\left(\underline{\text{not}}\quad cos\,\frac{n\,a}{v}\right) = \pi,\ 3\pi,\ 5\pi\ .\ .\ .\ \text{radians},$$

which is to be interpreted as the current-wave phase change over the whole antenna. Note that equation (1) is deduced from equation (4) by remembering that

$$sinh\,\frac{p\,a}{v} = j\,sin\,\frac{n\,a}{v} + cosh\,\frac{p\,a}{v} = cos\,\frac{n\,a}{v}$$

also that $$\frac{n\,a}{v} = \frac{\pi}{2} + Z_o = R_o$$

in the case under discussion.

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PROCEEDINGS

of

The Institute of Radio Engineers

(INCORPORATED)

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ALFRED N. GOLDSMITH, Ph.D.

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THE INSTITUTE OF RADIO ENGINEERS
announces with regret the deaths of

Walter Everett Chadbourne

and

Thomas Leo Murphy

Mr. Chadbourne was born in Waterboro, Maine, in 1882 and spent most of his early years in Dorchester, Massachusetts. He attended the Mechanic Arts High School, and later Massachusetts Institute of Technology, from which he received the degree of Bachelor of Science in Naval Architecture. Thereafter he studied electrical engineering at the Lowell Institute.

After working for a short time with the Edison Company, he entered the radio field. He worked with Mr. Fessenden of the National Electrical Signaling Company for some years at Brant Rock, and then with the Marconi Wireless Telegraph Company in New York. While with the latter company, he went to Europe to study radio stations.

In September, 1914, Mr. Chadbourne was assigned as Expert Radio Aide of the United States Navy to Charlestown Navy Yard. Apparatus of his design was used in the Naval stations. While still engaged in this work in May, 1918, Mr. Chadbourne fell ill and died. He was well known to radio workers in the United States, and was an Associate Member of The Institute of Radio Engineers.

Mr. Murphy was one of the early workers in radio in the United States Navy, and did much to further the development of radio for naval uses. He was affiliated with The Institute of Radio Engineers as an Associate Member.

While serving as a Radio Gunner in the Navy, he was seriously injured in a seaplane accident at Ravenna, Italy, on September 15, 1918. From the effects of these injuries he later died.

ON THE

ELECTRICAL OPERATION AND MECHANICAL DESIGN OF AN IMPULSE EXCITATION MULTI-SPARK-GROUP RADIO TRANSMITTER*

By

Bowden Washington

(Radio Engineer, Cutting and Washington, Cambridge, Massachusetts)

Electrical Operation of Apparatus

There are two phenomena that are characteristic of the type of radio transmitter here described which, tho not essentially new, have not, to the best of my knowledge, been put into thoroly practical operation before. These two phenomena are impact excitation and the multi-spark system of procuring a tonal group.

To obtain impact excitation, two requirements must be fulfilled: a suitable gap must be found, and the radio frequency circuits designed to have appropriate constants for this form of energy transfer. We have at present three forms of gap which are practically interchangeable. The first is the aluminum-copper gap in an air-tight chamber to which alcohol is fed thru a wick and is converted into vapor by the heat of the gap. The second is a copper-copper or silver-silver gap of similar construction. In the third gap, both electrodes are of thin tungsten thoroly welded to copper backs and operating in air. These gaps all have practically identical electrical properties. There are some small differences, however, which may be noted.

The copper-aluminum gap seems to have remarkable inherent quenching properties, and will operate successfully with a primary circuit of far less desirable constants (i. e., a "stiffer" circuit), than is usually possible with impact excitation. It shows a really beautiful regularity of operation. It is, however, somewhat less efficient than the other two. The copper and tungsten gaps are practically identical in operation—the copper requires a somewhat higher voltage. The copper-

* Received by the Editor, February 28, 1918.

aluminum gap is generally run with an opening of 0.006 to 0.014 inches (0.15 to 0.36 mm.), while the two latter gaps operate between 0.001 and 0.003 inches (0.02 to 0.06 mm.). The operation of these gaps has been taken up before. (See "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," August, 1916, page 34, Volume 4, Number 4, and December, 1916, Volume 4, Number 6.)

The operation of these gaps is as follows:

The gap is connected in the usual way with a primary condenser, and the primary inductance of the coupling coil. This circuit, however, is designed to have a very low persistence. In our type 4A 0.5 kilowatt set the condenser has the value of 0.16 μf. The inductance—a single turn of heavy copper tubing—is approximately 1.2 microhenries. For maximum energy transfer, this primary circuit should have a free period of from 1.2 to 1.7 that of the secondary. The primary condenser is connected to a source of potential, either direct or alternating, having a value of from two to four hundred volts. If direct current is used, a fairly large iron core inductance should be inserted in this line. If an inductor alternator is used, the inductance of a machine is found sufficient. The condenser charges up until it has reached a potential sufficient to break down the gap; it then discharges thru the gap in a single loop or half cycle which sets the antenna in oscillation. The condenser immediately begins to charge again, and when it has reached a potential almost sufficient to break down the gap, the slight counter e.m.f. induced in the primary by the still oscillating secondary adds just sufficient increment to "trigger" the gap off in the proper phase relation to maintain smoothly the antenna oscillations. If direct current is employed, this process continues at regular intervals and as the value of the feed current is increased the gap discharges more and more frequently. The number of antenna oscillations which occur between the discharges of the gap is called by us the "inverse charge frequency."

The following oscillograms may be helpful to a clearer understanding of these phenomena. The pictures were taken with the Braun tube at an antenna frequency of 500,000 cycles per second.

Figure 1 shows the *E-I* characteristic of the gap—the current vertical, voltage horizontal. A large residual charge is shown in the primary condenser.

FIGURE 1

Figure 2 shows the gap current. In this photograph, the beam was deflected vertically by the current thru the gap and the horizontal time axis was obtained from the antenna potential.

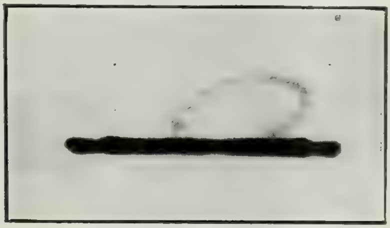


FIGURE 2

Figure 3 shows a highly damped train of oscillations in the antenna (the antenna resistance in this case was about 40 ohms) with an inverse charge frequency of 9.

FIGURE 3

Figures 4, 5, 6, and 7 show the antenna oscillation train with an antenna resistance of 5 ohms and an inverse charge frequency of six, four, three, and two, respectively. These wave-train pictures were taken by deflecting the beam vertically with the antenna current, and horizontally with the potential of the primary condenser. The gap discharges and the wave-train starts in the antenna. The gradual rise of potential in the primary condenser gives us our time axis until the gap discharges again when the spot returns to zero and traces the pattern over again. In Figures 5, 6, and 7 the return of the spot, which takes a time interval equal to nearly a whole antenna cycle, may be seen. It should be noted that these pictures were taken with exposures of from 0.1 to 0.4 seconds, so that the pattern was repeated several thousand times, showing a remarkable regularity of functioning of the gap.

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

Figures 8 and 9 are of interest principally as showing what can be done with the Braun tube. Figure 8 shows the damped train of Figure 3 in polar co-ordinates. Figure 9 shows two an-

tennas of slightly different period coupled to the same primary showing the re-transfer of energy between the two, and the production of beats.

Figure 8

Figure 9

The remaining oscillographs were taken with a special high speed Duddell oscillograph—some with direct current feed and others with a 60-cycle alternating current feed. The antenna in this case had a natural frequency of three or four thousand cycles per second.

In Figure 10, the upper curve shows the primary current and the lower curve the secondary. The inverse charge frequency is 3. The small loop on the lower side of the zero line is due to insufficient damping of the oscillograph vibrator and not to any reversal of current thru the gap. It will be easily seen that the envelope of these primary pulses in no sense approaches a logarithmic envelope, so that even without the Braun-tube oscillograms one would be reasonably sure of the uni-directional current pulse of this gap. (All the oscillograms have been retouched by tracing them carefully with white ink to enable them to be properly reproduced.)

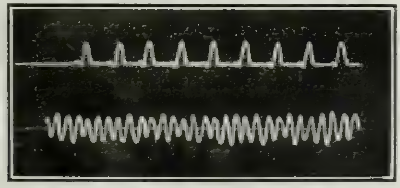


Figure 10

Figure 11 shows the antenna current in the upper curve and the primary current in the lower. The inverse charge frequency is one, and a practically undamped wave is emitted. This can be done quite successfully at even the shorter wave lengths. We have gotten fair "beat receiving" at a wave length of 450 meters, tho, of course, the adjustment of the receiving apparatus at this high period is extremely critical.

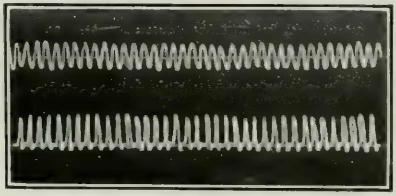


FIGURE 11

Figure 12 is taken with a 60-cycle feed current—the top curve being the primary curve, the bottom the secondary. It will be seen that the functioning of the gap is similar but the radiated energy is divided into tonal groups.

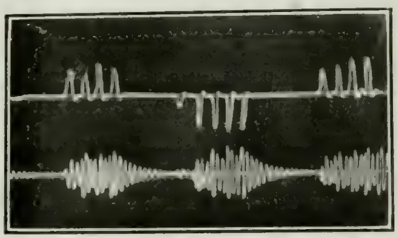


FIGURE 12

In Figure 13, the upper curve shows the primary or condenser voltage, the lower the primary current. A 60-cycle feed was employed.

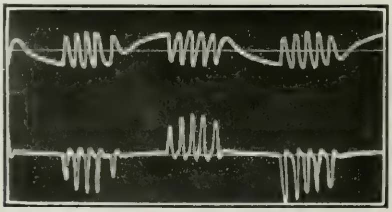


FIGURE 13

Figure 14 is a set of curves obtained partly from oscillograms and partly by calculation, and portrays, with I think fair accuracy, the operation of a 0.5 kilowatt set at a wave length of a thousand meters. The first curve shows the feed current, the peculiar shape of which will be explained later. The second curve shows the condenser voltage and will be seen to be similar to the top curve of Figure 13. The third curve shows the primary or gap current, the fourth the antenna current. It is obvious that the gap discharges at a rapidly increasing rate as the alternating feed current approaches its maximum and at a decreasing rate from maximum to zero, only to repeat this process, but with opposite polarity, during the next half cycle.

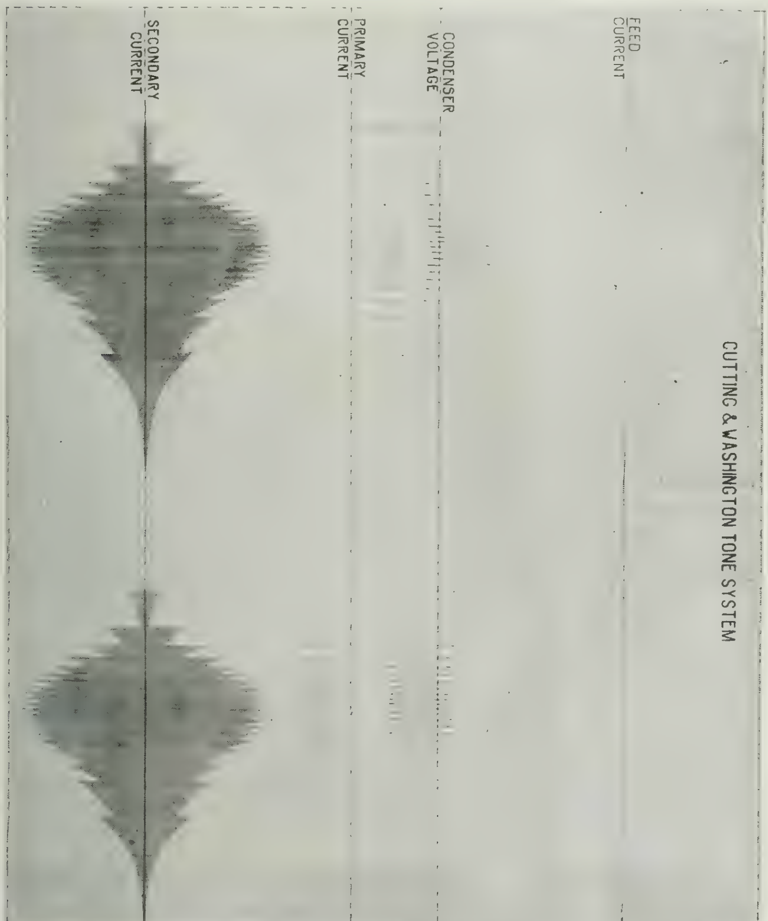


FIGURE 14

The envelope of the antenna current is a curve of substantially the same shape as the feed current.

This leads us to a consideration of the effect of this envelope, when rectified, upon the receiving telephones. It will be seen that if a sinusoidal e.m.f. is applied to the gap the results in the receiving telephones will be a wave which is approximately a rectified sine similar to that obtained by beat reception. This is a far from efficient way of exciting a telephone diafram, as there is thruout almost every moment a force applied to the diafram and it is not allowed to swing naturally thru the zero point and return; in other words, it might be said there is a large direct current component which is of no value as a sound producer.

In our first sets we found that while the antenna current is high, the audibilities obtained on any receiver but one of the regenerative type were only 50 or 60 per cent. of those obtained under similar conditions with a single-spark set. This, of course, led us to an investigation, and from this investigation we discovered the value (I might almost say the necessity), of properly grouping our sparks. When this grouping is correct, the tone efficiency is at least as high as that of any other group transmitter. This grouping may be accomplished in one or two ways, both of which appear thoroly satisfactory.

The first is by a special alternator which was developed thru several stages. Our first alternator in which any effort was made for spark grouping was a Holtzer-Cabot machine, of armature type, having considerable third harmonic in proper phase to produce a fairly peaked wave. This machine appeared to be very little better than a sinusoidal machine. When investigated with an oscillograph, it was found that the open circuit voltage was approximately as shown in Figure 15.

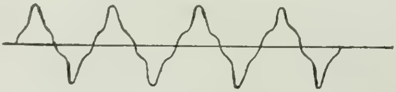


FIGURE 15

When the load was applied the voltage was roughly as shown in Figure 16 and the current practically sinusoidal, showing that the gap was in operation thruout far too long a period for good tone efficiency. Our next step was to build an inductor

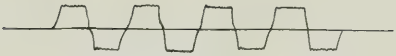


FIGURE 16

alternator of the Alexanderson type, having a single concentric field coil between two stators, and an unwound rotor. The pole spacing of this machine is shown roughly in the straight line sketch of Figure 17.

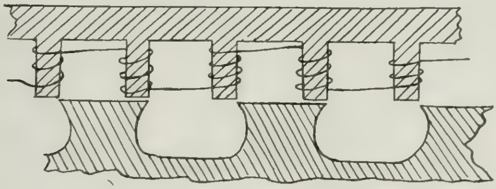
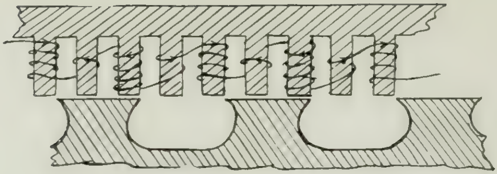


FIGURE 17

This gives a tremendously peaked no-load voltage, and both load voltage and audibility appeared to be somewhat better. A sketch of the load voltage is shown in Figure 18.

FIGURE 18

There was, however, a change of flux density thruout the whole machine due to the varying air gap, which caused exceedingly high losses in the cast iron frame-ring and in the center of the rotor. It occurred to us, as we had plenty of winding room, to intersperse poles between the wound poles purely to cut down this loss due to varying flux. It was then decided to wind these poles with a few turns (in practice about twenty-five per cent. of the turns on the main alternator poles), these turns to be in the opposite direction from those on the preceding pole, as shown in Figure 19.



Direction of rotation→

FIGURE 19

This gave a wave shape on open circuit approximately as shown in Figure 20 and a load voltage like Figure 21.

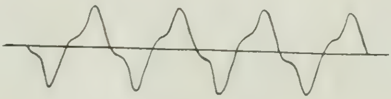


FIGURE 20

FIGURE 21

The portion of these load voltage curves shown by a ragged line occurs when the gap is in operation, as the discharges of the gap occur too rapidly, when using 500 cycles and 20 or 30 discharges per alternation, for the oscillograph vibrator to follow. This alternator is very satisfactory, for the purity of tone is absolutely independent of generator speed.

Our second method consists of placing an inductance and capacity in series across the primary condenser, the two having a period of approximately 1,500 cycles (called by us a "concentration circuit"). The operation of this circuit may be explained as follows. During the first sixty degrees of the alternating current pulse, this circuit acts almost as a short circuit on the machine and its condenser charges up. From sixty to one hundred twenty degrees, this circuit discharges into the primary condenser in conjunction with the machine. From one hundred twenty to one hundred eighty degrees, it is again a partial short circuit on the line. This circuit gives a remarkably desirable tone; and if of the right constants, it is much less critical

than would be supposed. In fact, with a standard Crocker-Wheeler, 500 cycle, 0.5-kilowatt, motor-generator set containing a shunt wound motor and a properly designed concentration circuit, the direct current voltage may be varied from 85 to 135 without a "break" in the note.

Figure 22 shows a curve of secondary current of one of these sets plotted against secondary wave length. The primary wave length remained fixed and had a value of about 850 meters. It should be noted that a good value of antenna current is obtained from 490 to 675 meters without change of primary. The advantages of a transmitter of this type seem to the writer quite numerous.

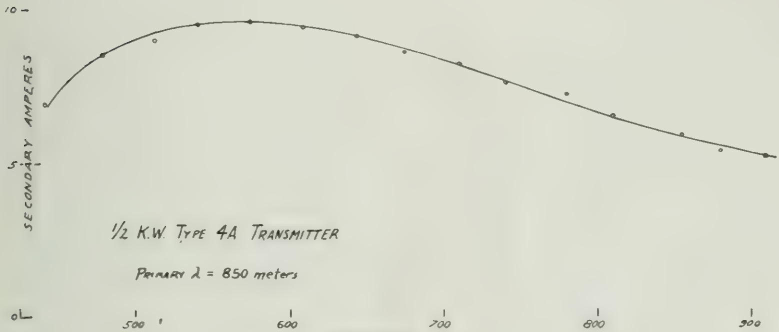


FIGURE 22

The main advantage perhaps is the almost entire lack of critical adjustment of both the audio and radio frequency circuits. Owing to the multi-spark system, a comparatively large change in the 500-cycle voltage will not affect the note with only 4 or 5 sparks more or less out of 20 or 30, and will not appreciably affect the shape of the tone envelope. With the interpole generator, there is no audio frequency resonance, and with the concentration circuit it is very broad. The note is stable thruout a large range of generator speeds. To be more specific, a properly designed concentration circuit will give a good note anywhere between 400 and 600 cycles. If the interpole generator is used, the note will remain good thruout any range of frequency in which the generator is likely to be operated. In fact, the one limiting factor of this case is that the speed must not be lowered to such an extent that the input is of such small

value as to give an insufficient number of sparks to produce a smooth tone envelope. Even in this case the note can be returned to its original purity by short-circuiting one of the two gaps usually supplied. A change of 4:1 in primary condenser and 2:1 in number of gaps does not ordinarily effect the note. As for the radio frequency adjustments, they are few. The gaps are screwed in until the electrodes are touching, and then opened about 0.002 inch (0.051 mm.). The primary inductance is fixed and the coupling is fixed. A 4:1 change is made in the primary condenser between 300 and 600 meters by the wave-shifting switch. The periods of the primary for these two wave lengths are approximately 425 and 850 meters respectively. If a third wave length is desired, such as 476 or 756, the 600 meter condenser can be so proportioned that 600 and one of these two wave lengths can be used with good efficiency in conjunction with the same primary. The only adjustment, therefore, in the radio-frequency circuit, apart from the length of gap, is the amount of inductance in the secondary or antenna circuit. As the apparatus emits one wave and that at the natural frequency of this secondary circuit, it is only necessary to tune this circuit for the desired wave lengths. With the exceedingly low voltages used—the total gap voltage on a 0.5-kilowatt set being about 900 maximum and 200 root-mean-square—the insulation can be of considerably less bulk than is usual in sets of the same power and yet the factor of safety may be much greater. The low voltage and the lack of adjustment make for a set that is compact, light, inexpensive to build, and very easy to operate.

It is the writer's opinion that the less radio apparatus is dependent upon the intelligence of the operator, the more satisfactory service it will give. Both the transmitter, and receiver, which will be described later, are designed with this point in view.

Another point of some importance is that the high rate of charging the primary condenser, which may be considered approximately 15 or 20 thousand cycles per second, reduces the losses of this piece of apparatus to a very great extent. This fact, in common with the low voltages, enables a remarkably small condenser to be used with a large factor of safety.

The space taken up by the 0.5-kilowatt set, type 4A, as shown by the detailed description which follows, is very limited, but the output appears to be about the same as that of the best quenched sets—five to nine antenna amperes being obtained, and the tone efficiency or audibility per ampere, as shown by

recent exceedingly careful tests conducted by ourselves and in a preliminary test by the Marconi Company, seems to be equal to that of the quenched spark set.

Mechanical Design of Type 4A Radio Transmitter and Type 8A Radio Receiver

Figures 23 and 24 show the front and rear views respectively of the complete transmitter, excepting, of course, the motor-generator set, which is a standard 0.5-kilowatt Crocker-Wheeler type. The dimensions of this transmitter are 14 inches (35.6

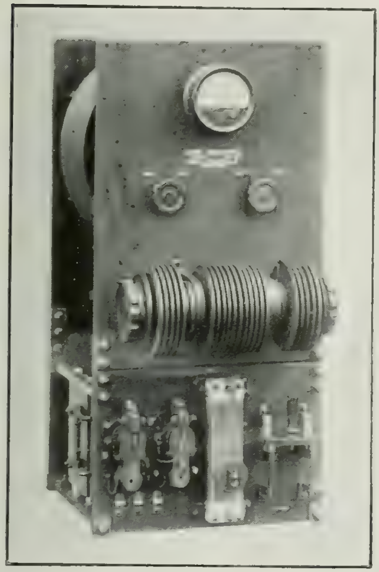


Figure 23

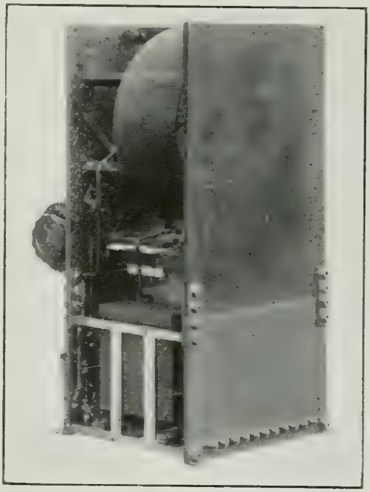


Figure 24

cm.) wide, 28 inches (71.1 cm.) high and 18 inches (45.7 cm.) deep over all, including gaps. The panel is divided into two parts, which may be roughly termed audio and radio frequency panels. These panels are of one-half inch "bakelite dilecto." It may be noted in passing that all insulation, with the exception of the field rheostat, transformer paper, and condenser dielectric, is bakelite, either moulded or sheet. On the lower part of the audio frequency panel will be seen the D. C. line switch, used for starting and stopping the motor generator, the motor field rheostat, and an automatic starter of the two step current

limit type. These two panels are complete mechanical units in themselves. The lower unit also contains a concentration circuit, the condenser of which is seen at the left (Figure 23), and the inductance at the left (Figure 24). A fixed resistance for the generator field, and another for the concentration circuit, is contained in this unit. At the rear of the lower unit is a terminal board.

On the upper or radio frequency unit are mounted the gaps. Suitable cooling fins are provided, and the current is led to the two movable sections by heavy multi-leaf brushes, which bear on the inside fin of this section. Locking screws are supplied in order to lock the two units in correct adjustment. On the right hand gap (Figure 23) a scale will be seen, which reads gap lengths directly in thousandths of an inch. Above the gaps are the antenna transfer switch and wave changer. The antenna transfer switch shifts the antenna from transmitter to receiver, and in the receiving position opens the generator field. The wave shifter picks off desired numbers of turns from the secondary of the coupling coil by means of flexible leads, and clips suitable inductance values for the 300 and 600 meter waves; and in the 600 meter position cuts in some additional primary condenser. These switches are mechanically identical. The radiation meter is of the hot band type, and 10 amperes are required for full scale deflection. The primary of the coupling coil is a single turn of 0.375 inch (0.95 cm.) copper tubing. The secondary consists of 30 turns of edgewise strip, 0.1875 inch (0.48 cm.) by 0.0625 inch (0.16 cm.), having a diameter of thirteen inches (33 cm.) and a total inductance of 320 microhenries, which is sufficient for antennas down to the value of 0.0003 microfarad. A good deal of work was entailed in the design of this coupling coil; the diameter and inductance were fixed upon, and strips of various cross-sections were wound with different spacing and the resistance determined with an oscillating, 3-element vacuum tube with a view to obtaining a coil of the lowest radio frequency resistance commensurate with this diameter and type.

The primary condenser is seen mounted on skids directly back of the gaps, and on the same skids is the transformer, mounted in a copper box as a shield from the high frequency field of the coupling coil. This transformer is of the closed core type with very low magnetic leakage. It has a short magnetic path and the primary and secondary are divided equally between the two legs. It is insulated to stand the full antenna

voltage between primary and secondary. The weight is in the neighborhood of five pounds (2.3 kg.). Its efficiency is 92 per cent., the losses being distributed equally between the iron and copper (about 20 watts each).

The complete connections of this transmitter, with the exception that but one unit of the starter is shown, are given in Figure 25.

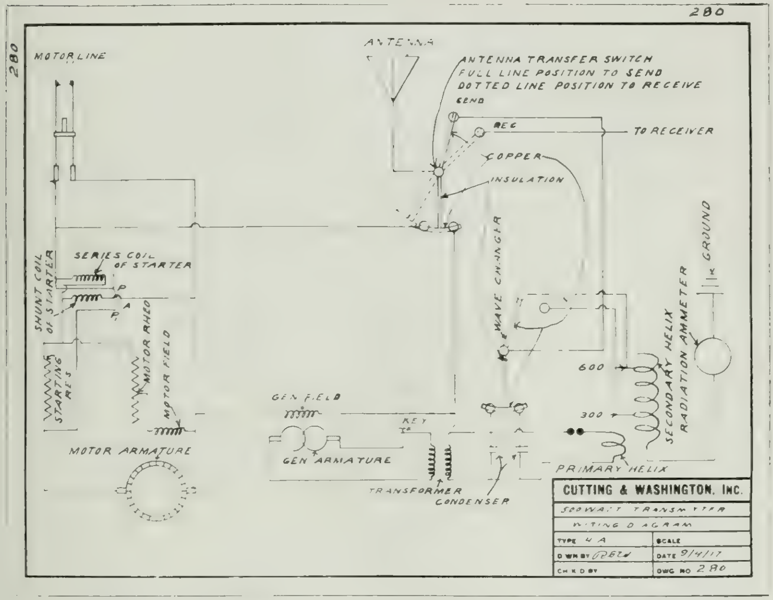


FIGURE 25

The gap handles, handle guards, gap supports, wave changer, antenna transfer switch handles, switch insulators and transformer supports are moulded. The radio frequency panel is an even one-quarter of a sheet of bakelite. The helix posts, skids and audio frequency panel are made of one-eighth of a sheet. The pointers, switch blades and connecting bars are punched. The angles supporting the helix posts and skids are identical, and these bakelite parts are identical, except that the helix posts have slots milled in them to take edgewise strip with a "gang miller." The gaps are made with cast aluminum flanges forced on a copper core. The thread on the movable sections is cut on a section of Shelby steel tubing, shrunk on the

inside fin, and working in a cast bronze ring. The transformer supports form the bottom half of the insulated bushing thru which the transformer terminals are brought, and the supporting rods form the terminals themselves. We have confined ourselves as much as possible to similar sizes of holes in these panels, so that a panel may be drilled with the aid of a jig in a multi-spindle press with great rapidity.

The insulation factor of safety has been kept very high thruout. The primary condenser, for instance, is subjected to a maximum working voltage of 1,000 volts, is provided with a safety gap of 0.016 inch (0.4 mm.) and tested at 6,000 volts with a spark gap of 0.032 inch (0.8 mm.). On an antenna of average size, the potential from the rat-tail to the ground is sufficient to break down a 0.25 inch (0.64 cm.) needle gap, and yet the insulation is comparable with that in some of the more compact types of quenched transmitter.

One of these transmitters, of the 2-kilowatt size, has been subjected in the laboratory to forty-eight hours of continuous operation, in eight-hour shifts, and to sixteen hours in two eight hour shifts of five minutes on and five minutes off, without showing signs of deterioration. The electrodes were cleaned off once with emery paper by hand during the two tests, and the radiation was maintained practically constant thruout the entire period. The set was, of course, operated at full power. Out of some ninety sets of this type, which have been given a one-hour key-locked run, but two have shown electrical or mechanical breakdown in any part, and this was in both cases due to defects of material rather than to faulty design or construction.

Figure 26 shows a 0.5 kilowatt set of the same type, mounted in a fiber chest.

Type 8A Receiver

The type 8A commercial receiver is shown in Figure 27, and its wiring in Figure 28.

All adjustments are made with the one handle. The receiver is of the untuned secondary type and is equal in efficiency and sharpness to those commercial receivers the writer has had opportunity to test. It has the one disadvantage that no "stand-by" is provided, but the simplicity of operation is such that we feel this to be not a great disadvantage. The multipoint switch shown in the lower portion of the panel operates on the first section of the primary coil, and cuts in inductance at a gradually increasing rate. When this switch is rotated

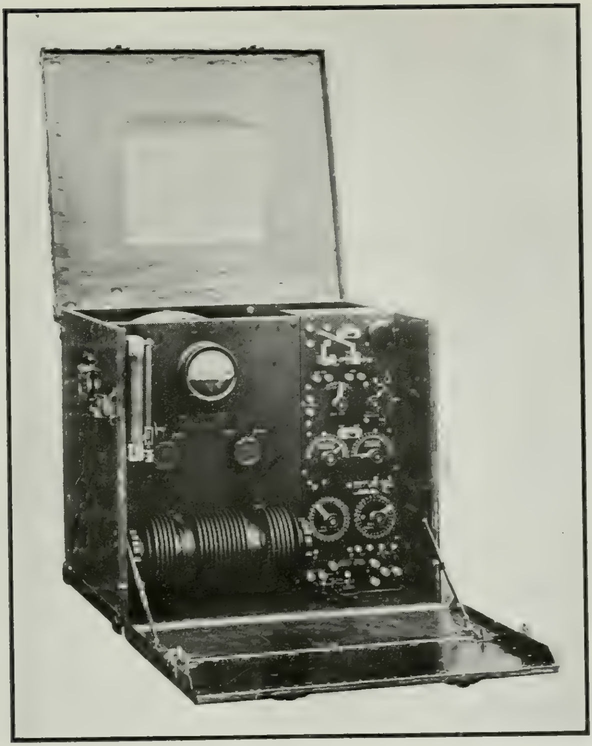


FIGURE 26

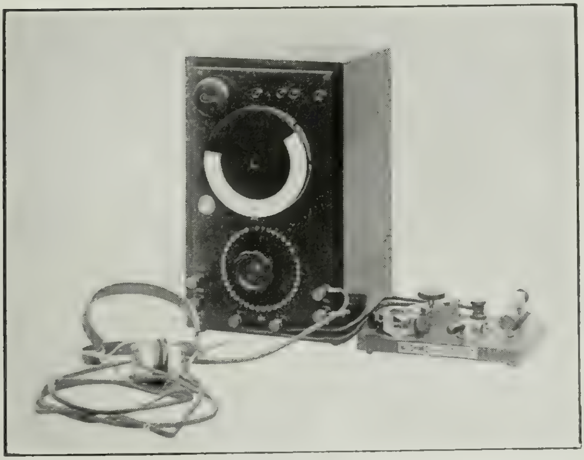


FIGURE 27

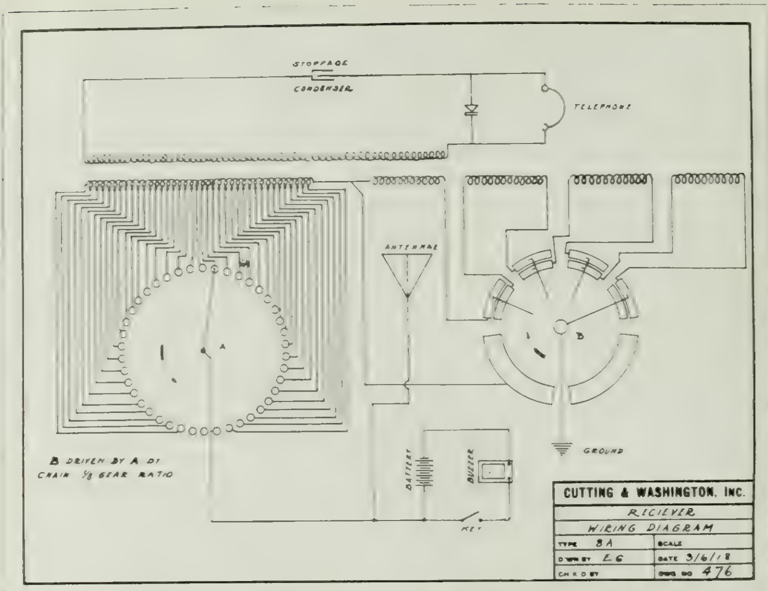


FIGURE 28

thru one complete revolution, the disc shown above the switch, which is connected to the switch-shaft by a light sprocket and chain and is geared down at a ratio of 8:1, cuts in the second section of the primary coil, which is equal in inductance to the whole of the first section. This process is repeated with the third section of the primary coil at the end of the third revolution of the switch handle and so on. This disc is also a dead-end switch. On the dull-silver plated scale the operator can mark in pencil the adjustment for certain wave lengths which are used frequently. The relative positions of the primary and secondary coils are fixed. The secondary coil consists of 25 turns of Number 30 B. and S. wire,* spaced 16 to the inch (6.3 to the cm.), and is a true untuned secondary, as its period is well below 100 meters which is the shortest wave length which we are equipped to measure easily. A test buzzer and silver chloride dry cells are provided in the cabinet. The detector is a combination of silicon, antimony, and galena, made by the Wireless Specialty Apparatus Company. The wave length range of the tuner on an antenna having a capacity of 0.0005 microfarad is fairly wide. On very large antennas, a small fixed series condenser is inserted in the primary circuit

* Diameter of number 30 wire = 0.010 inch = 0.25 mm.

The completely assembled set on shipboard is shown in Figure 29. In Figure 30, a larger set of the same type is illustrated. In this 2-kilowatt set, the panel design is again used. The increased number of gap sections is indicated.

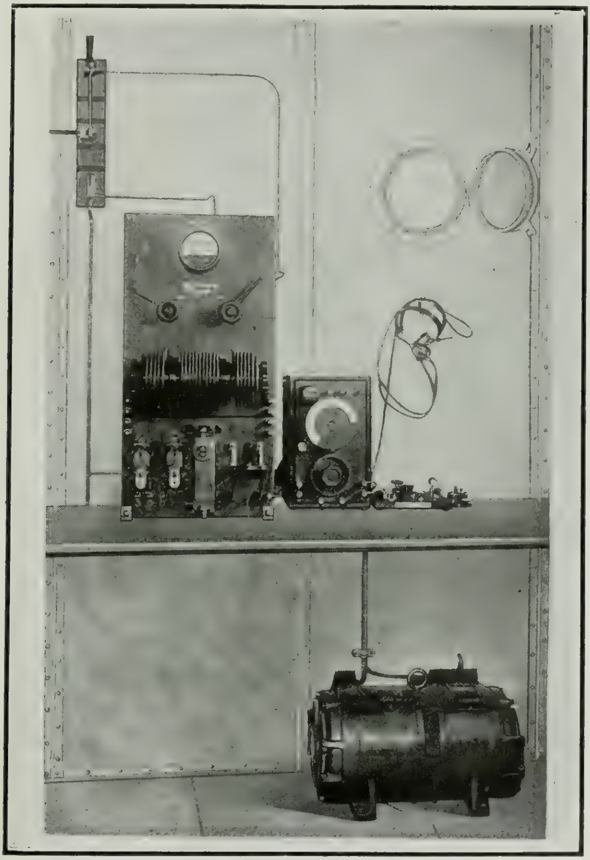


Figure 29

Antenna

As shown in erecting sketch, Figure 31, we have made an effort to standardize our antenna construction. The main insulators used are the Ohio Brass Company's compression strain type, about six inches (15.2 cm.) in diameter, one at each bridle being found sufficient. The spreaders are one and one-half inch (3.8 cm.) galvanized Shelby steel tubing, twelve feet

(3.66 m.) long, and are provided with clamps, as shown in the sketch at the lower left-hand corner of this figure. The screws which tighten these clamps on the spreader are provided with

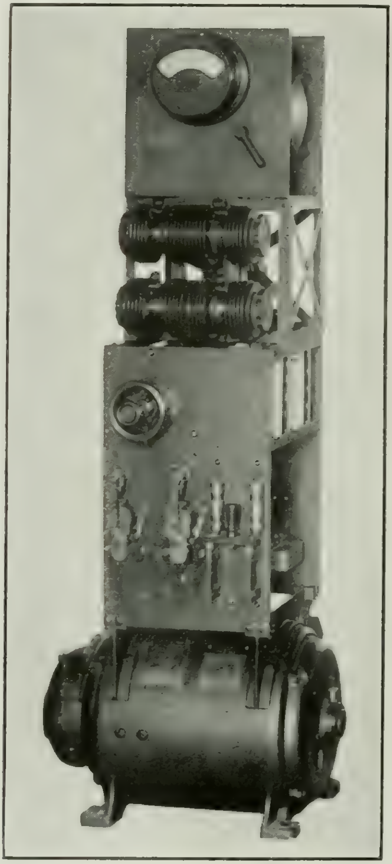


FIGURE 30

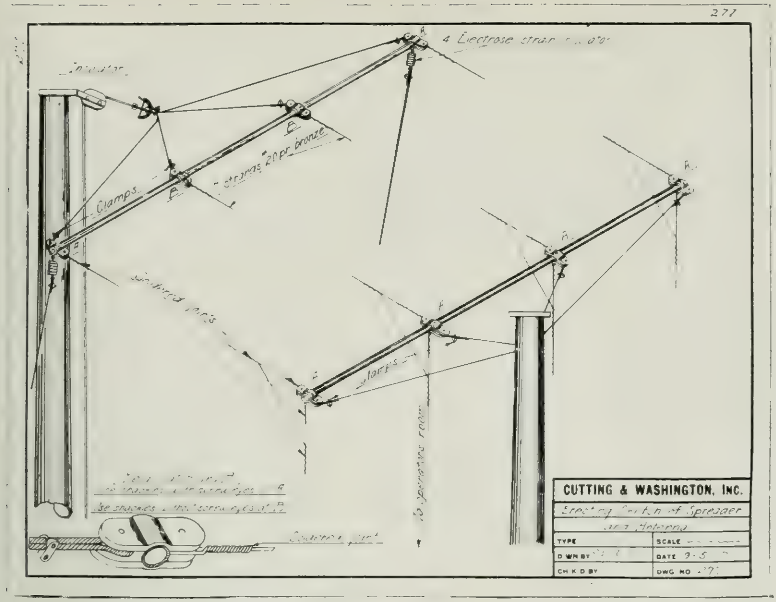


FIGURE 31

two small sheaves which give both the antenna wire, and the stranded ploughshare steel rope used for the bridle, a sufficient bending radius to reduce the chances of breakage at this point to a minimum.

SUMMARY: After a discussion of impulse excitation, three forms of gaps suitable for such extreme quenching are described. Braun tube oscillograms showing the operation of such gaps for various values of "inverse charge frequency" (number of secondary oscillations per primary discharge) are given. They show a remarkably regular gap-functioning over hundreds of thousands of cycles.

The problems of commercial construction of such sets are then considered. The necessity for using an alternator giving a special (highly non-sinusoidal) wave for feeding the gap transformer is demonstrated. The use of a modified tone or "concentration circuit" across the gap is justified on the basis of the increased receiving station audibility per ampere in the transmitting antenna.

The actual 0.5- and 2-kilowatt sets are then fully described, and the simple mode of wave changing explained. The receiver is also considered. A standardized antenna for these sets is shown.

THE VERTICAL GROUNDED ANTENNA AS A GENERALIZED BESSEL'S ANTENNA*

By

A. Press

(Consulting Engineer, New York, N. Y.)

In reality, the theory of the voltage and current distribution in a vertical grounded antenna should be based not only on the fact that the inductance and capacity are distributed; but, what is equally important, on the fact that such capacity and inductance are of variable distribution. It is this latter type of antenna that will be investigated.

In a former paper the following formula was developed giving the inductance in henrys per centimeter of a vertical grounded antenna where r was the radius of the wire and x the height of any given point from the ground, also in centimeters:

$$L = \frac{2}{10^9} \log_\epsilon \frac{\sqrt{x^2+r^2}+x}{r} \tag{1}$$

It is found that with a sufficient degree of exactness, up to 200 feet (60 m.), the above formula can be replaced by

$$L = \frac{7}{10^9} x^{0.13} \quad \text{henrys per cm.} \tag{2}$$

Evidently before one obtains a solution of the differential equations controlling the current and voltage relations in the aerial conductor, it will be necessary to obtain a formula similar to the above for the capacity per centimeter of the vertical grounded antenna as a function of the same distance x as before. The above differential equations of condition are

$$L \frac{d\,i}{d\,t} = - \frac{d\,v}{d\,x} \tag{3}$$

$$C \frac{d\,v}{d\,t} = - \frac{d\,i}{d\,x} \tag{4}$$

where L and C may be functions of x.

* Received by the Editor, December 27, 1917.

With wires having uniformly distributed inductance and capacity, the speed of propagation, neglecting the effect of resistance and conductance leakage, leads to the relation

$$L_o C_o = \frac{1}{V^2} \tag{5}$$

where V is the velocity of light in centimeters per second, and L_o and C_o are respectively the inductance and capacity in centimeters per centimeter. This is the same relation as obtains for free ether waves. The thought occurs to one that since the velocity of wave propagation along a straight vertical grounded antenna is practically the same quantity V, that, over the infinitesimal parts of the aerial wire, formula (5) should hold and be generalized[1] to read

$$LC = L_o C_o = \frac{1}{V^2} \tag{6}$$

which would give

$$C = \frac{1}{2 \times 9 \times 10^5 \log_e \dfrac{\sqrt{x^2+r^2}+x}{r}} \quad \mu\text{f. per cm.} \tag{7}$$

which is of the right order. Practically one may take over the same working range of 200 feet (60 m.)

$$C = \frac{1}{4.6 \times \log_{10}\left(\dfrac{4x}{d}\right) \cdot 9\,(10)^5} \quad \mu\text{f. per cm.} \tag{8}$$

The latter formula should account for a good deal of the work of of Professors Slaby and Howe.[2] If the formula (8) is compared with that obtaining for an antenna horizontally arranged with respect to the earth's surface, it will be seen that so far as vertical antennas are concerned, one can estimate antenna capacities on the assumption that the individual elements are made up of independent infinitesimal parts of horizontally disposed antennas.

The following set of expressions can therefore be said to prevail:

$$LC = \frac{1}{V^2} \tag{6}$$

[1] Compare Dr. Louis Cohen, "Calculation of Alternate Current Phenomena," page 108. In another place, I have indicated how for helices the equation (6) should be generalized to read $LC = L_o C_o = \frac{\mu}{V^2}$, where μ represents the equivalent magnetic loading of the medium giving an inductance axially to correspond with a straight wire having the same inductance per cm.

[2] See reference to above in Fleming's "Electric Wave Telegraphy," 1916, pages 204 and 642.

$$L = L_o x^n \tag{9}$$

$$C = \frac{C_o}{x^n} \tag{10}$$

$$L C = L_o C_o \tag{11}$$

Conditions (9) and (10) lead to the solution of a Bessel's antenna in precisely the same manner as Heaviside[3] has treated the corresponding Bessel's cable.

To arrive at the resultant characteristic from equations (3) and (4), following the symbolical methods of Heaviside let

$$t_1 = \frac{d}{d\,t}$$

$$x_1 = \frac{d}{d\,x}$$

then equations (3) and (4) can be written

$$L\,t_1\,i = -x_1\,v \tag{12}$$

$$C\,t_1\,v = -x_1\,i \tag{13}$$

and by non-commutative algebraic processes, so far as the coefficients L and C alone are concerned, one obtains by solving for v and i in succession

$$L\,t_1\,i = x_1 \frac{1}{C\,t_1} x_1\,i$$

which, interpreted back again, gives:

$$\frac{d^2\,i}{d\,t^2} = \frac{1}{L}\frac{d}{d\,x} \cdot \frac{1}{C}\frac{d}{d\,x} \cdot i \tag{14}$$

$$C\,t_1\,v = x_1 \frac{1}{L\,t_1} x_1\,v$$

$$\frac{d^2\,v}{d\,t^2} = \frac{1}{C}\frac{d}{d\,x} \cdot \frac{1}{L}\frac{d}{d\,x} \cdot v \tag{15}$$

It is the equations (14) and (15) that can be thrown into the usual Bessel's form with constant coefficients L_o, C_o by means of the relations (9) and (10) mentioned above and in consequence

$$\frac{d^2\,i}{d\,x^2} + \frac{n}{x} \cdot \frac{d\,i}{d\,x} = L_o C_o \frac{d^2\,i}{d\,t^2} = q^2\,i \tag{16}$$

$$\frac{d^2\,v}{d\,x^2} - \frac{n}{x} \cdot \frac{d\,v}{d\,x} = L_o C_o \frac{d^2\,v}{d\,t^2} = q^2\,v \tag{17}$$

with the understanding that the operation $L_o C_o \frac{d^2}{d\,t^2}$ has been

[3] Compare Heaviside, "Electromagnetic Theory," Volume II, page 239.

symbolized by q^2. These latter equations are to be solved in order to determine the true current and voltage relations which enable stationary waves to be set up in straight vertical grounded antennas with variably spaced nodes and antinodes.

The general solution of (16) is[4]

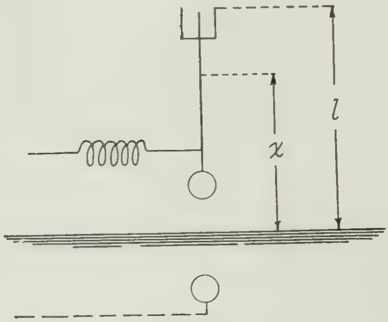
$$i=\frac{I_m(qx)}{x^m}A+\frac{I_{-m}(qx)}{x^m}\cdot B \tag{18}$$

given that $m=\frac{n-1}{2}$ and where

$$\frac{I_m(qx)}{x^m}=\frac{\left(\frac{q}{2}\right)^m}{0!\,m!}+\frac{\left(\frac{q}{2}\right)^{m+2}x^2}{1!\,(m+1)!}+\frac{\left(\frac{q}{2}\right)^{m+4}x^4}{2!\,(m+2)!}+\cdots \tag{19}$$

$$\frac{I_{-m}(qx)}{x^m}=\frac{\left(\frac{q}{2}\right)^{-m}x^{-2m}}{0!\,(-m)!}+\frac{\left(\frac{q}{2}\right)^{-m+2}x^{-2m+2}}{1!\,(-m+1)!}+\frac{\left(\frac{q}{2}\right)^{-m+4}x^{-2m+4}}{2!\,(-m+2)!}+\cdots \tag{20}$$

In the case at hand $n=0.13$, whence $m=\frac{0.13-1}{2}=-0.435$, showing that m is negative and therefore the indices of the x's in $\frac{I_{-m}(qx)}{x^m}$ are all plus.



To satisfy the simplest boundary conditions, by way of example, one may take

$$i=I \text{ when } x=0 \tag{21}$$

$$i=0 \text{ when } x=l, \text{ the antenna height} \tag{22}$$

[4] See Heaviside, previous citation, page 244 and page 240.

Thus for the first boundary condition when

$$x=0, \quad \frac{I_{-m}(q\,x)}{x^m}=0,$$

and therefore, by (18),

$$I=\frac{\left(\frac{q}{2}\right)^m}{0!\,m!}A$$

$$A=\frac{0!\,m!}{\left(\frac{q}{2}\right)^m}I \tag{23}$$

Again, for $x=l$,

$$0=\frac{I_m(q\,l)}{l^m}A+\frac{I_{-m}(ql)}{l^m}B$$

$$B=-\frac{I_m(q\,l)}{I_{-m}(q\,l)}A \tag{24}$$

and therefore, for the above set of boundary conditions, the complete solution is

$$i=\frac{0!\,m!}{\left(\frac{q}{2}\right)^m}\left\{\frac{I_m(q\,x)}{x^m}-\frac{I_m(q\,l)}{I_{-m}(q\,l)}\cdot\frac{I_{-m}(q\,x)}{x^m}\right\}\cdot I \tag{25}$$

and I is the impressed current function of the time for $x=0$. The current function I is that produced directly by the exciting or oscillation circuit, and therefore dependent on the constants in that circuit. On the other hand, the voltage distribution in the antenna is obtainable thru the fundamental equation:

$$C\frac{d\,v}{d\,t}=-\frac{d\,i}{d\,x} \tag{4}$$

and thus one may obtain a relation between the current at $x=0$ and the maximum antenna voltage at $x=l$, that is, at the top of the antenna. Another way would be to assume a definite impressed voltage function where the lead of the primary circuit attaches to the antenna.

Substituting (25) in (4), there obtains[5]

$$C\frac{d\,v}{d\,t}=\frac{0!\,m!}{\left(\frac{q}{2}\right)^m}\left\{\frac{I_m(q\,l)}{I_{-m}(ql)}\cdot\frac{I_{-(m+1)}(qx)}{x^m}-\frac{I_{m+1}(q\,x)}{x^m}\right\}\cdot I \tag{26}$$

[5] By formula (27) in Heaviside, previous citation, page 245.

and therefore since

$$q^2 = LC\frac{d^2}{dt^2}$$

$$\frac{d}{dt} = \frac{q}{\sqrt{LC}} = Vq \tag{27}$$

whence

$$I = \frac{CVq\left(\frac{q}{2}\right)^m}{0!\,m!}\left\{\frac{x^m I_{-m}(ql)}{I_m(ql)\,.\,I_{-(m+1)}(ql) - I_{-m}(ql)\,.\,I_{m+1}(qx)}\right\}v \tag{28}$$

If then we write v_l for the maximum voltage at $x = l$ and C_l for the capacity per centimeter at $x = l$ the relationship between the maximum voltage and the maximum current is as follows

$$I = \frac{C_l Vq\left(\frac{q}{2}\right)^m}{0!\,m!}\left\{\frac{l^m I_{-m}(ql)}{I_m(ql)\,.\,I_{-(m+1)}(ql) - I_{-m}(ql)\,.\,I_{m+1}(ql)}\right\}v_l$$

$$I = \frac{C_l Vq\left(\frac{ql}{2}\right)^{m+1}}{0!\,m!}\left\{\frac{-\pi}{2\sin m\pi}\right\}I_{-m}(ql)\,v_l \tag{29}$$

For steady impressed sinusoids the function $I_{-m}(ql)$ becomes a real Bessel's of oscillating character and then, because q is imaginary, one may use the formula[6]

$$i^{2m} = (\cos + j\sin)\,m\pi$$

This would indicate that for Poulsen circuits, the maximum current amplitude is in general always out of time phase with the maximum voltage.[7]

SUMMARY: By taking account of the variable distribution of inductance and capacity along a vertical grounded antenna, the general expression for the current at any point of the antenna is obtained.

For the case of an antenna having zero current at the top and maximum current at the (unloaded) bottom, the particular solution for current and voltage distribution is obtained.

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[6] Heaviside, previous citation, page 245.
[7] Heaviside, previous citation, page 253.

ON THE POSSIBILITY OF TONE PRODUCTION BY ROTARY AND STATIONARY SPARK GAPS*

By

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Introduction

In a previous paper on resonance transformers,[1] the author has shown that the tone phenomena are possible only for certain values of discharge voltage, and he has considered how the possible range varies with the ratio of the natural frequency β of the circuit to the forced frequency ω of the source.

A remarkable result was obtained, namely, that with a stationary spark gap, the possibility range vanishes at absolute resonance, $\beta=\omega$, if the damping of the circuit is negligibly small.

The possibility range will naturally be quite different when a rotary spark gap is employed.

An admirable paper has been published by Lieutenant L. Bouthillon[2] on the combined system of a high voltage, direct current generator and a rotary spark gap. So the author finds it hardly necessary to go into the details of the fundamental principles of tone production both for the alternating current resonance transformer method and for the high tension direct current method.

In what follows, an attempt is made to determine the possibility range of the regular discharge, or the tone phenomenon, in the two systems above mentioned when equipped with a rotary or a stationary spark gap. Part I deals with the discharge characteristic of a rotary gap, and enables the drawing of conclusions concerning the stationary gap since this is but one particular case of the rotary gap. In Part II, the possibility ranges are considered for the resonance transformer method

* Received by the Editor, September 20, 1917.

[1] H. Yagi, Proceedings of the Institute of Radio Engineers, Volume 5, Number 6, December, 1917.

[2] L. Bouthillon, Proceedings of the Institute of Radio Engineers, June, Volume 5, Number 3, 1917.

with both kinds of spark gap; and in Part III, the same is done for the high tension, direct current method.

Part I. Characteristics of a Rotary Spark Gap

Whatever the shape of the rotary gap, the variation of the gap length with time may be deduced from the following simple formulas.

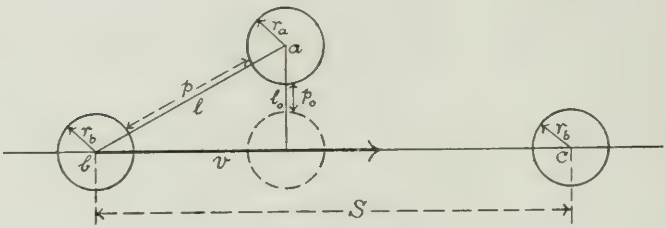

Figure 1

Suppose that the point a is fixed and b moves along a straight line $b\ c$ with a constant velocity v. Other points c, d, and so on, similar to b, move in unison with b and with a constant distance S between them.

If b, c, d, and so on, form one electrode and a the other, then the gap length between two poles varies periodically. Let t_o be the time at the moment when b lies at the minimum distance l_o from a, and t the time at any other position distant l from a. Then

$$l^2 = l_o^2 + v^2\,(t_o - t)^2$$

This is an hyperbola like A in Figure 2, the asymptote of which has an inclination v to the horizontal axis.

When a, b, c, and so on, are mere points, or the gap is a needle gap, l denotes the gap length itself. In actual gaps, however, the electrodes are not points but have various shapes which may for convenience be taken to be spheres. Assuming their radii to be r_a and r_b respectively, the true gap length p is

$$p = l - r_a - r_b$$

and the minimum gap length is given by

$$p_o = l_o - r_a - r_b.$$

Or, $$(p + r_a + r_b)^2 = l_o^2 + v^2\,(t_o - t)^2 \qquad (1)$$

Now r_a, r_b, l_o, v, and t_o are all constants, and p as a function

of t traces an hyperbola with its center C displaced r_a+r_b from the t axis. (B in Figure 2.)

The period T is given by

$$T=\frac{S}{v}.$$

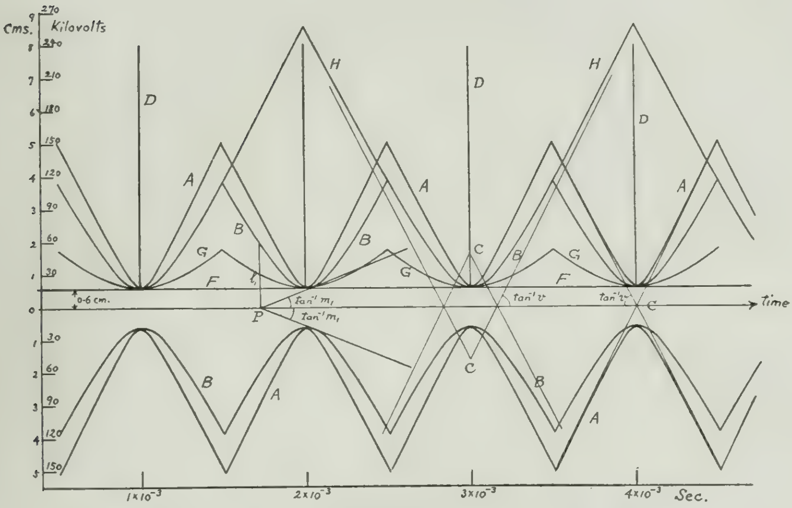

FIGURE 2—Characteristics of Rotary Spark Gaps

T is determined by the proposed spark frequency and then S is proportional to v. As v is proportional to the diameter of the wheel, or the diameter of the path of b, the number of knobs (or studs) around the circumference for this condition is independent of the diameter when the number of revolutions per minute of the wheel is constant.

With increasing v, the hyperbola becomes more and more peaked and the characteristic feature of a rotary gap becomes more manifest. It has usually been supposed that a rotary gap is an arrangement which enables the discharge to take place with ideal exactness at intervals independent of the terminal potential at the moment. However, from the consideration of the mechanical stress due to centrifugal force, there is a practical limit to the peripheral speed v, and the rotary gap can never be so ideal an arrangement as is usually supposed. For an ideal gap, the characteristic curve must coincide with the straight lines D in Figure 2.

The most efficient way of improving the discharge characteristic is to divide the gap into many series gaps, each one being a rotary gap of the highest permissible speed.

When $v=0$, as a particular case, S must also be zero, and the gap is nothing but a stationary gap. (F in Figure 2.)

In Figure 2, A and B represent the discharge characteristics for $r_a=r_b=0$, or $l_o=0.6$ cm. (needle gap) and for $r_a=r_b=0.8$ cm., or $l_o=2.2$ cm. (spherical gap) respectively. For both of them

$v=10^4$ cm. per sec. (100 m. per sec. or 330 feet per sec.)
$p_o=0.6$ cm.
$T=0.001$ sec.
$S=T\,v=10$ cm. (4 inches).

G is the curve of the spherical gap for $v=5\times10^3$ cm. per sec. (50 m. per sec. or 165 feet per sec.) or $S=5$ cm. (2 inches). H is drawn for $T=0.002$ sec. and $v=10^4$ cm. per sec.

The relation between the air gap length and the disruptive voltage is not truly linear, but they may practically be assumed to be proportional to each other, and Figure 2 may at the same time be taken as a diagram representing the relation between the sparking voltage and the time.

In cases of alternating current operation, in which alternate discharges may take place, the image of the characteristic curves must be drawn below the horizontal axis as in Figure 2. Whenever the terminal voltage passes beyond these upper and lower limiting curves, there will be a spark discharge across the gap.

It is plain from these examples that even at the high speed of 100 meters per second (330 feet per second), with 10 cm. (4 inches) distance between consecutive knobs, the rotary gap is not an arrangement which causes the discharge to occur as sharply as presupposed.

Part II. Possibility Range of Tone Phenomena with the Resonance Transformer Method

The performance of a spark gap is not to be defined by the magnitude of current thru it, but is determined simply by its disruptive voltage: and consequently the explanation of its operation should be more suitably developed from voltage considerations. It is true that the introduction of current terms in the equations makes the forms of some expressions simpler: nevertheless, a method of solution that might be called a "current method" is no more convenient than the

"potential method" used by the author to deduce directly the most important relations.

Let us restrict the subject as hitherto[3] to the two sorts of regular discharge, namely:

Alternate discharge—one discharge per half cycle;

Unidirectional discharge—one discharge per cycle.

The terminal voltage e_2 of the condenser is given by the following expression for alternate discharge:

$$\left\{\begin{aligned} & e_2 = E_c \sin(\omega t+\phi) - K E_o \varepsilon^{-\alpha t} \cos\left(\beta t - \theta - \tan^{-1}\frac{\alpha}{\beta}\right) && (2) \\ & K = \frac{1}{\sqrt{1+2\left(\varepsilon^{-\alpha\frac{\pi}{\omega}}\right)\cos\beta\frac{\pi}{\omega}+\left(\varepsilon^{-\alpha\frac{\pi}{\omega}}\right)^2}} && (3) \\ & \tan\theta = \frac{\varepsilon^{-\alpha\frac{\pi}{\omega}}\sin\beta\frac{\pi}{\omega}}{1+\varepsilon^{-\alpha\frac{\pi}{\omega}}\cos\beta\frac{\pi}{\omega}} && (4) \end{aligned}\right.$$

and for unidirectional discharge:

$$\left\{\begin{aligned} & e_2 = E_c \sin(\omega t+\phi) - K E_o \varepsilon^{-\alpha t} \cos\left(\beta t + \theta - \tan^{-1}\frac{\alpha}{\beta}\right) && (5) \\ & K = \frac{1}{\sqrt{1-2\left(\varepsilon^{-\alpha\frac{2\pi}{\omega}}\right)\cos\beta\frac{2\pi}{\omega}+\left(\varepsilon^{-\alpha\frac{2\pi}{\omega}}\right)^2}} && (6) \\ & \tan\theta = \frac{\varepsilon^{-\alpha\frac{2\pi}{\omega}}\sin\beta\frac{2\pi}{\omega}}{1-\varepsilon^{-\alpha\frac{2\pi}{\omega}}\cos\beta\frac{2\pi}{\omega}} && (7) \end{aligned}\right.$$

where t is measured from the instant of a discharge.

When α is negligibly small in comparison with β, we have

$$\left(\frac{d e_2}{d t}\right)_{t=0} = \left(\frac{d e_2}{d t}\right)_{t=T}$$

This corresponds to the assumption in the current method that the current i_o at discharge must remain constant during the momentary short circuit. It will later be seen that $\left(\frac{d e_2}{d t}\right)_o$ is an important term in deciding the possibility of tone phenomena from the characteristic of the spark gap.

[3] H. Yagi. Previous citation.

Another initial condition is

$$e_2 = 0 \text{ at } t = 0,$$

or

$$\sin\phi = \frac{E_o}{E_c} K \cos\theta \tag{8}$$

K and θ are constants for a given $\frac{\beta}{\omega}$,and when E_c is held constant, there is a value of $\sin\phi$ corresponding to each E_o. As $\frac{1}{K\cos\theta}$ is never greater than 2, E_o can never be larger than $2E_c$[4].

In actual rotary gaps, tho not in ideal ones, in order to have discharges at a definite E_o, the wheel must be rotated in such phase relation that the gap length becomes that corresponding to E_o at the proper instants.

From equation (1),

$$E_o = q\,p = q\sqrt{l_o^2 + v^2\,(t_o - t)^2} - q\,(r_a + r_b) \tag{9}$$

where q is the disruptive strength per cm. or nearly 30.000 volts per cm. (75,000 volts per inch) in the air.

If t is measured from the instant of a discharge, then t_o is the time interval between the discharge and the time corresponding to the minimum length of the rotary gap. Putting $\psi = \omega t_o$, we know that there is a definite t_o or ψ corresponding to the given E_o.

Now the time interval between the discharge and the zero of the fundamental sine wave, $E_c \sin(\omega t + \phi)$, is $\frac{\phi}{\omega}$, and therefore the time interval between this zero condition and the minimum length of gap condition is

$$\frac{\phi + \psi}{\omega}$$

where

$$\phi = \sin^{-1}\frac{E_o}{E_c} K \cos\theta$$

and

$$\psi = \frac{\omega}{v}\sqrt{\left[\frac{E_o}{q} + (r_a + r_b)\right]^2 - l_o^2} \tag{10}$$

Hence the phase difference between the alternator position[5]

[4] In Lieutenant Bouthillon's paper, the coefficient in his equation (19) and the value in Figure 8 correspond to what is denoted by $\frac{1}{K\cos\theta}$ in the present paper.

[5] The phase difference between $E_c \sin(\omega t + \phi)$ and the alternator E.M.F. is determined by the circuit constants.

and the position of the rotary gap must be adjusted to accord with E_o. ψ and ϕ are plotted in Figure 3 as the functions of E_o. It may be recognized therefrom that $\frac{\psi+\phi}{\omega}$ is nearly proportional to E_o within a certain intermediate region of E_o when the discharge takes place at $o < \phi < \frac{\pi}{2}$, whereas $\frac{\psi+\phi}{\omega}$ is nearly constant (equal to T) when $\frac{\pi}{2} < \phi < \pi$.

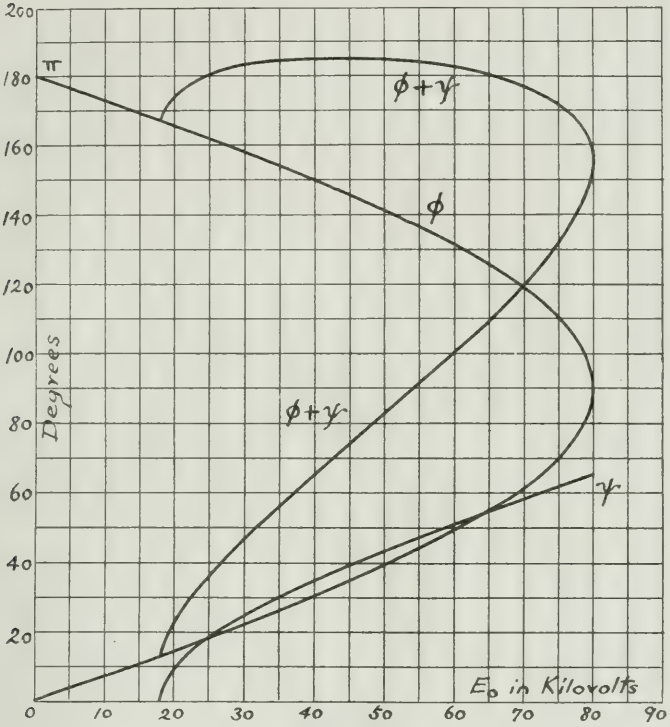

FIGURE 3

POSSIBLE RANGE

From (2) and (5) we have

$$\left(\frac{d\,e_2}{d\,t}\right)_o = \omega E_c \cos\phi \mp \sqrt{\alpha^2+\beta^2}\, K E_o \sin\theta,$$

or, substituting in (8),

$$\left(\frac{d\,e_2}{d\,t}\right)_o = \omega\sqrt{E_c^2 - E_o^2\, K^2 \cos^2\theta} \mp \beta\, K E_o \sin\theta. \qquad (11)$$

taking the $-$ sign for alternate discharge and the $+$ sign for unidirectional discharge.

When the circuit constants are all given and E_c and ω are fixed, there will be a definite $\left(\frac{d\,e_2}{d\,t}\right)_o$ corresponding to E_o, and when E_o is varied from zero to $2E_c$, $\left(\frac{d\,e_2}{d\,t}\right)_o$ varies also according to the equation (11). (See Figures 5 and 6.)

Whether this $\left(\frac{d\,e_2}{d\,t}\right)_o$ is permissible or not will be determined by the characteristic of the spark gap.

As we have too many variable quantities, it is not easy to determine very accurately the boundary of the possibility region of tone phenomena. In the following, conventional methods, tho not strictly exact, are resorted to; thus enabling us to solve the problem very simply.

Rotary Gap

The permissible range of $\left(\frac{d\,e_2}{d\,t}\right)_o$ is limited by the following relations:—

(1) Suppose a discharge to take place at P (Figure 2). The condenser potential e_2 becomes instantly zero and then rises from zero, whereby the curve of e_2 against t must not cut the characteristic curve before the next regular discharge. The limit is reached when these two curves touch each other. One of the curves is an hyperbola and the other a sine curve, so that the solution is not quite simple. It is assumed that the upper and the lower limits of $\left(\frac{d\,e_2}{d\,t}\right)_o$ are given by the inclination of the tangents (straight lines) drawn to the hyperbolas thru the point P.

If P is the point corresponding to the gap length l_1, the inclination m_1 which the tangent to the hyperbola makes with the horizontal axis is given by

$$m_1 = \frac{r}{l_1^2}\left\{(r_a + r_b)\sqrt{l_1^2 - l_o^2} \pm l_o\sqrt{l_1^2 - (r_a + r_b)^2}\right\} \qquad (12)$$

This value of m_1 is graphically represented in Figure 4 as a function of E_o, E_o being equal to $q\,p_1$, or $= q\,(l_1 - r_a - r_b)$.

The curves above the horizontal axis are for $r_a + r_b = 0$, 0.8, 1.6, and 2.4 cms. (0, 0.3, 0.6, and 0.9 inch). Below the axis are drawn only two of them, i. e., $r_a + r_b = 0$ and $= 1.6$ cm. (0.6 inch).

These curves show that the possible range is more restricted for larger electrodes $(r_a + r_b)$, when the minimum gap length is to be constant. If v is made smaller, the relative proportion is unaltered but all the limits as a whole are lowered in proportion to v.

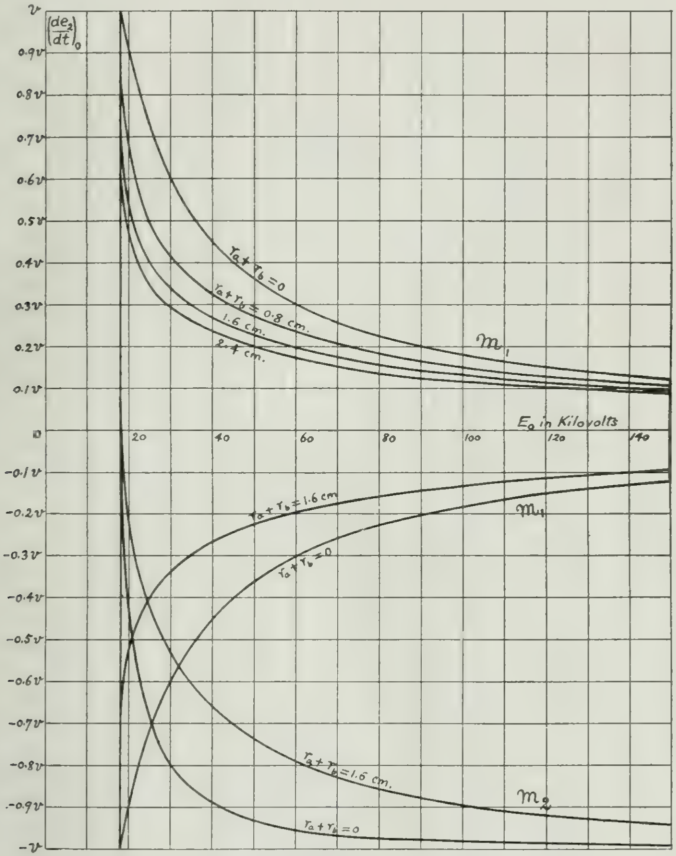

FIGURE 4—A. C. Method

(2) $\left(\frac{d e_2}{d t}\right)_o$, which is always positive in stationary gaps may be negative in the case of rotary gaps, and its negative limit is given by the condition that $\left(\frac{d e_2}{d t}\right)_T$ coincides with the inclination of the tangent to the hyperbola at the moment of

discharge. This inclination m_2 is given by

$$m_2 = v\sqrt{1-\left(\frac{l_o}{l_1}\right)^2} \tag{13}$$

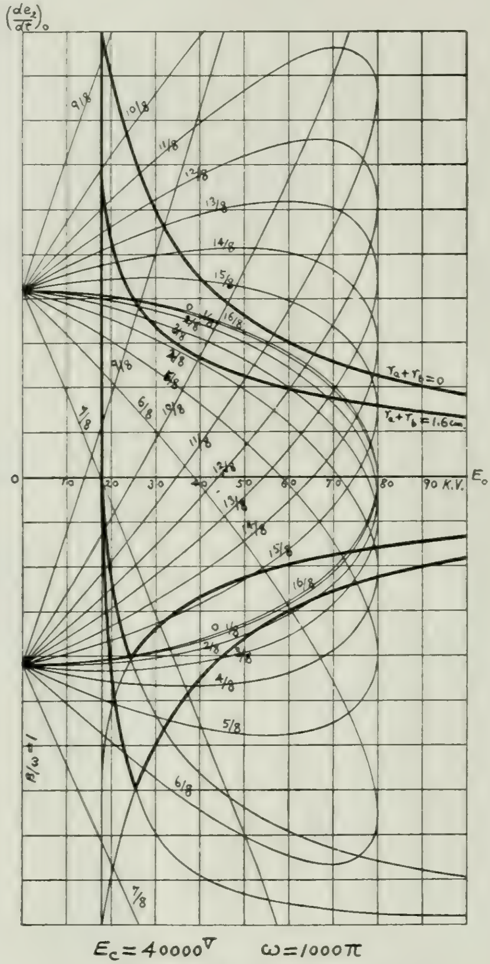

FIGURE 5—Alternate Discharge

The m_2 curves in Figure 4 are plotted for $r_a+r_b=0$ and $=1.6$ cm. (0.6 inch). The large part of these m_2 curves lie beyond the first limit m_1.

The same limits m_1 and m_2 apply also for unidirectional discharges, provided that the stud number is halved and the wheel rotated at the same speed as before.

On the other hand, the curves of $\left(\frac{d e_2}{d t}\right)_o$ corresponding to E_o are computed by equation (11) for various β's on both sides of absolute resonance, assuming $\omega = 1000\pi$ and $E_c = 40{,}000$ volts.

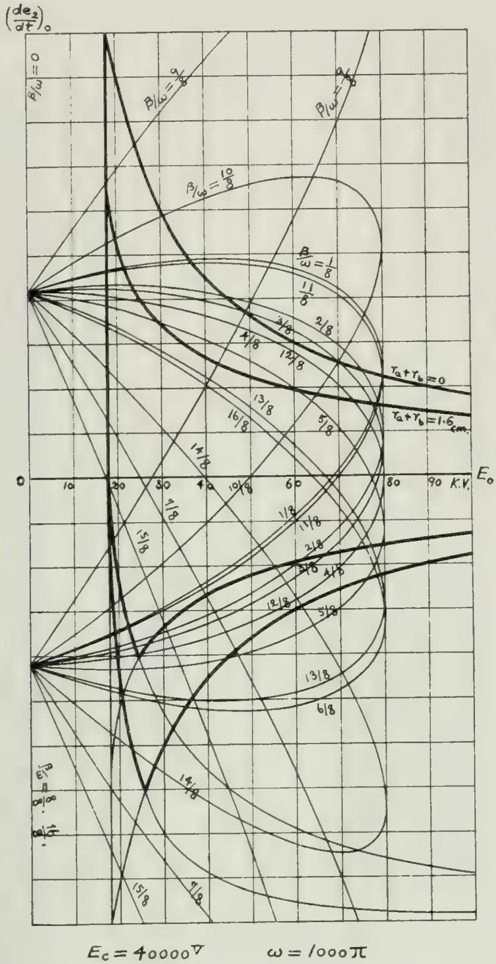

FIGURE 6—Unidirectional Discharge

These are plotted in Figure 5 and Figure 6.

In computing the values of K, $\sin\theta$, and $\cos\theta$, e^{-aT} has always been taken equal to 1, so that $K\cos\theta = \frac{1}{2}$, and the first term of (11) is independent of β (which is not rigorously true).

Now if $\left(\frac{d\,e_2}{d\,t}\right)_o$ thus determined is without the possible limits m_1 and m_2, the presupposed sorts of regular discharge will be impossible.

For two kinds of gaps, namely $r_a+r_b=0$ (needle gap) and $r_a+r_b=1.6$cm. (0.6 inch) (spherical gap), the range of E_o has been determined from Figure 5 and Figure 6, which makes the tone phenomena possible.

In Figures 7, 8, 9, and 10, two classes of possible areas are distinguished by their hatchings, one corresponding to discharges at $0<\phi<\frac{\pi}{2}$ and the other to $\frac{\pi}{2}<\phi<\pi$. That is, they differ according as the discharge takes place on one or the other side of the maximum of the fundamental sine wave, $E_c \sin(\omega t+\phi)$.

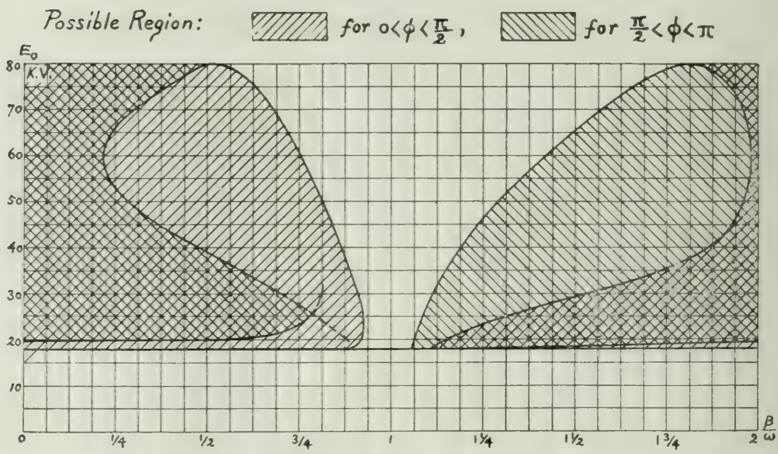

FIGURE 7—Alternate Discharge—Rotary Ideal (Needle) Gap

As seen from these figures, the possibility range is more restricted in actual rotary gaps with spherical electrodes. At $\beta<\omega$, the possible area is much larger for $0<\phi<\frac{\pi}{2}$ and at $\beta<\omega$, for $\frac{\pi}{2}<\phi<\pi$.

Stationary Gap

The limiting condition is somewhat different in stationary gap operation; and there is only one condition, namely,

$$\left(\frac{d\,e_2}{d\,t}\right)_o \overline{\overline{>}}\, 0$$

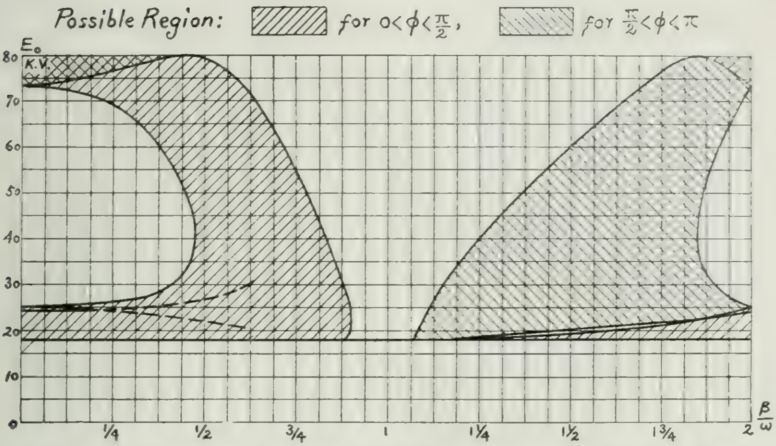

Figure 8—Alternate Discharge—Rotary Spherical Gap ($r_a = r_b = 0.8$ cm.)

The possible ranges of E_o corresponding to $\frac{\beta}{\omega}$ are shown in Figures 11 and 12. These curves, representing the limit

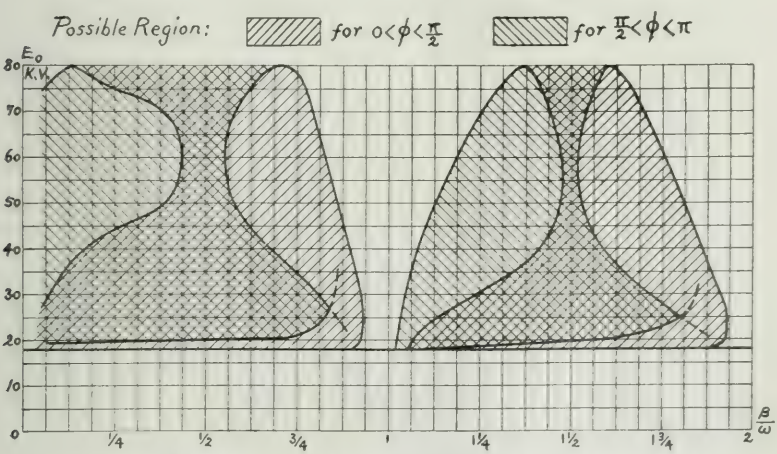

Figure 9—Unidirectional Discharge—Rotary Ideal (Needle) Gap

$\left(\frac{d e_2}{d t}\right)_o = 0$, have been determined from Figures 5 and 6, and coincide exactly with the curves obtained by the author in his previous paper.

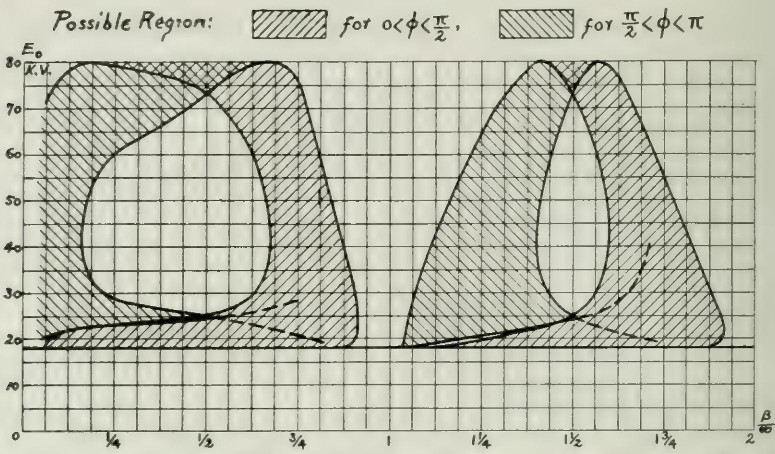

FIGURE 10—Unidirectional Discharge—Rotary Spherical Gap ($r_a = r_b = 0.8$ cm.)

Another limiting factor was considered in connection therewith, which is reproduced here, for comparison, by the dotted curves. The possibility areas given on that occasion lie entirely within the possible range determined in the present paper.

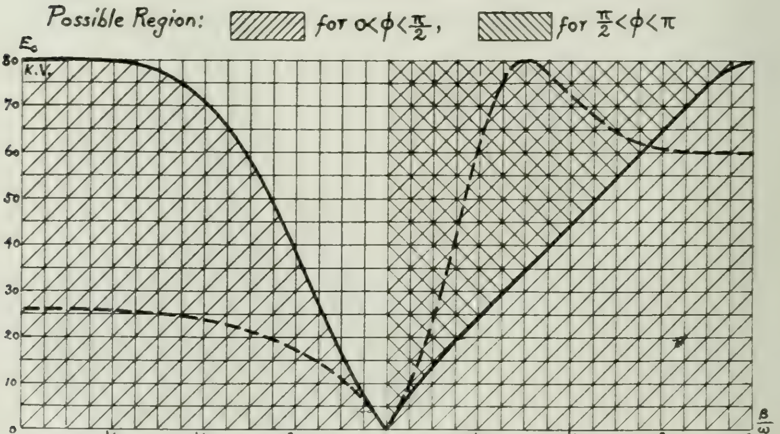

FIGURE 11—Alternate Discharge—Stationary Gap

Part III. Possibility Range of Tone Phenomena in the High Voltage D. C. Method

Altho Lieutenant Bouthillon's solution is made without neglecting the damping, the author nevertheless regards it as convenient and important to treat the d. c. operation by a potential method analogous to that of the above calculation, even tho some approximation are introduced for the sake of simplicity.

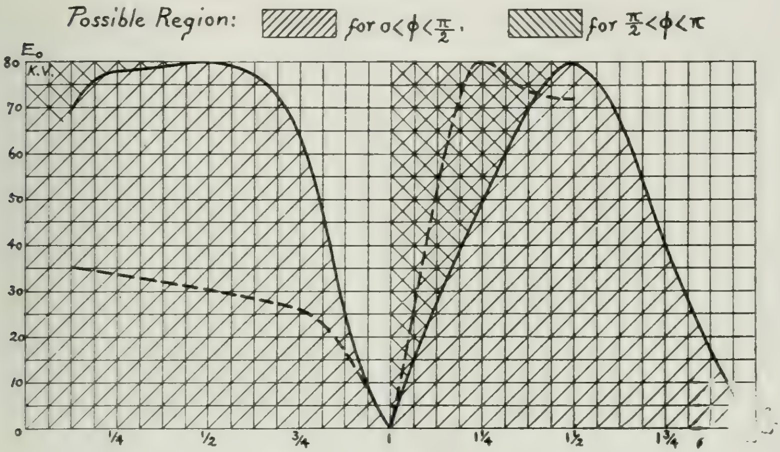

Figure 12—Unidirectional Discharge—Stationary Gap

The transient term due to one spark discharge is given by[6]

$$\begin{cases} i_2 = -\beta C E_o \varepsilon^{-a t} \sin \beta t \\ e_2 = E_o \varepsilon^{-a t} \cos \left(\beta t - \tan^{-1} \dfrac{a}{\beta} \right) \end{cases}$$

and the sum of the transient terms becomes

$$\sum_{n=0}^{\infty} E_o \varepsilon^{-a(t+nT)} \left(\cos \beta (t+nT) - \tan^{-1} \frac{a}{\beta} \right)$$

$$= K E_o \varepsilon^{-a t} \cos \left(\beta t + \theta - \tan^{-1} \frac{a}{\beta} \right) \tag{14}$$

where

$$K = \frac{1}{\sqrt{1 - 2 (\varepsilon^{-aT}) \cos \beta T + (\varepsilon^{-aT})^2}} \tag{15}$$

$$= P_o (\cos \beta T) + \varepsilon^{-aT} P_1 (\cos \beta T) + \varepsilon^{-2aT} P_2 (\cos \beta T) + \varepsilon^{-3aT} P_3 (\cos \beta T) + \ \ldots \tag{16}$$

[6] H. Yagi, previous citation.

and
$$\tan\theta = \frac{\varepsilon^{-aT}\sin\beta T}{1-\varepsilon^{-aT}\cos\beta t} \tag{17}$$

Hence, if E_c is the E. M. F. of the source, the terminal voltage e of the condenser becomes

$$e = E_c - KE_o\,\varepsilon^{-at}\cos\left(\beta t+\theta-\tan^{-1}\frac{\alpha}{\beta}\right) \tag{18}$$

Curves of e are drawn in Figure 13 for various β's assuming $\alpha=0$.

From the terminal conditions

$$(e)_{t=0}=0$$

and
$$(e)_{t=T}=E_o,$$

we obtain

$$\frac{E_c}{E_o}=K\cos\theta \tag{19}$$

When βT and E_c are definitely given, the assumed form of regular discharge may occur only at a definite E_o as given by the above equation. That is, in the case of rotary gaps, which require T to be constant, only one E_o is possible for a given β. It must be noted at the same time that a single value of $\left(\frac{de}{dt}\right)_o$ or $\left(\frac{de}{dt}\right)_T$ is determined corresponding to this E_o and β; for

$$\left(\frac{de}{dt}\right)=\sqrt{a^2+\beta^2}\,KE_o\,\varepsilon^{-at}\sin(\beta t+\theta)$$

and
$$\left(\frac{de}{dt}\right)_o=\beta KE_o\sin\theta \tag{20}$$

Whether or not the tone phenomenon is possible at this E_o and $\left(\frac{de}{dt}\right)_o$ has to be decided with reference to the characteristic of the spark gap.

Rotary Gap

(I) After leaving zero potential, the e curve must not cut across the characteristic curve of the gap before the next discharge, so the same m_1 as in the a. c. method gives the upper limit of $\left(\frac{de}{dt}\right)_o$.

(II) Exactly as in the a. c. method, the negative limit of $\left(\frac{de}{dt}\right)_T$ is given by m_2 which is the inclination of the tangent drawn to the hyperbola.

(III) As shown in Figure 13, the e curve approaches a straight line when βT becomes very small. Therefore, $\left(\frac{de}{dt}\right)_o$ may never become larger than $m_3 = \frac{E_o}{T}$.

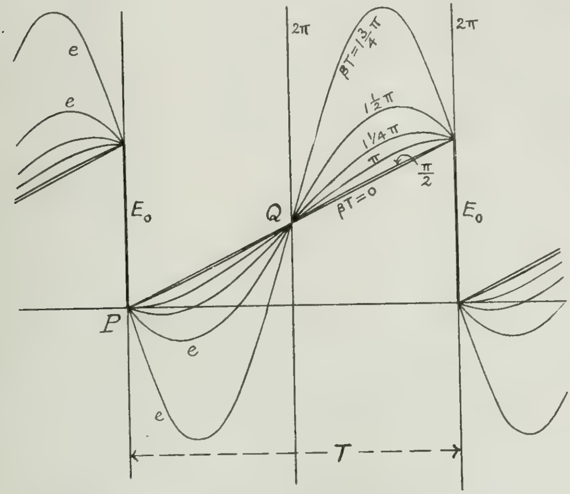

FIGURE 13—Curves of Condenser Potential e with Regular Discharge—High Tension D. C. Method

(IV) In a. c. operation, the curvature of the e_2 curve after a discharge is comparatively small and the positive and negative limits of $\left(\frac{de_2}{dt}\right)_o$ were obtained by assuming e_2 to vary nearly along a straight line.

The same relation does not hold for the e curve in d. c. operation when $\left(\frac{de}{dt}\right)_o$ is negative, because the e curve will soon reach a minimum and reverse its direction, and consequently the same m_1 cannot be considered to indicate the negative limit of $\left(\frac{de}{dt}\right)_o$.

It was determined by trial that the curve of e will touch the gap characteristic on the negative side when $\left(\frac{de}{dt}\right)_o$ exceeds the values m_4 shown in Figure 14.

As this occurs at $\beta T > \frac{3}{2}\pi$, it does not give rise to any very serious restriction in practice.

(V) When E_o is chosen larger than a certain value E_o', the point Q in Figure 13 will lie beyond the gap characteristic and the tone phenomena are impossible beyond E_o', irrespective of the value of βT. This limit is much higher for a needle gap than for a spherical gap.

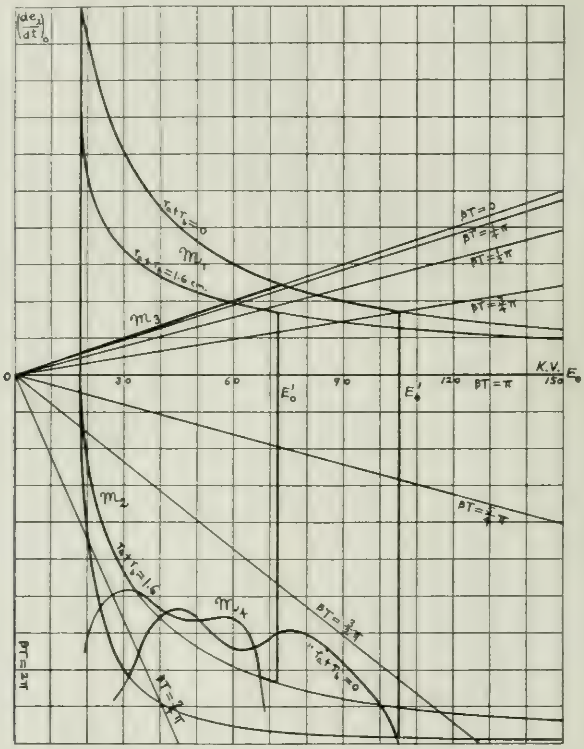

Figure 14—High Tension D. C. Method

All these limits are shown in Figure 14. The straight lines radiating from a point in this diagram show the relation between E_o and $\left(\frac{de}{dt}\right)_o$ as given by equation (20) for different values of βT.

From Figure 14, the permissible E_o for tone phenomena

may be determined at various values of βT both for the case of a needle gap and that of a spherical gap.

The results are plotted in Figure 15, and Figure 16.

When E_c is fixed, E_o must also have a definite value

$$E_o = \frac{E_c}{K \cos\theta} \left(=2E_c\right),$$

and whether or not this E_o is well suited to tone production must be checked by these diagrams. If the answer be negative, E_c or the gap ought to be adjusted.

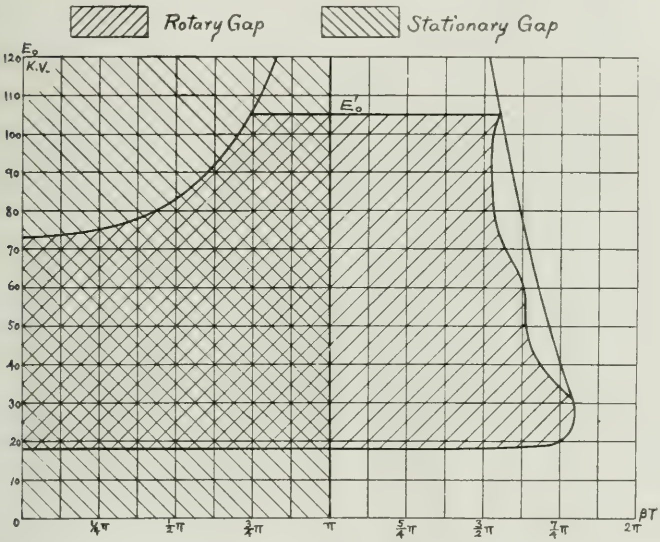

FIGURE 15—D. C. Method—Ideal (Needle) Gap

STATIONARY GAP

The special condition for stationary gaps is

$$\left(\frac{de}{dt}\right)_o \geqq 0$$

or, by equation (20) $\quad \sin\theta \geqq 0$

i. e., the tone phenomena are possible within the region

$$0 < \beta T < \pi$$
$$2\pi < \beta T < 3\pi$$
$$\cdots\cdots$$

and no "late sparking" is possible for stationary gaps.

Any value whatever may be assigned to E_o, provided that E_c is so adjusted that $E_c = E_o K \cos\theta$, or in other words, when E_c is held constant, the tone phenomenon is possible with $E_o = \dfrac{E_c}{K \cos\theta}$ only.

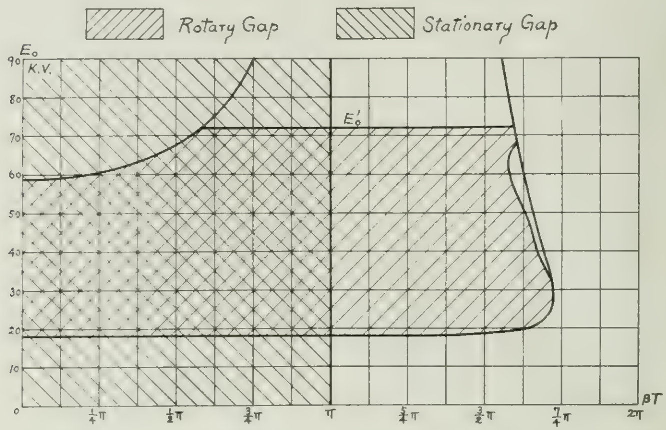

FIGURE 16—D. C. Method—Spherical Gap ($r_a = r_b = 0.8$ cm.) (0.3 inch)

TRANSIENT STATE

No attempt is made in this paper to determine the phenomena in the transient state preceding the establishment of the true tone phenomenon. Nor is it easy to do so, especially for rotary gap operation. A short survey, at least, will not be out of place when appended to this paper.

The only condition which determines the phenomena during this transient state is that "a spark discharge will take place without fail as soon as the terminal potential difference reaches the value determined by the characteristic of the spark gap, no matter what the current at the moment."

With stationary gaps, this disruptive voltage is constant and the time interval T between consecutive discharges is variable. With rotary gaps, T is supposed to be nearly constant, nevertheless, if the solution were developed merely from the current relations without considering the above-stated voltage condition, there would be the possibility of an entirely false assumption, since the discharge might have missed when e was too small.

Again, the conclusions that regular discharge will follow

the first spark if the initial current is equal to that corresponding to the tone phenomenon, and that the tone phenomenon will be established automatically regardless of the current at the beginning of the first charge need to be justified from the above mentioned voltage condition, because a rotary gap is not an appliance which guarantees that the spark discharge shall occur at regular intervals regardless of the potential at the moment.

The phase relation between the charging oscillation and the rotation of the rotary gap has an important bearing upon the matter, and unless the stud comes to its proper position at the very moment when the potential difference has reached the assigned value, the events will not continue as desired.

Figure 17 shows the plainest example of this sort. If, in this case, $E_c < V$, where V = minimum disruptive voltage of the gap, then the tone phenomenon has failed to start, and the main switch or the key must be closed anew at a proper instant.

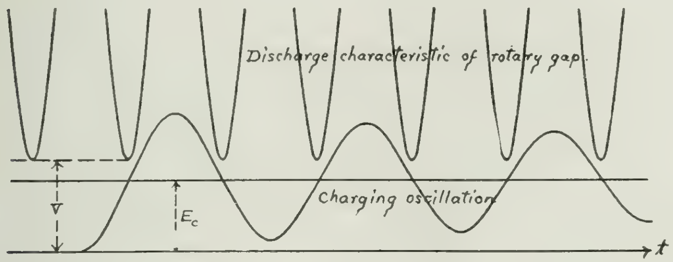

FIGURE 17

Thus, in order to work with E_o nearly equal to $2E_c$, V must be smaller than E_c, otherwise some special device is necessary to start the discharge. On the other hand, it is not desirable to make E_o too much greater than V.

The author has already stated in his previous paper, on the a.c. method, that unless some special means is provided to start the discharge in stationary gap operation, E_o cannot be made larger than E_c, while it is quite desirable to work with E_o ranging between E_c and $2E_c$.

SUMMARY: The possibilities of securing tone phenomena with the a. c. resonance transformer, spark gap method, and with the high tension d. c. spark gap method, are considered.

There are treated a. c. rotary spark gaps, resonance transformer effects on tone production, and the effects produced respectively by needle and spherical gaps. The results of using rotary, needle, and spherical gaps with high tension d. c. are also considered.

Finally a brief treatment is given of the transient conditions existing before the establishment of a stable tone regime.

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All references to any individual, company, or radio station are fully listed in the index. Answers to points brought out in the Discussions will be listed under the name of questioner. With the exception of the names of companies, all topics will be listed in general under the ***noun*** referred to. The numbers correspond to pages in the text. The following abbreviations are used: r. f.—radio frequency; a. f.—audio frequency; l. v.—low voltage; h. v.—high voltage.